Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 1

Question:

A particle is moving in a straight line with constant acceleration 3 m s^{-2}. At time $t = 0$, the speed of the particle is 2 m s^{-1}. Find the speed of the particle at time $t = 6$ s.

Solution:

$$
\begin{aligned}
a &= 3\ ,\ u = 2\ ,\ t = 6\ ,\ v = \ ?\\
v &= u + at\\
&= 2 + 3 \times 6 = 2 + 18 = 20
\end{aligned}
$$

The speed of the particle at time $t = 6$ s is 20 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 2

Question:

A particle is moving in a straight line with constant acceleration. The particle passes a point with speed 1.2 m s $^{-1}$. Four seconds later the particle has speed 7.6 m s $^{-1}$. Find the acceleration of the particle.

Solution:

$$u = 1.2 \ , \ t = 4 \ , \ v = 7.6 \ , \ a = \ ?$$
$$v = u + at$$
$$7.6 = 1.2 + a \times 4$$
$$a = \frac{7.6 - 1.2}{4} = 1.6$$

The acceleration of the particle is 1.6 m s $^{-2}$.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 3

Question:

A car is approaching traffic lights. The car is travelling with speed 10 m s $^{-1}$. The driver applies the brakes to the car and the car comes to rest with constant deceleration in 16 s. Modelling the car as a particle, find the deceleration of the car.

Solution:

$$u = 10 \ , \ v = 0 \ , \ t = 16 \ , \ a = \ ?$$
$$v = u + at$$
$$0 = 10 + a \times 16$$
$$a = -\frac{10}{16} = -0.625$$

The deceleration of the car is 0.625 m s $^{-2}$.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 4

Question:

A particle moves in a straight line from a point A to point B with constant acceleration. The particle passes A with speed 2.4 m s^{-1}. The particle passes B with speed 8 m s^{-1}, five seconds after it passed A. Find the distance between A and B.

Solution:

$u = 2.4$, $v = 8$, $t = 5$, $s = ?$

$$s = \left(\frac{u+v}{2}\right) t$$

$$= \frac{2.4+8}{2} \times 5 = 5.2 \times 5 = 26$$

The distance between A and B is 26 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 5

Question:

A car accelerates uniformly while travelling on a straight road. The car passes two signposts 360 m apart. The car takes 15 s to travel from one signpost to the other. When passing the second signpost, it has speed 28 m s^{-1}. Find the speed of the car at the first signpost.

Solution:

$s = 360$, $t = 15$, $v = 28$, $u =$?

$$s = \left(\frac{u + v}{2} \right) t$$

$$360 = \frac{u + 28}{2} \times 15$$

$$u + 28 = \frac{360 \times 2}{15} = 48$$

$$u = 48 - 28 = 20$$

The speed of the car at the first sign post is 20 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 6

Question:

A particle is moving along a straight line with constant deceleration. The points X and Y are on the line and $XY = 120$ m. At time $t = 0$, the particle passes X and is moving towards Y with speed 18 m s $^{-1}$. At time $t = 10$ s, the particle is at Y. Find the velocity of the particle at time $t = 10$ s.

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Kinematics of a particle moving in a straight line
Exercise A, Question 7

Question:

A cyclist is moving along a straight road from *A* to *B* with constant acceleration 0.5 m s^{-2}. Her speed at *A* is 3 m s^{-1} and it takes her 12 seconds to cycle from *A* to *B*. Find **a** her speed at *B*, **b** the distance from *A* to *B*.

Solution:

a

$a = 0.5$, $u = 3$, $t = 12$, $v = ?$

$$v = u + at$$
$$= 3 + 0.5 \times 12 = 3 + 6 = 9$$

The speed of the cyclist at *B* is 9 m s^{-1}.

b

$u = 3$, $v = 9$, $t = 12$, $s = ?$

$$s = \left(\frac{u + v}{2} \right) t$$
$$= \left(\frac{3 + 9}{2} \right) \times 12 = 6 \times 12 = 72$$

The distance from *A* to *B* is 72 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 8

Question:

A particle is moving along a straight line with constant acceleration from a point A to a point B, where $AB = 24$ m. The particle takes 6 s to move from A to B and the speed of the particle at B is 5 m s^{-1}. Find **a** the speed of the particle at A, **b** the acceleration of the particle.

Solution:

a

$s = 24$, $t = 6$, $v = 5$, $u =$?

$$s = \left(\frac{u + v}{2} \right) t$$

$$24 = \left(\frac{u + 5}{2} \right) \times 6$$

$$u + 5 = \frac{24 \times 2}{6} = 8$$

$$u = 8 - 5 = 3$$

The speed of the particle at A is 3 m s^{-1}.

b

$u = 3$, $v = 5$, $t = 6$, $a =$?

$$v = u + at$$

$$5 = 3 + 6a$$

$$a = \frac{5 - 3}{6} = \frac{1}{3}$$

The acceleration of the particle is $\frac{1}{3}$ m s^{-2}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 9

Question:

A particle moves in a straight line from a point A to a point B with constant deceleration 1.2 m s^{-2}. The particle takes 6 s to move from A to B. The speed of the particle at B is 2 m s^{-1} and the direction of motion of the particle has not changed. Find **a** the speed of the particle at A, **b** the distance from A to B.

Solution:

a

$$a = -1.2,\ t = 6,\ v = 2,\ u = ?$$
$$v = u + at$$
$$2 = u - 1.2 \times 6 = u - 7.2$$
$$u = 2 + 7.2 = 9.2$$

The speed of the particle at A is 9.2 m s^{-1}.

b

$$u = 9.2,\ v = 2,\ t = 6,\ s = ?$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$= \left(\frac{9.2+2}{2}\right) \times 6 = 5.6 \times 6 = 33.6$$

The distance from A to B is 33.6 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 10

Question:

A train, travelling on a straight track, is slowing down with constant deceleration 0.6 m s^{-2}. The train passes one signal with speed 72 km h^{-1} and a second signal 25 s later. Find **a** the speed, in km h^{-1}, of the train as it passes the second signal, **b** the distance between the signals.

Solution:

a

$72 \text{ km h}^{-1} = 72 \times 1000 \text{ m h}^{-1} = \frac{72 \times 1000}{3600} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$

$$u = 20,\ a = -0.6,\ t = 25,\ v = ?$$
$$v = u + at$$
$$= 20 - 0.6 \times 25 = 20 - 15 = 5\ (\text{m s}^{-1})$$

$5 \text{ m s}^{-1} = 5 \times 3600 \text{ m h}^{-1} = \frac{5 \times 3600}{1000} \text{ km h}^{-1} = 18 \text{ km h}^{-1}$

The speed of the train as it passes the second signal is 18 km h^{-1}.

b

$$u = 20,\ v = 5,\ t = 25,\ s = ?$$
$$s = \left(\frac{u+v}{2}\right) t$$
$$= \left(\frac{20+5}{2}\right) \times 25 = 12.5 \times 25 = 312.5$$

The distance between the signals is 312.5 m.

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Kinematics of a particle moving in a straight line
Exercise A, Question 11

Question:

A particle moves in a straight line from a point *A* to a point *B* with a constant deceleration of 4 m s $^{-2}$. At *A* the particle has speed 32 m s $^{-1}$ and the particle comes to rest at *B*. Find **a** the time taken for the particle to travel from *A* to *B*, **b** the distance between *A* and *B*.

Solution:

a

$$\begin{aligned} a &= -4,\ u = 32,\ v = 0,\ t = ? \\ v &= u + at \\ 0 &= 32 - 4t \\ t &= \frac{32}{4} = 8 \end{aligned}$$

The time taken for the particle to move from *A* to *B* is 8 *s*.

b

$$\begin{aligned} u &= 32,\ v = 0,\ t = 8,\ s = ? \\ s &= \left(\frac{u+v}{2}\right)t \\ &= \left(\frac{32+0}{2}\right) \times 8 = 16 \times 8 = 128 \end{aligned}$$

The distance between *A* and *B* is 128 m.

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Kinematics of a particle moving in a straight line
Exercise A, Question 12

Question:

A skier travelling in a straight line up a hill experiences a constant deceleration. At the bottom of the hill, the skier has a speed of 16 m s^{-1} and, after moving up the hill for 40 s, he comes to rest. Find **a** the deceleration of the skier, **b** the distance from the bottom of the hill to the point where the skier comes to rest.

Solution:

a

$$u = 16 ,\ t = 40 ,\ v = 0 ,\ a = ?$$
$$v = u + at$$
$$0 = 16 + 40a$$
$$a = -\frac{16}{40} = -0.4$$

The deceleration of the skier is 0.4 m s^{-2}.

b

$$u = 16 ,\ t = 40 ,\ v = 0 ,\ s = ?$$
$$s = \left(\frac{u+v}{2} \right) t$$
$$= \left(\frac{16+0}{2} \right) \times 40 = 8 \times 40 = 320$$

The distance from the bottom of the hill to the point where the skier comes to rest is 320 m.

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Kinematics of a particle moving in a straight line
Exercise A, Question 13

Question:

A particle is moving in a straight line with constant acceleration. The points *A*, *B* and *C* lie on this line. The particle moves from *A* through *B* to *C*. The speed of the particle at *A* is 2 m s^{-1} and the speed of the particle at *B* is 7 m s^{-1}. The particle takes 20 s to move from *A* to *B*.

a Find the acceleration of the particle.

The speed of the particle is *C* is 11 m s^{-1}. Find

b the time taken for the particle to move from *B* to *C*,

c the distance between *A* and *C*.

Solution:

a

$$\begin{aligned} u &= 2 , v = 7 , t = 20 , a = ? \\ v &= u + at \\ 7 &= 2 + 20a \\ a &= \frac{7-2}{20} = 0.25 \end{aligned}$$

The acceleration of the particle is 0.25 m s^{-2}.

b
From *B* to *C*

$$\begin{aligned} u &= 7 , v = 11 , a = 0.25 , t = ? \\ v &= u + at \\ 11 &= 7 + 0.25t \\ t &= \frac{11-7}{0.25} = 16 \end{aligned}$$

The time taken for the particle to move from *B* to *C* is 16 s.

c The time taken to move from *A* to *C* is $(20 + 16)$ s = 36 s

From *A* to *C*

$$\begin{aligned} u &= 2 , v = 11 , t = 36 , s = ? \\ s &= \left(\frac{u+v}{2} \right) t \\ &= \left(\frac{2+11}{2} \right) \times 36 = 6.5 \times 36 = 234 \end{aligned}$$

The distance between *A* and *C* is 234 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 14

Question:

A particle moves in a straight line from *A* to *B* with constant acceleration 1.5 m s^{-2}. It then moves, along the same straight line, from *B* to *C* with a different acceleration. The speed of the particle at *A* is 1 m s^{-1} and the speed of the particle at *C* is 43 m s^{-1}. The particle takes 12 s to move from *A* to *B* and 10 s to move from *B* to *C*. Find

a the speed of the particle at *B*,

b the acceleration of the particle as it moves from *B* to *C*,

c the distance from *A* to *C*.

Solution:

a
From *A* to *B*

$$a = 1.5 , u = 1 , t = 12 , v = ?$$
$$v = u + at$$
$$= 1 + 1.5 \times 12 = 1 + 18 = 19$$

The speed of the particle at *B* is 19 m s^{-1}.

b
From *B* to *C*

$$u = 19 , v = 43 , t = 10 , a = ?$$
$$v = u + at$$
$$43 = 19 + 10a$$
$$a = \frac{43 - 19}{10} = 2.4$$

The acceleration from *B* to *C* is 2.4 m s^{-2}.

c
The distance from *A* to *B*

$$u = 1 , v = 19 , t = 12 , s = ?$$
$$s = \left(\frac{u + v}{2} \right) t$$
$$= \left(\frac{1 + 19}{2} \right) \times 12 = 10 \times 12 = 120$$

The distance from *B* to *C*

$$u = 19 , v = 43 , t = 10 , s = ?$$
$$s = \left(\frac{u + v}{2} \right) t$$
$$= \left(\frac{19 + 43}{2} \right) \times 10 = 31 \times 10 = 310$$

The distance from *A* to *C* is $(120 + 310)$ m = 430 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 15

Question:

A cyclist travels with constant acceleration x m s^{-2}, in a straight line, from rest to 5 m s^{-1} in 20 s. She then decelerates from 5 m s^{-1} to rest with constant deceleration $\frac{1}{2}x$ m s^{-2}. Find **a** the value of x, **b** the total distance she travelled.

Solution:

a

$u = 0, v = 5, t = 20, a = x$

$v = u + at$

$5 = 0 + 20x$

$x = \frac{5}{20} = 0.25$

b
While accelerating

$u = 0, v = 5, t = 20, s = ?$

$s = \left(\frac{u+v}{2}\right)t$

$= \left(\frac{0+5}{2}\right) \times 20 = 2.5 \times 20 = 50$

While decelerating

The value of t is found first.

$u = 5, v = 0, a = -\frac{1}{2}x = -0.125, t = ?$

$v = u + at$

$0 = 5 - 0.125t$

$t = \frac{5}{0.125} = 40$

To find the value of s while decelerating.

$u = 5, v = 0, t = 40, s = ?$

$s = \left(\frac{u+v}{2}\right)t$

$= \left(\frac{5+0}{2}\right) \times 40 = 2.5 \times 40 = 100$

The total distance travelled is the distance travelled while accelerating added to the distance travelled while decelerating = $(50 + 100)$ m = 150 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 16

Question:

A particle is moving with constant acceleration in a straight line. It passes through three points, A, B and C with speeds 20 m s^{-1}, 30 m s^{-1} and 45 m s^{-1} respectively. The time taken to move from A to B is t_1 seconds and the time taken to move from B to C is t_2 seconds.

a Show that $\frac{t_1}{t_2} = \frac{2}{3}$.

Given also that the total time taken for the particle to move from A to C is 50 s,

b find the distance between A and B.

Solution:

a
From A to B

$$v = u + at$$
$$30 = 20 + at_1$$
$$at_1 = 10 \qquad (1)$$

From B to C

$$v = u + at$$
$$45 = 30 + at_2$$
$$at_2 = 15 \qquad (2)$$

Dividing equation (1) by equation (2)

$$\frac{\cancel{a}t_1}{\cancel{a}t_2} = \frac{\cancel{10}^2}{\cancel{15}_3}$$
$$\frac{t_1}{t_2} = \frac{2}{3} \text{, as required}$$

b
From the result in part **a**

$$t_2 = \frac{3}{2}t_1$$
$$t_1 + t_2 = t_1 + \frac{3}{2}t_1 = \frac{5}{2}t_1 = 50 \text{, given.}$$
$$t_1 = \frac{2}{5} \times 50 = 20$$

From A to B

$u = 20$, $v = 30$, $t = 20$, $s = ?$

$$s = \left(\frac{u + v}{2} \right) t$$

$$= \left(\frac{20 + 30}{2} \right) \times 20 = 25 \times 20 = 500$$

The distance from *A* to *B* is 500 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 1

Question:

A particle is moving in a straight line with constant acceleration 2.5 m s^{-2}. It passes a point A with speed 3 m s^{-1} and later passes through a point B, where AB = 8 m. Find the speed of the particle as it passes through B.

Solution:

$$
\begin{aligned}
a &= 2.5, u = 3, s = 8, v = \ ? \\
v^2 &= u^2 + 2as \\
&= 3^2 + 2 \times 2.5 \times 8 = 9 + 40 = 49 \\
v &= \sqrt{49} = 7
\end{aligned}
$$

The speed of the particle as it passes through B is 7 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 2

Question:

A car is accelerating at a constant rate along a straight horizontal road. Travelling at 8 m s $^{-1}$, it passes a pillar box and 6 s later it passes a sign. The distance between the pillar box and the sign is 60 m. Find the acceleration of the car.

Solution:

$$u = 8, t = 6, s = 60, a = \ ?$$

$$s = ut + \frac{1}{2}at^2$$

$$60 = 8 \times 6 + \frac{1}{2} \times a \times 6^2 = 48 + 18a$$

$$a = \frac{60 - 48}{18} = \frac{12}{18} = \frac{2}{3}$$

The acceleration of the car is $\frac{2}{3}$ m s $^{-2}$.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 3

Question:

A cyclist travelling at 12 m s $^{-1}$ applies her brakes and comes to rest after travelling 36 m in a straight line. Assuming that the brakes cause the cyclist to decelerate uniformly, find the deceleration.

Solution:

$$
\begin{aligned}
u &= 12, v = 0, s = 36, a = \ ?\\
v^2 &= u^2 + 2as\\
0^2 &= 12^2 + 2 \times a \times 36 = 144 + 72a\\
a &= -\frac{144}{72} = -2
\end{aligned}
$$

The deceleration is 2 m s $^{-2}$.

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise B, Question 4

Question:

A particle moves along a straight line from P to Q with constant acceleration 1.5 m s^{-2}. The particle takes 4 s to pass from
P to Q and PQ = 22 m. Find the speed of the particle at Q.

Solution:

$$a = 1.5, t = 4, s = 22, v = ?$$

$$s = vt - \frac{1}{2}at^2$$

$$22 = 4v - \frac{1}{2} \times 1.5 \times 4^2 = 4v - 12$$

$$v = \frac{22 + 12}{4} = \frac{34}{4} = 8.5$$

The speed of the particle at Q is 8.5 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise B, Question 5

Question:

A particle is moving along a straight line *OA* with constant acceleration 2 m s $^{-2}$. At *O* the particle is moving towards *A* with speed 5.5 m s $^{-1}$. The distance *OA* is 20 m. Find the time the particle takes to move from *O* to *A*.

Solution:

$$a = 2, u = 5.5, s = 20, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$20 = 5.5t + \frac{1}{2} \times 2 \times t^2$$

$$t^2 + 5.5t - 20 = 0$$

$$2t^2 + 11t - 40 = (2t - 5)(t + 8) = 0$$

$$t = 2.5, -8$$

$\boxed{\text{Reject the negative answer.}}$

The time the particle takes to move from *O* to *A* is 2.5 s.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 6

Question:

A train is moving along a straight horizontal track with constant acceleration. The train passes a signal at 54 km h^{-1} and a second signal at 72 km h^{-1}. The distance between the two signals is 500 m. Find, in m s^{-2}, the acceleration of the train.

Solution:

$$54 \text{ km h}^{-1} = \frac{54 \times 1000}{3600} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$$

$$72 \text{ km h}^{-1} = \frac{72 \times 1000}{3600} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$$

$$u = 15, v = 20, s = 500, a = \ ?$$
$$v^2 = u^2 + 2as$$
$$20^2 = 15^2 + 2 \times a \times 500$$
$$400 = 225 + 1000a$$
$$a = \frac{400 - 225}{1000} = 0.175$$

The acceleration of the train is 0.175 m s^{-2}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 7

Question:

A particle moves along a straight line, with constant acceleration, from a point A to a point B where $AB = 48$ m. At A the particle has speed 4 m s^{-1} and at B it has speed 16 m s^{-1}. Find **a** the acceleration of the particle, **b** the time the particle takes to move from A to B.

Solution:

a

$$\begin{aligned} s &= 48, u = 4, v = 16, a = \ ? \\ v^2 &= u^2 + 2as \\ 16^2 &= 4^2 + 2 \times a \times 48 \\ 256 &= 16 + 96a \\ a &= \frac{256 - 16}{96} = 2.5 \end{aligned}$$

The acceleration of the particle is 2.5 m s^{-2}.

b

$$\begin{aligned} u &= 4, v = 16, a = 2.5, t = \ ? \\ v &= u + at \\ 16 &= 4 + 2.5t \\ t &= \frac{16 - 4}{2.5} = 4.8 \end{aligned}$$

The time taken to move from A to B is 4.8 s.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 8

Question:

A particle moves along a straight line with constant acceleration 3 m s^{-2}. The particle moves 38 m in 4 s. Find **a** the initial speed of the particle, **b** the final speed of the particle.

Solution:

a

$$a = 3, s = 38, t = 4, u = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$38 = 4u + \frac{1}{2} \times 3 \times 4^2 = 4u + 24$$

$$u = \frac{38 - 24}{4} = 3.5$$

The initial speed of the particle is 3.5 m s^{-1}.

b

$$v = u + at$$
$$= 3.5 + 3 \times 4 = 15.5$$

The final speed of the particle is 15.5 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 9

Question:

The driver of a car is travelling at 18 m s^{-1} along a straight road when she sees an obstruction ahead. She applies the brakes and the brakes cause the car to slow down to rest with a constant deceleration of 3 m s^{-2}. Find **a** the distance travelled as the car decelerates, **b** the time it takes for the car to decelerate from 18 m s^{-1} to rest.

Solution:

a

$$\begin{aligned} u &= 18, v = 0, a = -3, s = ? \\ v^2 &= u^2 + 2as \\ 0^2 &= 18^2 - 2 \times 3 \times s = 324 - 6s \\ s &= \frac{324}{6} = 54 \end{aligned}$$

The distance travelled as the car decelerates is 54 m.

b

$$\begin{aligned} u &= 18, v = 0, a = -3, t = ? \\ v &= u + at \\ 0 &= 18 - 3t \\ t &= \frac{18}{3} = 6 \end{aligned}$$

The time taken for the car to decelerate is 6 s.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 10

Question:

A stone is sliding across a frozen lake in a straight line. The initial speed of the stone is 12 m s $^{-1}$. The friction between the stone and the ice causes the stone to slow down at a constant rate of 0.8 m s $^{-2}$. Find **a** the distance moved by the stone before coming to rest, **b** the speed of the stone at the instant when it has travelled half of this distance.

Solution:

a

$$\begin{aligned} u &= 12, v = 0, a = -0.8, s = ? \\ v^2 &= u^2 + 2as \\ 0^2 &= 12^2 - 2 \times 0.8 \times s = 144 - 1.6s \\ s &= \frac{144}{1.6} = 90 \end{aligned}$$

The distance moved by the stone is 90 m.

b

$$u = 12, a = -0.8, s = 45, v = ?$$

Half the distance in **a** is 45 m.

$$\begin{aligned} v^2 &= u^2 + 2as \\ &= 12^2 - 2 \times 0.8 \times 45 = 144 - 72 = 72 \\ v &= \sqrt{72} \approx 8.49 \end{aligned}$$

The speed of the stone is 8.49 m s $^{-1}$ (3 s.f.).

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Kinematics of a particle moving in a straight line
Exercise B, Question 11

Question:

A particle is moving along a straight line OA with constant acceleration 2.5 m s^{-2}. At time $t = 0$, the particle passes through O with speed 8 m s^{-1} and is moving in the direction OA. The distance OA is 40 m. Find **a** the time taken for the particle to move from O to A, **b** the speed of the particle at A. Give your answers to one decimal place.

Solution:

a

$$a = 2.5, u = 8, s = 40, t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$40 = 8t + 1.25t^2$$

$1.25t^2 + 8t - 40 = 0$

$$t = \frac{-8 + \sqrt{(8^2 + 4 \times 1.25 \times 40)}}{2.5} = \frac{-8 + \sqrt{264}}{2.5}$$
$$= 3.299 \ldots$$

$\boxed{\text{The negative solution is discounted.}}$

The time taken for the particle to move from O to A 3.3 s (1 d.p.).

b

$$a = 2.5, u = 8, s = 40, v = ?$$
$$v^2 = u^2 + 2as$$
$$= 8^2 + 2 \times 2.5 \times 40 = 264$$
$$v = \sqrt{264} = 16.24 \ldots$$

The speed of the particle at A is 16.2 m s^{-1} (1 d.p.).

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise B, Question 12

Question:

A particle travels with uniform deceleration 2 m s^{-2} in a horizontal line. The points A and B lie on the line and AB = 32 m. At time t = 0, the particle passes through A with velocity 12 m s^{-1} in the direction AB. Find **a** the values of t when the particle is at B, **b** the velocity of the particle for each of these values of t.

Solution:

a

$$a = -2, s = 32, u = 12, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$32 = 12t - t^2$$

$$t^2 - 12t + 32 = (t-4)(t-8) = 0$$

$$t = 4, 8$$

b

$$\text{When } t = 4, v = u + at$$
$$= 12 - 2 \times 4 = 4$$

The velocity is 4 m s^{-1} in the direction AB.

$$\text{When } t = 8, v = u + at$$
$$= 12 - 2 \times 8 = -4$$

The velocity is 4 m s^{-1} in the direction BA.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 13

Question:

A particle is moving along the x-axis with constant deceleration 5 m s^{-2}. At time $t = 0$, the particle passes through the origin O with velocity 12 m s^{-1} in the positive direction. At time t seconds the particle passes through the point A with x-coordinate 8. Find **a** the values of t, **b** the velocity of the particle as it passes through the point with x-coordinate -8.

Solution:

a

$$a = -5, u = 12, s = 8, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$8 = 12t - 2.5t^2$$

$$2.5t^2 - 12t + 8 = 0$$

$$5t^2 - 24t + 16 = (5t - 4)(t - 4) = 0$$

$$t = 0.8, 4$$

b

$$a = -5, u = 12, s = -8, v = ?$$

$$v^2 = u^2 + 2as$$

$$= 12^2 + 2 \times (-5) \times (-8) = 144 + 80 = 224$$

$$v = \sqrt{224} = 14.966 \ldots$$

The speed at $x = -8$ is 15.0 m s^{-1} (3 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 14

Question:

A particle P is moving on the x-axis with constant deceleration 4 m s^{-2}. At time $t = 0$, P passes through the origin O with velocity 14 m s^{-1} in the positive direction. The point A lies on the axis and OA = 22.5 m. Find **a** the difference between the times when P passes through A, **b** the total distance travelled by P during the interval between these times.

Solution:

a

$$\begin{aligned} a &= -4, u = 14, s = 22.5, t = ? \\ s &= ut + \frac{1}{2}at^2 \\ 22.5 &= 14t - 2t^2 \\ 2t^2 &- 14t + 22.5 = 0 \\ 4t^2 &- 28t + 45 = (2t-5)(2t-9) = 0 \\ t &= 2.5, 4.5 \end{aligned}$$

The difference between the times is $(4.5 - 2.5)$ s = 2 s.

b

Find the time when P reverses direction.

$$\begin{aligned} a &= -4, u = 14, v = 0, t = ? \\ v &= u + at \\ 0 &= 14 - 4t \Rightarrow t = \frac{14}{4} = 3.5 \end{aligned}$$

Find the displacement when $t = 3.5$.

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ &= 14 \times 3.5 - 2 \times 3.5^2 = 24.5 \end{aligned}$$

Between the times when $t = 2.5$ and $t = 4.5$ the particle moves $2(24.5 - 22.5)$ m = 4 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 15

Question:

A car is travelling along a straight horizontal road with constant acceleration. The car passes over three consecutive points A, B and C where $AB = 100$ m and $BC = 300$ m. The speed of the car at B is 14 m s^{-1} and the speed of the car at C is 20 m s^{-1}. Find **a** the acceleration of the car, **b** the time taken for the car to travel from A to C.

Solution:

a From B to C

$$
\begin{aligned}
u &= 14, v = 20, s = 300, a = \ ? \\
v^2 &= u^2 + 2as \\
20^2 &= 14^2 + 2 \times a \times 300 \\
a &= \frac{20^2 - 14^2}{600} = 0.34
\end{aligned}
$$

The acceleration of the car is 0.34 m s^{-2}.

b From A to C

$$
\begin{aligned}
v &= 20, s = 400, a = 0.34, u = \ ? \\
v^2 &= u^2 + 2as \\
20^2 &= u^2 + 2 \times 0.34 \times 400 = u^2 + 272 \\
u^2 &= 400 - 272 = 128 \\
u &= \sqrt{128} = 8\sqrt{2} \quad \text{Assuming the car is not in reverse at } A \\
v &= u + at \\
20 &= 8\sqrt{2} + 0.34t \\
t &= \frac{20 - 8\sqrt{2}}{0.34} \approx 25.5
\end{aligned}
$$

The time taken for the car to travel from A to C is 25.5 s (3 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 16

Question:

Two particles P and Q are moving along the same straight horizontal line with constant accelerations 2 m s $^{-2}$ and 3.6 m s $^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed 4 m s $^{-1}$. One second later Q passes through A with speed 3 m s $^{-1}$, moving in the same direction as P.

a Write down expressions for the displacements of P and Q from A, in terms of t, where t seconds is the time after P has passed through A.

b Find the value of t where the particles meet.

c Find the distance of A from the point where the particles meet.

Solution:

a For P:

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = 4t + t^2$$

The displacement of P is $(4t + t^2)$ m.

For Q:

Q has been moving for $(t - 1)$ seconds since passing through A.

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = 3(t - 1) + 1.8(t - 1)^2$$

The displacement of Q is $[3(t - 1) + 1.8(t - 1)^2]$ m.

b

$$4t + t^2 = 3(t - 1) + 1.8(t - 1)^2$$
$$4t + t^2 = 3t - 3 + 1.8t^2 - 3.6t + 1.8$$
$$0.8t^2 - 4.6t - 1.2 = 0$$

Divide throughout by 0.2.

$$4t^2 - 23t - 6 = 0$$
$$(t - 6)(4t + 1) = 0$$

$t = 6$, the negative solution is rejected (t is the time after passing through A).

c

Substitute $t = 6$ into the expression for the displacement of P.

$s = 4t + t^2 = 4 \times 6 + 6^2 = 60$

The distance of *A* from the point where the particles meet is 60 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 1

Question:

A ball is projected vertically upwards from a point O with speed 14 m s^{-1}. Find the greatest height above O reached by the ball.

Solution:

Take upwards as the positive direction.

$u = 14$, $v = 0$, $a = -9.8$, $s = ?$

$v^2 = u^2 + 2as$

$0^2 = 14^2 - 2 \times 9.8 \times s$

$s = \frac{14^2}{2 \times 9.8} = 10$

The greatest height above O reached by the ball is 10 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 2

Question:

A well is 50 m deep. A stone is released from rest at the top of the well. Find how long the stone takes to reach the bottom of the well.

Solution:

Take downwards as the positive direction.

$s = 50$, $u = 0$, $a = 9.8$, $t =$?

$s = ut + \frac{1}{2}at^2$

$50 = 0 + 4.9t^2$

$t^2 = \frac{50}{4.9} = 10.204 \ldots \Rightarrow t = 3.194 \ldots \approx 3.2$

The stone takes 3.2 s (2 s.f.) to reach the bottom of the well.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 3

Question:

A book falls from the top shelf of a bookcase. It takes 0.6 s to reach the floor. Find how far it is from the top shelf to the floor.

Solution:

Take downwards as the positive direction.

$u = 0$, $t = 0.6$, $a = 9.8$, $s =$?

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + 4.9 \times 0.6^2 = 1.764 \approx 1.8$$

The top shelf is 1.8 m (2 s.f.) from the floor.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 4

Question:

A particle is projected vertically upwards with speed 20 m s^{-1} from a point on the ground. Find the time of flight of the particle.

Solution:

Take upwards as the positive direction.

$$u = 20\ ,\ a = -9.8\ ,\ s = 0\ ,\ t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 20t - 4.9t^2 = t(20 - 4.9t)\quad ,\ t \neq 0$$

$$t = \frac{20}{4.9} = 4.081\ \dots \approx 4.1$$

The time of flight of the particle is 4.1 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 5

Question:

A ball is thrown vertically downward from the top of a tower with speed 18 m s^{-1}. It reaches the ground in 1.6 s. Find the height of the tower.

Solution:

Take downwards as the positive direction.

$u = 18$, $a = 9.8$, $t = 1.6$, $s =$?

$$s = ut + \frac{1}{2}at^2$$

$$= 18 \times 1.6 + 4.9 \times 1.6^2 = 41.344 \approx 41$$

The height of the tower is 41 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 6

Question:

A pebble is catapulted vertically upwards with speed 24 m s^{-1}. Find **a** the greatest height above the point of projection reached by the pebble, **b** the time taken to reach this height.

Solution:

a

Take upwards as the positive direction.

$$u = 24 \, , \, a = -9.8 \, , \, v = 0 \, , \, s = ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 24^2 - 2 \times 9.8 \times s$$
$$s = \frac{24^2}{2 \times 9.8} = 29.387 \ldots \approx 29$$

The greatest height above the point of projection reached by the pebble is 29 m (2 s.f.).

b

$$u = 24 \, , \, a = -9.8 \, , \, v = 0 \, , \, t = ?$$
$$v = u + at$$
$$0 = 24 - 9.8t$$
$$t = \frac{24}{9.8} = 2.448 \ldots \approx 2.4$$

The time taken to reach the greatest height is 2.4 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 7

Question:

A ball is projected upwards from a point which is 4 m above the ground with speed 18 m s^{-1}. Find **a** the speed of the ball when it is 15 m above its point of projection, **b** the speed with which the ball hits the ground.

Solution:

a

Take upwards as the positive direction.

$$\begin{aligned} u &= 18,\ a = -9.8,\ s = 15,\ v = ? \\ v^2 &= u^2 + 2as \\ &= 18^2 - 2 \times 9.8 \times 15 = 30 \\ v &= \sqrt{30} = \pm 5.477\ldots \approx \pm 5.5 \end{aligned}$$

The speed of the ball when it is 15 m above its point of projection is 5.5 m s^{-1} (2 s.f.).

b

$$\begin{aligned} u &= 18,\ a = -9.8,\ s = -4,\ v = ? \\ v^2 &= u^2 + 2as \\ &= 18^2 + 2 \times (-9.8) \times (-4) \\ &= 324 + 78.4 = 402.4 \\ v &= -\sqrt{402.2} = -20.059\ldots \approx -20 \end{aligned}$$

The speed with which the ball hits the ground is 20 m s^{-1} (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 8

Question:

A particle P is projected vertically downwards from a point 80 m above the ground with speed 4 m s^{-1}. Find **a** the speed with which P hits the ground, **b** the time P takes to reach the ground.

Solution:

a

Take downwards as the positive direction.

$$
\begin{aligned}
s &= 80\ ,\ u = 4\ ,\ a = 9.8\ ,\ v = \ ?\\
v^2 &= u^2 + 2as\\
&= 4^2 + 2 \times 9.8 \times 80 = 1584\\
v &= \sqrt{1584} = 39.799\ \ldots\ \approx 40
\end{aligned}
$$

The speed with which P hits the ground is 40 m s^{-1} (2 s.f.).

b

$$
\begin{aligned}
&u = 4\ ,\ a = 9.8\ ,\ v = \sqrt{1584}\ ,\ t = \ ?\\
&v = u + at\\
&\sqrt{1584} = 4 + 9.8t\\
&t = \frac{\sqrt{1584} - 4}{9.8} = 3.653\ \ldots\ \approx 3.7
\end{aligned}
$$

The time P takes to reach the ground is 3.7 s (2 s.f.).

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise C, Question 9

Question:

A particle P is projected vertically upwards from a point X. Five seconds later P is moving downwards with speed 10 m s^{-1}. Find **a** the speed of projection of P, **b** the greatest height above X attained by P during its motion.

Solution:

a

Take upwards as the positive direction.

$$\begin{aligned} v &= -10,\ a = -9.8,\ t = 5,\ u = ? \\ v &= u + a t \\ -10 &= u - 9.8 \times 5 \\ u &= 9.8 \times 5 - 10 = 39 \end{aligned}$$

The speed of projection of P is 39 m s^{-1}.

b

$$\begin{aligned} u &= 39,\ v = 0,\ a = -9.8,\ s = ? \\ v^2 &= u^2 + 2a s \\ 0^2 &= 39^2 - 2 \times 9.8 \times s \\ s &= \frac{39^2}{2 \times 9.8} = 77.602 \ldots \approx 78 \end{aligned}$$

The greatest height above X attained by P during its motion is 78 m (2 s.f.).

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 10

Question:

A ball is thrown vertically upwards with speed 21 m s $^{-1}$. It hits the ground 4.5 s later. Find the height above the ground from which the ball was thrown.

Solution:

Take upwards as the positive direction.

$u = 21$, $t = 4.5$, $a = -9.8$, $s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$= 21 \times 4.5 - 4.9 \times 4.5^2 = -4.725 \approx -4.7$$

The height above the ground from which the ball was thrown is 4.7 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 11

Question:

A stone is thrown vertically upward from a point which is 3 m above the ground, with speed 16 m s^{-1}. Find **a** the time of flight of the stone, **b** the total distance travelled by the stone.

Solution:

a
Take upwards as the positive direction.

$$s = -3,\ u = 16,\ a = -9.8,\ t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$-3 = 16t - 4.9t^2$$

$4.9t^2 - 16t - 3 = 0$

Using $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$$t = \frac{16 \pm \sqrt{(16^2 + 4 \times 4.9 \times 3)}}{2 \times 4.9}$$ Only the positive solution need be considered.
$$= 3.443\ldots \approx 3.4$$

The time of flight of the stone is 3.4 s (2 s.f.).

b
Find the greatest height, say h, reached by the stone.

$$u = 16,\ v = 0,\ a = -9.8,\ s = h$$
$$v^2 = u^2 + 2as$$
$$0^2 = 16^2 - 2 \times 9.8 \times h$$
$$h = \frac{16^2}{2 \times 4.9} = 13.061\ldots \approx 13$$

The total distance travelled by the stone is $(2 \times 13 + 3)$ m = 29 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 12

Question:

A particle is projected vertically upwards with speed 24.5 m s^{-1}. Find the total time for which it is 21 m or more above its point of projection.

Solution:

Take upwards as the positive direction.

$u = 24.5$, $a = -9.8$, $s = 21$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$21 = 24.5t - 4.9t^2$

$4.9t^2 - 24.5t + 21 = 0$

Using $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$t = \frac{24.5 \pm \sqrt{(24.5^2 - 4 \times 4.9 \times 21)}}{2 \times 9.8}$

$= 1.0984 \ldots , 3.9015 \ldots$

The difference between these times is $(3.9015 \ldots - 1.0984 \ldots)$ s $= 2.803 \ldots$ s

The total time for which the particle is 21 m or more above its point of projection is 2.8 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 13

Question:

A particle is projected vertically upwards from a point *O* with speed u m s^{-1}. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u$ m s^{-1}. Find **a** the value of u, **b** the time from the instant that the particle leaves *O* to the instant that it returns to *O*.

Solution:

a

Take upwards as the positive direction.

$$u = u,\ v = \frac{1}{3}u,\ a = -9.8,\ t = 2$$
$$v = u + at$$
$$\frac{1}{3}u = u - 9.8 \times 2$$
$$\frac{2}{3}u = 19.6 \Rightarrow u = \frac{3}{2} \times 19.6 = 29.4$$
$$u = 29\ \ (2 \text{ s.f.})$$

b

$$u = 29.4,\ s = 0,\ a = -9.8,\ t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$0 = 29.4t - 4.9t^2 = t(29.4 - 4.9t),\ t \neq 0$$
$$t = \frac{29.4}{4.9} = 6$$

The time from the instant that the particle leaves *O* to the instant that it returns to *O* is 6 s.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 14

Question:

A ball A is thrown vertically downwards with speed 5 m s $^{-1}$ from the top of a tower block 46 m above the ground. At the same time as A is thrown downwards, another ball B is thrown vertically upwards from the ground with speed 18 m s $^{-1}$. The balls collide. Find the distance of the point where A and B collide from the point where A was thrown.

Solution:

For A, take downwards as the positive direction.

$s_A = u\ t + \frac{1}{2}at^2 = 5t + 4.9t^2 \ldots\ \ast$

For B, take upwards as the positive direction.

$s_B = u\ t + \frac{1}{2}at^2 = 18t - 4.9t^2$

$s_A + s_B = 46$

$5t + 4.9t^2 + 18t - 4.9t^2 = 46$

$23t = 46 \Rightarrow t = 2$

Substitute into $\ast$

$s_A = 5 \times 2 + 4.9 \times 2^2 = 29.6$

The distance of the point where A and B collide from the point where A was thrown is 30 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 15

Question:

A ball is released from rest at a point which is 10 m above a wooden floor. Each time the ball strikes the floor, it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball **a** the first time it rebounds from the floor, **b** the second time it rebounds from the floor.

Solution:

a

Find the speed, u_1 say, immediately before the ball strikes the floor.

$$u = 0 , a = 9.8 , s = 10 , v = u_1$$
$$v^2 = u^2 + 2as$$
$$u_1{}^2 = 0^2 + 2 \times 9.8 \times 10 = 196$$
$$u_1 = \sqrt{196} = 14$$

The speed of the first rebound, u_2 say, is given by

$$u_2 = \frac{3}{4}u_1 = \frac{3}{4} \times 14 = 10.5$$

Find the maximum height, h_1 say, reached after the first rebound.

$$u = 10.5 , v = 0 , a = -9.8 , s = h_1$$
$$v^2 = u^2 + 2as$$
$$0^2 = 10.5^2 - 2 \times 9.8 \times h_1 \Rightarrow h_1 = \frac{10.5^2}{2 \times 9.8} = 5.625$$

The greatest height above the floor reached by the ball the first time it rebounds from the floor is 5.6 m (2 s.f.).

b

Immediately before the ball strikes the floor for the second time, its speed is again $u_2 = 10.5$.

The speed of the second rebound, u_3 say, is given by

$$u_3 = \frac{3}{4}u_2 = \frac{3}{4} \times 10.5 = 7.875$$

Find the maximum height, h_2 say, reached after the second rebound.

$u = 7.875$, $v = 0$, $a = -9.8$, $s = h_2$

$v^2 = u^2 + 2as$

$0^2 = 7.875^2 - 2 \times 9.8 \times h_2 \Rightarrow h_2 = \frac{7.875^2}{2 \times 9.8} = 3.164 \ldots$

The greatest height above the floor reached by the ball the second time it rebounds from the floor is 3.2 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 16

Question:

A particle P is projected vertically upwards from a point O with speed 12 m s $^{-1}$. One second after P has been projected from O, another particle Q is projected vertically upwards from O with speed 20 m s $^{-1}$. Find **a** the time between the instant that P is projected from O and the instant when P and Q collide, **b** the distance of the point where P and Q collide from O.

Solution:

a

Take upwards as the positive direction.

For P,

$$s = ut + \frac{1}{2}at^2$$

$$s_P = 12t - 4.9t^2 \ \ldots \ \ldots \ *$$

For Q,

$s = ut + \frac{1}{2}at^2$ Q has been moving for one less second than P.

$$s_Q = 20(t-1) - 4.9(t-1)^2$$

At the point of collision $s_P = s_Q$.

$$12t - 4.9t^2 = 20(t-1) - 4.9(t-1)^2$$
$$12t - 4.9t^2 = 20t - 20 - 4.9t^2 + 9.8t - 4.9$$
$$24.9 = 17.8t \Rightarrow t = \frac{24.9}{17.8} = 1.3988 \ldots \approx 1.4$$

The time between the instant that P is projected from O and the instant when P and Q collide is 1.4 s (2 s.f.).

b

Substitute for t in $*$

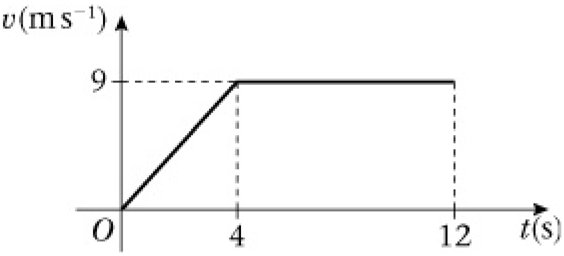
$$s_P = 12t - 4.9t^2 \approx 12 \times 1.4 - 4.9 \times 1.4^2 \approx 7.2$$

The distance of the point where P and Q collide from O is 7.2 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 1

Question:



The diagram shows the speed-time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a speed of 9 m s $^{-1}$. This speed is then maintained for a further 8 s. Find

a the rate at which the athlete accelerates,

b the total distance travelled by the athlete in 12 s.

Solution:

a $a = \frac{9}{4} = 2.25$

The athlete accelerates at a rate of 2.25 m s $^{-2}$.

b

$$s = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2}(8+12) \times 9 = 90$$

The total distance travelled by the athlete is 90 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 2

Question:

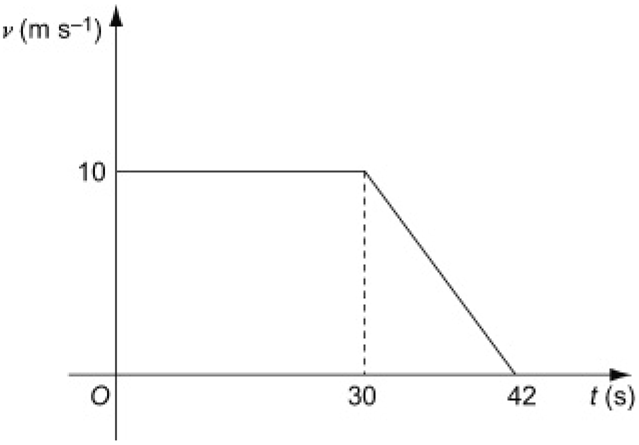
A car is moving along a straight road. When $t = 0$ s, the car passes a point A with speed 10 m s^{-1} and this speed is maintained until $t = 30$ s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point B when $t = 42$ s.

a Sketch a speed-time graph to illustrate the motion of the car.

b Find the distance from A to B.

Solution:

a



b

$$s = \frac{1}{2}(a+b)h$$

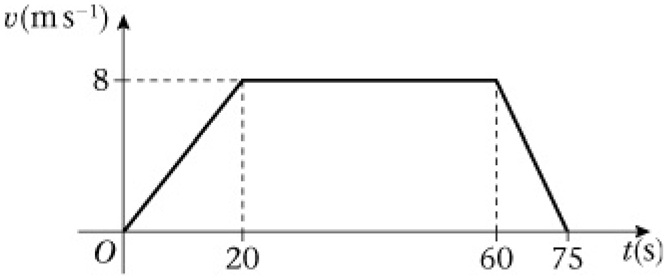
$$= \frac{1}{2}(30+42) \times 10 = 360$$

The distance from A to B is 360 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 3

Question:



The diagram shows the speed-time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to 8 m s^{-1} in 20 s. She then travels at a constant speed of 8 m s^{-1} for 40 s. She then decelerates uniformly to rest in 15 s. Find

a the acceleration of the cyclist in the first 20 s of motion,

b the deceleration of the cyclist in the last 15 s of motion,

c the total distance travelled in 75 s.

Solution:

a $a = \frac{8}{20} = 0.4$

The acceleration of the cyclist is 0.4 m s^{-2}.

b $a = \frac{-8}{15}\ (= -0.53)$

The deceleration of the cyclist is $\frac{8}{15}$ m s^{-2}.

c

$$s = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2}(40+75) \times 8 = 460$$

The total distance travelled is 460 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 4

Question:

A car accelerates at a constant rate, starting from rest at a point *A* and reaching a speed of 45 km h^{-1} in 20 s. This speed is then maintained and the car passes a point *B* 3 minutes after leaving *A*.

a Sketch a speed-time graph to illustrate the motion of the car.

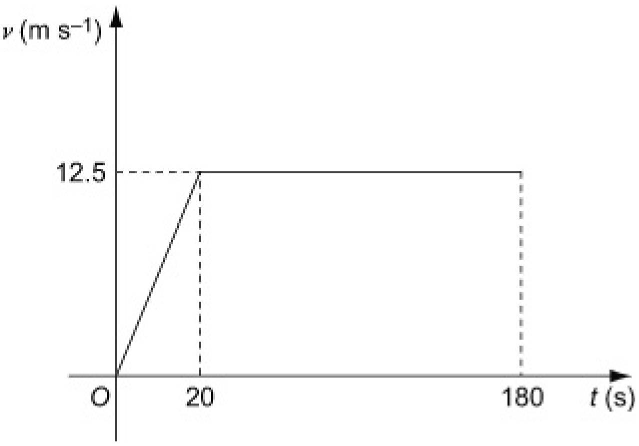
b Find the distance from *A* to *B*.

Solution:

a

$$45 \text{ km h}^{-1} = \frac{45 \times 1000}{3600} \text{ m s}^{-1} = 12.5 \text{ m s}^{-1}$$

3 min = 180 s



b

$$s = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(160+180) \times 12.5 = 2125$$

The distance from *A* to *B* is 2125 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 5

Question:

A motorcyclist starts from rest at a point *S* on a straight race track. He moves with constant acceleration for 15 s, reaching a speed of 30 m s^{-1}. He then travels at a constant speed of 30 m s^{-1} for *T* seconds. Finally he decelerates at a constant rate coming to rest at a point *F*, 25 s after he begins to decelerate.

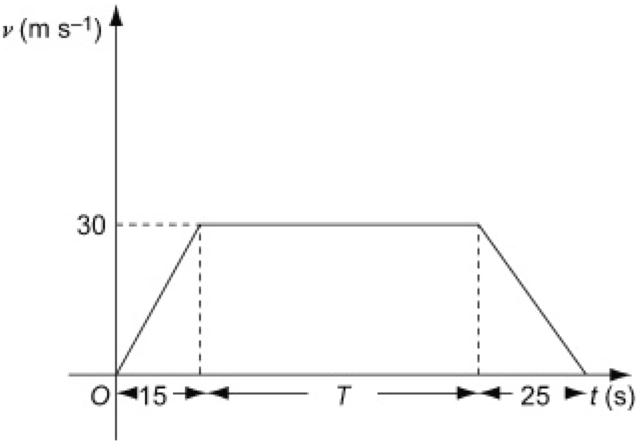
a Sketch a speed-time graph to illustrate the motion.

Given that the distance between *S* and *F* is 2.4 km,

b calculate the time the motorcyclist takes to travel from *S* to *F*.

Solution:

a



b

$$s = \frac{1}{2}(a+b)h$$

$$2400 = \frac{1}{2}(T + (15 + T + 25)) \times 30 = 15(2T + 40)$$

$$2T + 40 = \frac{2400}{15} = 160$$

$$T = \frac{160 - 40}{2} = 60$$

The taken to travel from *S* to *F* is $(15 + T + 25)$ s = 100 s.

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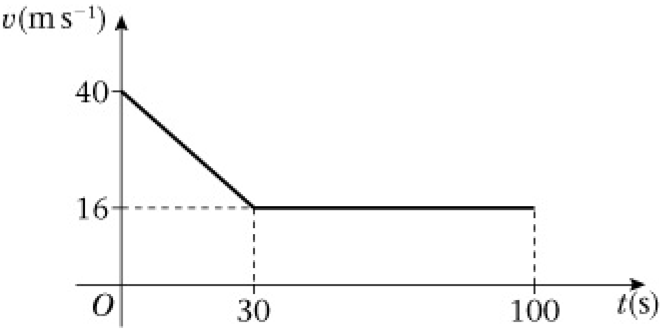
Kinematics of a particle moving in a straight line
Exercise D, Question 6

Question:



A train is travelling along a straight track. To obey a speed restriction, the brakes of the train are applied for 30 s reducing the speed of the train from 40 m s^{-1} to 16 m s^{-1}. The train then continues at a constant speed of 16 m s^{-1} for a further 70s. The diagram shows a speed-time graph illustrating the motion of the train for the total period of 100 s. Find

a the retardation of the train in the first 30 s,

b the total distance travelled by the train in 100 s.

Solution:

a $a = \frac{16 - 40}{30} = - \frac{24}{30} = -0.8$

The retardation (deceleration) is 0.8 m s^{-2}.

b The area under the graph is made up of a triangle, with sides 30 × 24, and a rectangle, with sides 100 × 16.

$s = \frac{1}{2} \times 30 \times 24 + 100 \times 16 = 360 + 1600 = 1960$

The total distance travelled by the train is 1960 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 7

Question:

A train starts from a station X and moves with constant acceleration 0.6 m s $^{-2}$ for 20 s. The speed it has reached after 20 s is then maintained for T seconds. The train then decelerates from this speed to rest in a further 40 s, stopping at a station Y.

a Sketch a speed-time graph to illustrate the motion of the train.

Given that the distance between the stations is 4.2 km, find

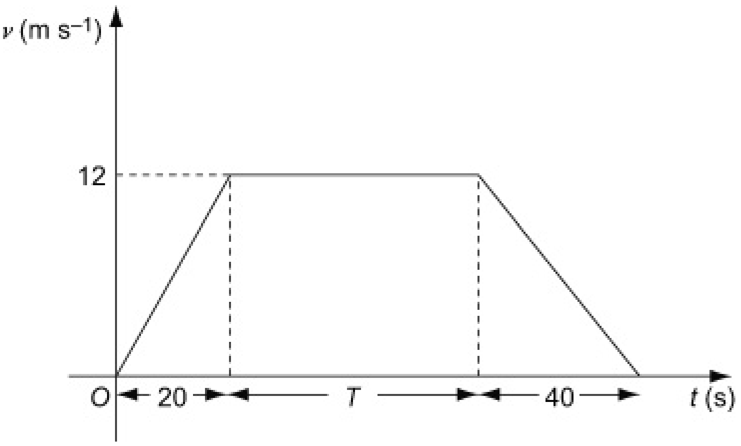
b the value of T,

c the distance travelled by the train while it is moving with constant speed.

Solution:

a The speed after 20 s is given by

$$\begin{aligned} v &= u + at \\ &= 0 + 0.6 \times 20 = 12 \end{aligned}$$



b

$$s = \frac{1}{2}(a+b)h$$

$$4200 = \frac{1}{2}(T + (20 + T + 40)) \times 12$$

$$4200 = 6(2T + 60) = 12T + 360$$

$$T = \frac{4200 - 360}{12} = 320$$

c at constant speed, distance = speed × time

$$= 12 \times 320 = 3840$$

The distance travelled at a constant speed is 3840 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 8

Question:

A particle moves along a straight line. The particle accelerates from rest to a speed of 10 m s^{-1} in 15 s. The particle then moves at a constant speed of 10 m s^{-1} for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant speed is 4 times the period of time for which it is decelerating.

a Sketch a speed-time graph to illustrate the motion of the particle.

Given that the total distance travelled by the particle is 480 m,

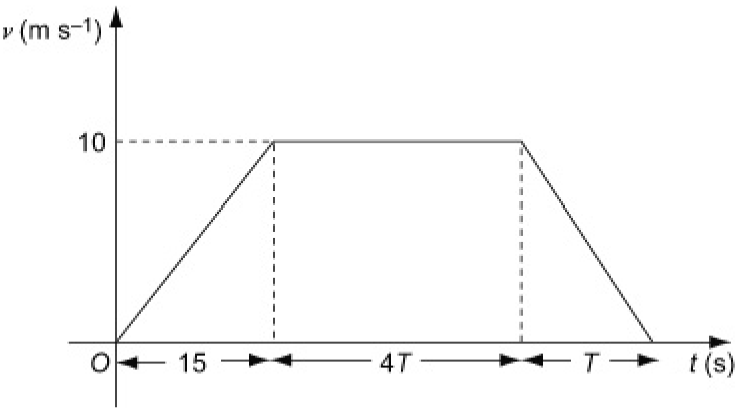
b find the total time for which the particle is moving,

c sketch an acceleration-time graph illustrating the motion of the particle.

Solution:

a Let the time for which the particle decelerates be T seconds.

Then the time for which the particle moves at a constant speed is $4T$ seconds.



b

$$s = \frac{1}{2}(a+b)h$$

$$480 = \frac{1}{2}(4T + (15 + 4T + T)) \times 10 = 5(9T + 15)$$

$$45T + 75 = 480$$

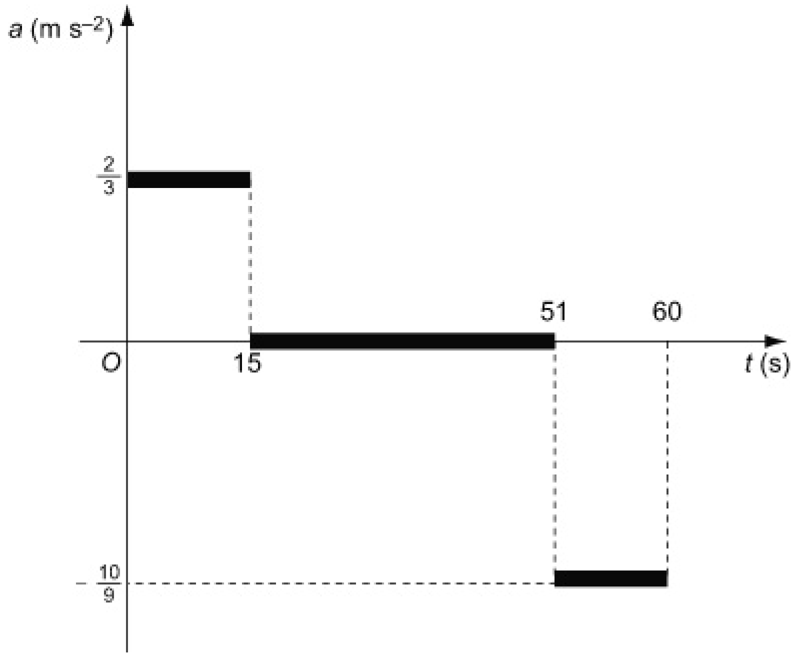
$$T = \frac{480 - 75}{45} = 9$$

The total time for which the particle is moving is

$(5T + 15)$ s $= (5 \times 9 + 15)$ s $= 60$ s

c While accelerating, $a = \frac{10}{15} = \frac{2}{3}$

While decelerating, $a = -\frac{10}{9}$



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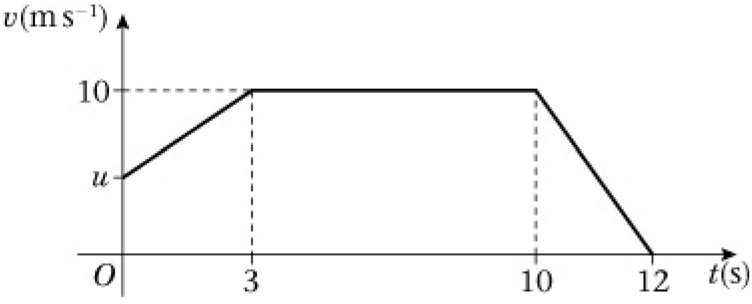
Kinematics of a particle moving in a straight line
Exercise D, Question 9

Question:



A particle moves 100 m in a straight line. The diagram is a sketch of a speed-time graph of the motion of the particle. The particle starts with speed u m s^{-1} and accelerates to a speed 10 m s^{-1} in 3 s. The speed of 10 m s^{-1} is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find

a the value of u,

b the acceleration of the particle in the first 3 s of motion.

Solution:

a Area = trapezium + rectangle + triangle

$$100 = \frac{1}{2}(u + 10) \times 3 + 7 \times 10 + \frac{1}{2} \times 2 \times 10$$

$$100 = \frac{3}{2}(u + 10) + 70 + 10$$

$$\frac{3}{2}(u + 10) = 100 - 70 - 10 = 20$$

$$u + 10 = 20 \times \frac{2}{3} = \frac{40}{3}$$

$$u = \frac{40}{3} - 10 = \frac{10}{3}$$

b $a = \frac{10 - \frac{10}{3}}{3} = \frac{20}{9}$

The acceleration of the particle is $\frac{20}{9}$ m s^{-2}.

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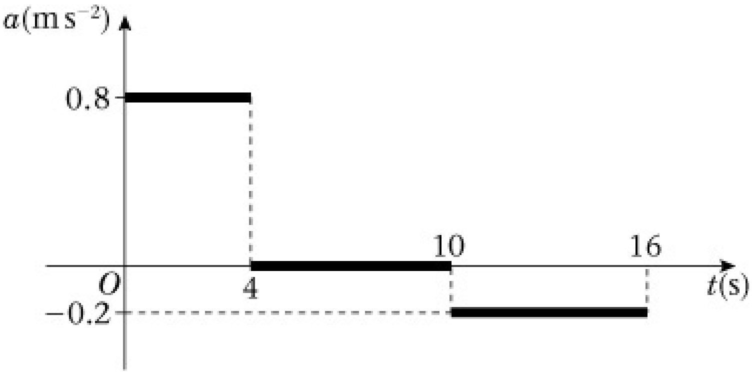
Kinematics of a particle moving in a straight line
Exercise D, Question 10

Question:

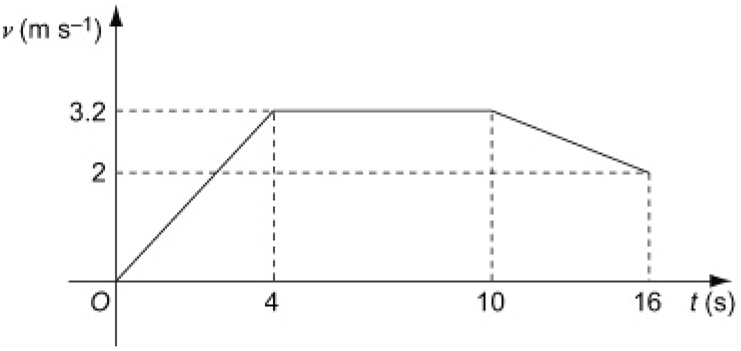


The diagram is an acceleration-time graph to show the motion of a particle. At time $t = 0$ s, the particle is at rest. Sketch a speed-time graph for the motion of the particle.

Solution:

From $t = 0$ to $t = 4$ $\quad v = u + at$
$\quad = 0 + 0.8 \times 4 = 3.2$

From $t = 10$ to $t = 16$ $\quad v = u + at$
$\quad = 3.2 - 0.2 \times 6 = 2$



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Kinematics of a particle moving in a straight line
Exercise D, Question 11

Question:

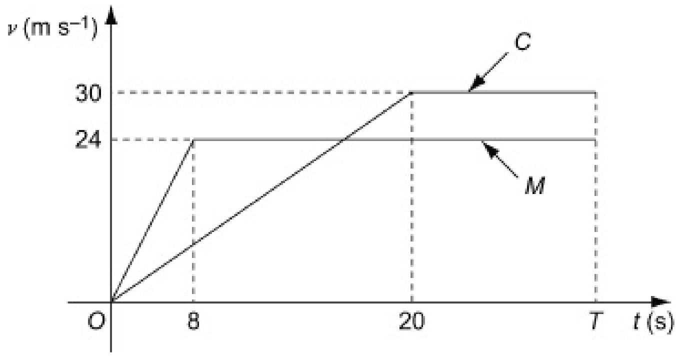
A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates at a rate of 3 m s^{-2} for 8 s and then maintains the speed she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to 30 m s^{-1} in 20 s and then maintains the speed of 30 m s^{-1}. C passes M as they both pass a pedestrian.

a On the same diagram, sketch speed-time graphs to illustrate the motion of M and C.

b Find the distance of the pedestrian from the road junction.

Solution:

a For M, $v = u + at = 0 + 3 \times 8 = 24$



b Let C overtake M at time T seconds.

The distance travelled by M is given by

$$s = \frac{1}{2}(a+b)h = \frac{1}{2}(T - 8 + T) \times 24 = 12(2T - 8)$$

The distance travelled by C is given by

$$s = \frac{1}{2}(a+b)h = \frac{1}{2}(T - 20 + T) \times 30 = 15(2T - 20)$$

At the point of overtaking the distances are equal

$$\begin{aligned} 12(2T-8) &= 15(2T-20) \\ 24T - 96 &= 30T - 300 \\ 6T &= 204 \\ T &= \frac{204}{6} = 34 \end{aligned}$$

$s = 12(2T - 8) = 12(2 \times 34 - 8) = 720$

The distance of the pedestrian from the road junction is 720 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 12

Question:

A particle is moving on an axis Ox. From time $t = 0$ s to time $t = 32$ s, the particle is travelling with constant speed 15 m s^{-1}. The particle then decelerates from 15 m s^{-1} to rest in T seconds.

a Sketch a speed-time graph to illustrate the motion of the particle.

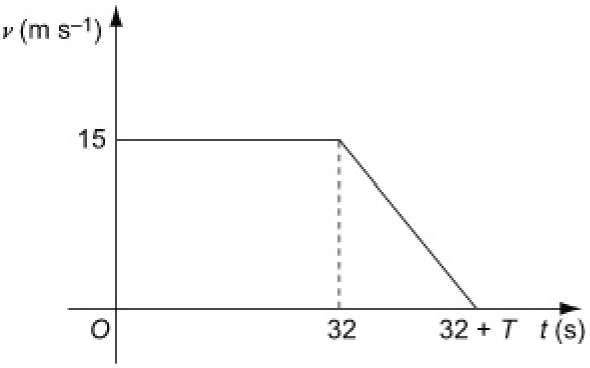
The total distance travelled by the particle is 570 m.

b Find the value of T.

c Sketch a distance-time graph illustrating the motion of the particle.

Solution:

a



b

$$s = \frac{1}{2}(a+b)h$$

$$570 = \frac{1}{2}(32+32+T) \times 15$$

$$\frac{15}{2}(T+64) = 570$$

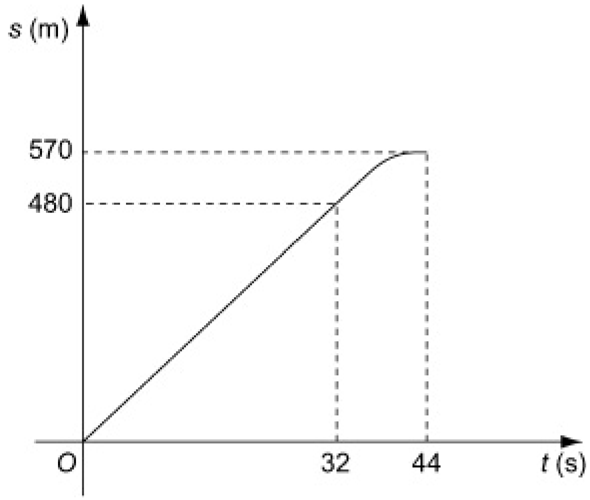
$$T+64 = \frac{570 \times 2}{15} = 76$$

$$T = 76 - 64 = 12$$

c At $t = 32$, $s = 32 \times 15 = 480$

At $t = 44$, $s = 480 +$ area of the triangle

$$= 480 + \frac{1}{2} \times 12 \times 15 = 570$$


s (m)
570
480
O
32
44
t (s)

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 1

Question:

A car travelling along a straight road at 14 m s $^{-1}$ is approaching traffic lights. The driver applies the brakes and the car comes to rest with constant deceleration. The distance from the point where the brakes are applied to the point where the car comes to rest is 49 m. Find the deceleration of the car.

Solution:

$$u = 14, v = 0, s = 49, a = \ ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 14^2 + 2 \times a \times 49$$
$$a = -\frac{14^2}{2 \times 49} = -2$$

The deceleration of the car is 2 m s $^{-2}$.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 2

Question:

A ball is thrown vertically downward from the top of a tower with speed 6 m s $^{-1}$. The ball strikes the ground with speed 25 m s $^{-1}$. Find the time the ball takes to move from the top of the tower to the ground.

Solution:

$\boxed{\text{Take downwards as the positive direction.}}$

$$u = 6, v = 25, a = 9.8, t = ?$$
$$v = u + at$$
$$25 = 6 + 9.8t$$
$$t = \frac{25 - 6}{9.8} = 1.938 \ldots \approx 1.9$$

The ball takes 1.9 s (2 s.f.) to move from the top of the tower to the ground.

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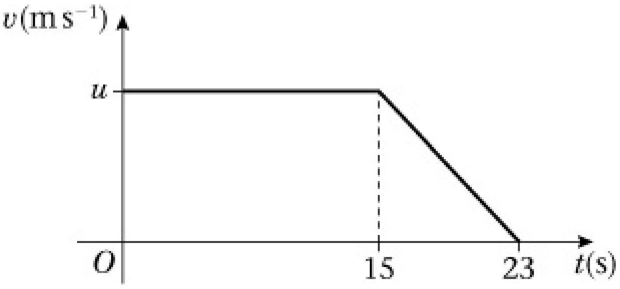
Kinematics of a particle moving in a straight line
Exercise E, Question 3

Question:



The diagram is a speed-time graph representing the motion of a cyclist along a straight road. At time $t = 0$ s, the cyclist is moving with speed u m s^{-1}. The speed is maintained until time $t = 15$ s, when she slows down with constant deceleration, coming to rest when $t = 23$ s. The total distance she travels in 23 s is 152 m. Find the value of u.

Solution:

$$s = \frac{1}{2}(a + b)h$$

$$152 = \frac{1}{2}(15 + 23)u = 19u$$

$$u = \frac{152}{19} = 8$$

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 4

Question:

A stone is projected vertically upwards with speed 21 m s^{-1}. Find

a the greatest height above the point of projection reached by the stone,

b the time between the instant that the stone is projected and the instant that it reaches its greatest height.

Solution:

a

Take upwards as the positive direction.

$$\begin{aligned} u &= 21, v = 0, a = -9.8, s = ? \\ v^2 &= u^2 + 2as \\ 0^2 &= 21^2 - 2 \times 9.8 \times s \\ s &= \frac{21^2}{2 \times 9.8} = 22.5 \end{aligned}$$

The greatest height above the point of projection reached is 23 m (2 s.f.).

b

$$\begin{aligned} u &= 21, v = 0, a = -9.8, t = ? \\ v &= u + at \\ 0 &= 21 - 9.8t \\ t &= \frac{21}{9.8} = 2.14 \ldots \approx 2.1 \end{aligned}$$

The time between the instant that the stone is projected and the instant that it reaches its greatest height is 2.1 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 5

Question:

A train is travelling with constant acceleration along a straight track. At time $t = 0$ s, the train passes a point O travelling with speed 18 m s^{-1}. At time $t = 12$ s, the train passes a point P travelling with speed 24 m s^{-1}. At time $t = 20$ s, the train passes a point Q. Find

a the speed of the train at Q,

b the distance from P to Q.

Solution:

a From O to P

$$\begin{aligned} u &= 18, v = 24, t = 12, a = ? \\ v &= u + at \\ 24 &= 18 + 12a \\ a &= \frac{24 - 18}{12} = \frac{1}{2} \end{aligned}$$

From O to Q

$$\begin{aligned} u &= 18, t = 20, a = \frac{1}{2}, v = ? \\ v &= u + at \\ &= 18 + \frac{1}{2} \times 20 = 28 \end{aligned}$$

The speed of the train at Q is 28 m s^{-1}.

b From P to Q

$$\begin{aligned} u &= 24, v = 28, t = 8, s = ? \\ s &= \left(\frac{u + v}{2}\right) t \\ &= \left(\frac{24 + 28}{2}\right) \times 8 = 208 \end{aligned}$$

The distance from P to Q is 208 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 6

Question:

A car travelling on a straight road slows down with constant deceleration. The car passes a road sign with speed 40 km h^{-1} and a post box with speed of 24 km h^{-1}. The distance between the road sign and the post box is 240 m. Find, in m s^{-2}, the deceleration of the car.

Solution:

$$40 \text{ km h}^{-1} = \frac{40 \times 1000}{3600} \text{ m s}^{-1} = \frac{100}{9} \text{ m s}^{-1}$$

$$24 \text{ km h}^{-1} = \frac{24 \times 1000}{3600} \text{ m s}^{-1} = \frac{20}{3} \text{ m s}^{-1}$$

$$u = \frac{100}{9}, v = \frac{20}{3}, s = 240, a = \ ?$$

$$v^2 = u^2 + 2as$$

$$\left(\frac{20}{3}\right)^2 = \left(\frac{100}{9}\right)^2 + 2 \times a \times 240$$

$$a = \left(\frac{\left(\frac{20}{3}\right)^2 - \left(\frac{100}{9}\right)^2}{2 \times 240}\right) = -0.1646 \ldots$$

The deceleration of the car is 0.165 m s^{-2} (3 d.p.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 7

Question:

A skier is travelling downhill along a straight path with constant acceleration. At time $t = 0$ s, she passes a point A with speed 6 m s^{-1}. She continues with the same acceleration until she reaches a point B with speed 15 m s^{-1}. At B, the path flattens out and she travels from B to a point C at the constant speed of 15 m s^{-1}. It takes 20 s for the skier to travel from B to C and the distance from A to C is 615 m.

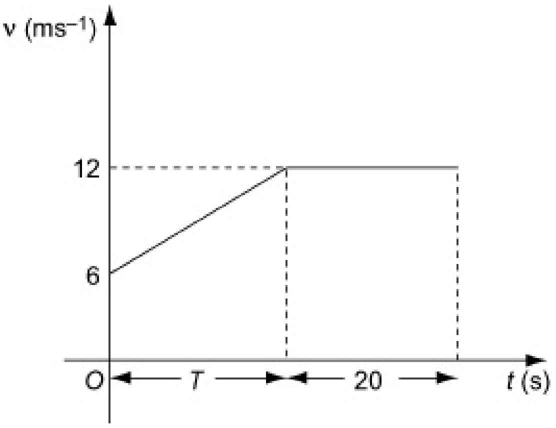
a Sketch a speed-time graph to illustrate the motion of the skier.

b Find the distance from A to B.

c Find the time the skier took to travel from A to B.

Solution:

a



b The distance from B to C is given by distance = speed × time = 15 × 20 = 300 (m)

The distance from A to B is (615 − 300) m = 315 m.

c

Let T seconds be the time for which the skier is accelerating.

$$s = \frac{1}{2}(a+b)h$$

$$315 = \frac{1}{2}(6+15)T = \frac{21}{2}T$$

$$T = \frac{315 \times 2}{21} = 30$$

The time the skier took to travel from A to B is 30 s.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 8

Question:

A child drops a ball from a point at the top of a cliff which is 82 m above the sea. The ball is initially at rest. Find

a the time taken for the ball to reach the sea,

b the speed with which the ball hits the sea.

c State one physical factor which has been ignored in making your calculation.

Solution:

a

Take downwards as the positive direction.

$$
\begin{aligned}
u &= 0, s = 82, a = 9.8, t = \ ?\\
s &= ut + \frac{1}{2}at^2\\
82 &= 0 + 4.9t^2\\
t^2 &= \frac{82}{4.9} = 16.73\ldots\\
t &= \sqrt{16.73\ldots} \approx 4.1
\end{aligned}
$$

The time taken for the ball to reach the sea is 4.1 s (2 s.f.).

b

$$
\begin{aligned}
u &= 0, s = 82, a = 9.8, v = \ ?\\
v^2 &= u^2 + 2as\\
&= 0^2 + 2 \times 9.8 \times 82 = 1607.2\\
v &= \sqrt{1607.2} \approx 40.09
\end{aligned}
$$

The speed with which the ball hits the sea is 40 m s^{-1} (2 s.f.).

c Air resistance

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 9

Question:

A particle moves along a straight line, from a point X to a point Y, with constant acceleration. The distance from X to Y is 104 m. The particle takes 8 s to move from X to Y and the speed of the particle at Y is 18 m s $^{-1}$. Find

a the speed of the particle at X,

b the acceleration of the particle.

The particle continues to move with the same acceleration until it reaches a point Z. At Z the speed of the particle is three times the speed of the particle at X.

c Find the distance XZ.

Solution:

a

$$s = 104, t = 8, v = 18, u = ?$$

$$s = \left(\frac{u+v}{2} \right) t$$

$$104 = \left(\frac{u+18}{2} \right) \times 8 = (u+18)\, 4 = 4u + 72$$

$$u = \frac{104-72}{4} = 8$$

The speed of the particle at X is 8 m s $^{-1}$.

b

$$s = 104, t = 8, v = 18, a = ?$$

$$s = vt - \frac{1}{2}at^2$$

$$104 = 18 \times 8 - \frac{1}{2}a \times 8^2 = 144 - 32a$$

$$a = \frac{144-104}{32} = 1.25$$

The acceleration of the particle is 1.25 m s $^{-2}$.

c From X to Z

$u = 8, v = 24, a = 1.25, s = ?$

$v^2 = u^2 + 2as$

$24^2 = 8^2 + 2 \times 1.25 \times s$

$s = \frac{24^2 - 8^2}{2.5} = 204.8$

$XZ = 204.8$ m

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 10

Question:

A pebble is projected vertically upwards with speed 21 m s $^{-1}$ from a point 32 m above the ground. Find

a the speed with which the pebble strikes the ground,

b the total time for which the pebble is more than 40 m above the ground.

Solution:

a

Take upwards as the positive direction.

$$u = 21, s = -32, a = -9.8, v = ?$$
$$v^2 = u^2 + 2as$$
$$= 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2$$
$$v = \sqrt{1068.2} = 32.68 \ldots$$

the speed with which the pebble strikes the ground is 33 m s $^{-1}$ (2 s.f.).

b

40 m above the ground is 8 m above the point of projection.

$$u = 21, s = 8, a = -9.8, t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$8 = 21t - 4.9t^2$$

$$4.9t^2 - 21t + 8 = 0$$
$$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = \frac{21 \pm \sqrt{(21^2 - 4 \times 4.9 \times 8)}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86, 0.42$$

the pebble is above 40 m between these times; 3.86 − 0.42 = 3.44

the pebble is more than 40 m above the ground for 3.4 s (2 s.f.)

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 11

Question:

A particle P is moving along the x-axis with constant deceleration 2.5 m s^{-2}. At time $t = 0$ s, P passes through the origin with velocity 20 m s^{-1} in the direction of x increasing. At time $t = 12$ s, P is at the point A. Find

a the distance OA,

b the total distance P travels in 12 s.

Solution:

a

$a = -2.5, u = 20, t = 12, s = ?$

$s = ut + \frac{1}{2}at^2$

$= 20 \times 12 - \frac{1}{2} \times 2.5 \times 12^2 = 240 - 180 = 60$

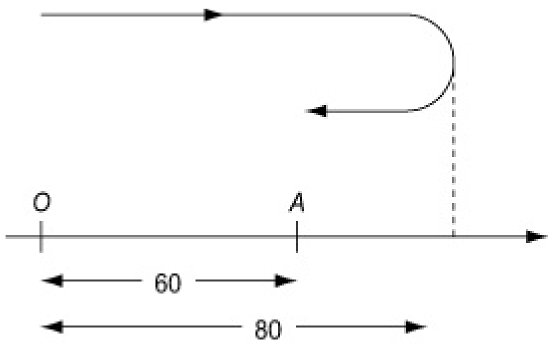
$OA = 60$ m

b The particle will turn round when $v = 0$.

$a = -2.5, u = 20, v = 0, s = ?$

$v^2 = u^2 + 2as$

$0^2 = 20^2 - 5s \Rightarrow s = 80$



The total distance P travels is $(80 + 20)$ m = 100 m.

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Kinematics of a particle moving in a straight line
Exercise E, Question 12

Question:

A train starts from rest at a station *P* and moves with constant acceleration for 45 s reaching a speed of 25 m s^{-1}. The train then maintains this speed for 4 minutes. The train then uniformly decelerates, coming to rest at a station *Q*.

a Sketch a speed-time graph illustrating the motion of the train from *P* to *Q*.

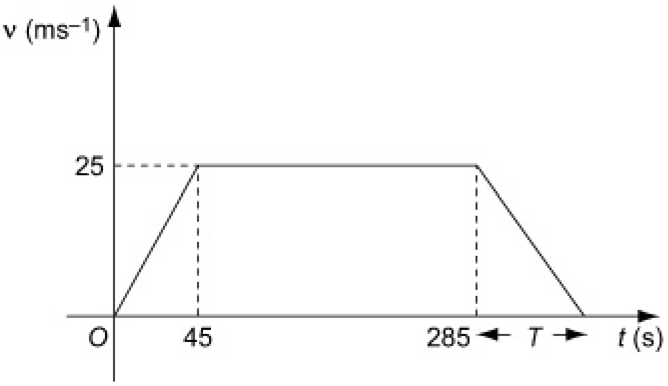
The distance between the stations is 7 km.

b Find the deceleration of the train.

c Sketch an acceleration-time graph illustrating the motion of the train from *P* to *Q*.

Solution:

a



b

$$s = \frac{1}{2}(a+b)h$$

$$7000 = \frac{1}{2}(240+285+T) \times 25$$

$$525 + T = \frac{7000 \times 2}{25} = 560 \Rightarrow T = 35$$

The deceleration in the last 35 s is given by the gradient of the line.

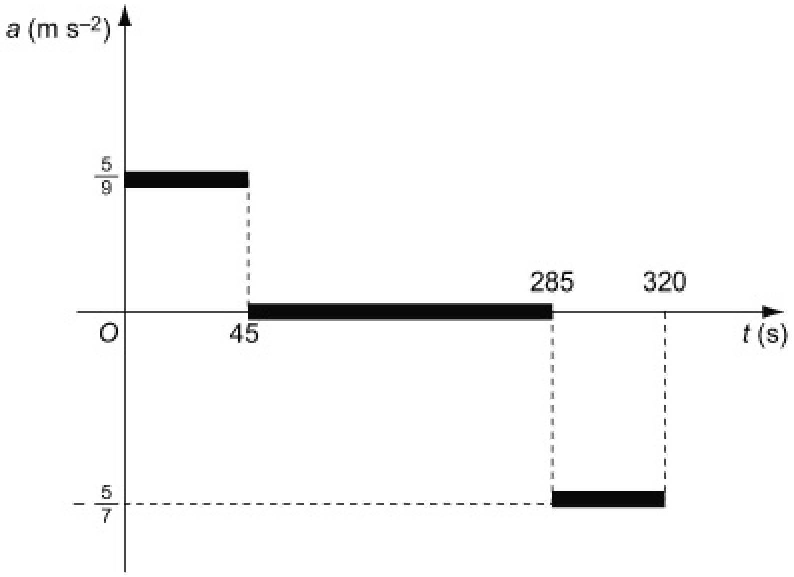
$$a = \frac{-25}{35} = -\frac{5}{7}$$

the deceleration of the train is $\frac{5}{7}$ m s^{-2}

c

The acceleration of the train in the first 40 s is given by the gradient of the line.

$a = \frac{25}{45} = \frac{5}{9}$



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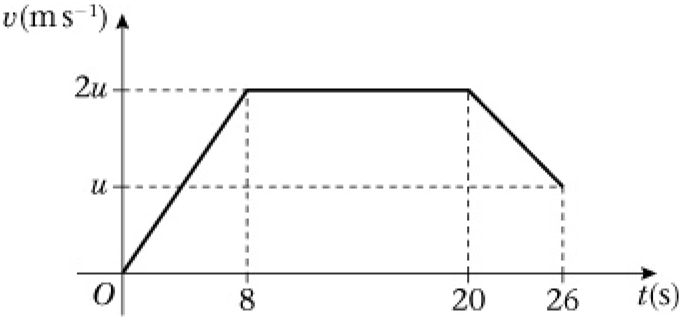
Kinematics of a particle moving in a straight line
Exercise E, Question 13

Question:



A particle moves 451 m in a straight line. The diagram shows a speed-time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u$ m s^{-1}. This speed is then maintained until $t = 20$ s. The particle then decelerates to a speed of u m s^{-1} at time $t = 26$ s. Find

a the value of u,

b the distance moved by the particle while its speed is less than u m s^{-1}.

Solution:

a

distance = area of triangle + area of rectangle + area of trapezium

$$451 = \frac{1}{2} \times 8 \times 2u + 12 \times 2u + \frac{1}{2} \times (u + 2u) \times 6$$
$$= 8u + 24u + 9u = 41u$$
$$u = \frac{451}{41} = 11$$

b

The particle is moving with speed less than u m s^{-1} for the first 4 s.

$$s = \frac{1}{2} \times 4 \times 11 = 22$$

The distance moved with speed less than 11 m s^{-1} is 22 m.

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Kinematics of a particle moving in a straight line
Exercise E, Question 14

Question:

A particle is moving in a straight line. The particle starts with speed 5 m s^{-1} and accelerates at a constant rate of 2 m s^{-1} for
8 s. It then decelerates at a constant rate coming to rest in a further 12 s.

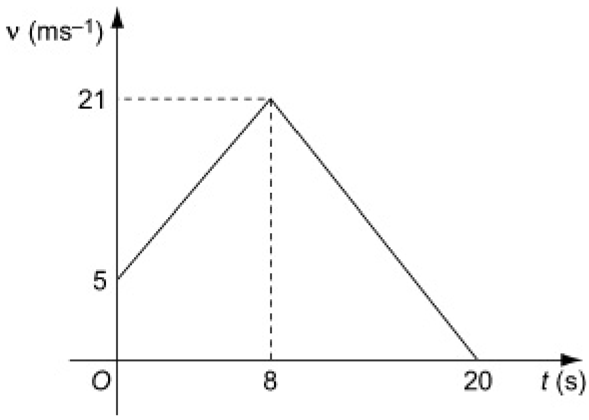
a Sketch a speed-time graph illustrating the motion of the particle.

b Find the total distance moved by the particle during its 20 s of motion.

c Sketch a distance-time graph illustrating the motion of the particle.

Solution:

a



$$v = u + at$$
$$v = 5 + 2 \times 8$$
$$= 21 \text{ m s}^{-1}$$

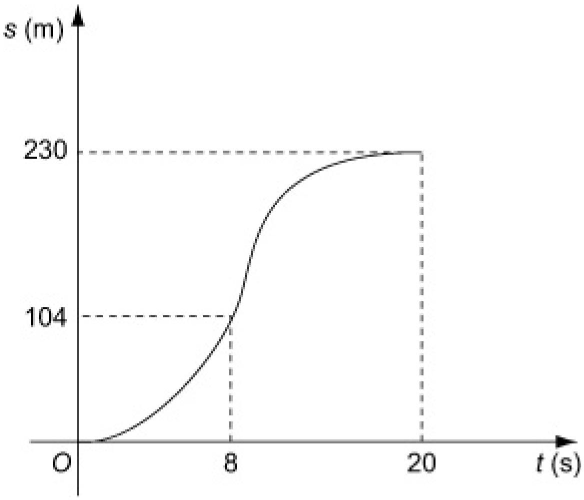
b

distance = area of trapezium + area of triangle

$$s = \frac{1}{2}(5 + 21) \times 8 + \frac{1}{2} \times 12 \times 21 = 104 + 126 = 230$$

The distance moved in 20 s is 230 m.

c After 8 s, $s = 104$


s (m)
230
104
O
8
20
t (s)

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Kinematics of a particle moving in a straight line
Exercise E, Question 15

Question:

A boy projects a ball vertically upwards with speed 10 m s^{-1} from a point *X*, which is 50 m above the ground. *T* seconds after the first ball is projected upwards, the boy drops a second ball from *X*. Initially the second ball is at rest. The balls collide
25 m above the ground. Find the value of *T*.

Solution:

Find the time taken by the first ball to reach 25 m below its point of projection, take upwards as the positive direction.

$u = 10, s = -25, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$-25 = 10t - 4.9t^2$

$4.9t^2 - 10t - 25 = 0$

$t = \frac{10 + \sqrt{(10^2 + 4 \times 4.9 \times 25)}}{9.8} = 3.4989\ldots$

The negative solution is not possible.

Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

$u = 0, s = 25, a = 9.8, t = t'$

$s = ut + \frac{1}{2}at^2$

$25 = 4.9t'^2 \Rightarrow t' \approx 2.2587\ldots$

$T = t - t' = 1.240\ldots = 1.2\text{ s } (2\text{ s.f.})$

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Kinematics of a particle moving in a straight line
Exercise E, Question 16

Question:

A car is moving along a straight road with uniform acceleration. The car passes a check-point A with speed 12 m s^{-1} and another check-point C with speed 32 m s^{-1}. The distance between A and C is 1100 m.

a Find the time taken by the car to move from A to C.

Given that B is the mid-point of AC,

b find the speed with which the car passes B.

Solution:

a

$$u = 12, v = 32, s = 1100, t = ?$$
$$s = \left(\frac{u+v}{2}\right)t$$

$$1100 = \frac{1}{2}(12 + 32)t = 22t$$

$$t = \frac{1100}{22} = 50$$

The time taken by the car to move from A to C is 50 s.

b

$\boxed{\text{Find } a \text{ first.}}$

From A to C

$$u = 12, v = 32, s = 1100, a = ?$$
$$v^2 = u^2 + 2as$$
$$32^2 = 12^2 + 2a \times 1100$$
$$a = \frac{32^2 - 12^2}{2200} = 0.4$$

From A to B

$$u = 12, s = 550, a = 0.4, v = ?$$
$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times 0.4 \times 550 = 584 \Rightarrow v = 24.166\ldots$$

The car passes C with speed 24.2 m s^{-1} (3 s.f.).

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Kinematics of a particle moving in a straight line
Exercise E, Question 17

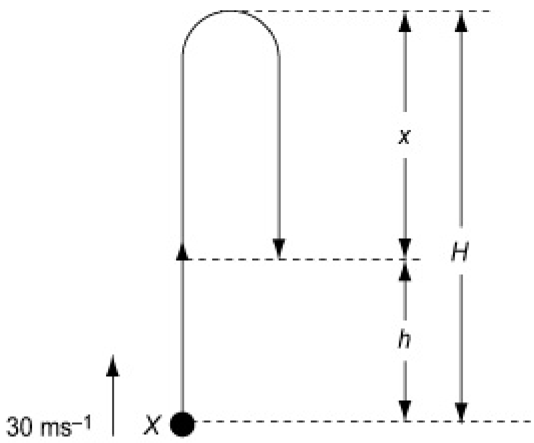
Question:

A particle is projected vertically upwards with a speed of 30 m s $^{-1}$ from a point *A*. The point *B* is *h* metres above *A*. The particle moves freely under gravity and is above *B* for a time 2.4 s. Calculate the value of *h*.

Solution:

To find greatest height reached *H*, take upwards as the positive direction.

$$\begin{aligned} u &= 30, v = 0, a = -9.8, s = H \\ v^2 &= u^2 + 2a\ s \\ 0^2 &= 30^2 - 2 \times 9.8 \times H \\ H &= \frac{30^2}{2 \times 9.8} \approx 45.918\ \ldots \end{aligned}$$



By symmetry, the particle takes $\frac{2.4}{2} = 1.2$ s to fall the distance, say *x*, from the highest point to a point *h* m above the point of projection.

Take downwards as the positive direction.

$$\begin{aligned} u &= 0, t = 1.2, a = 9.8, s = x \\ s &= u\ t + \frac{1}{2}at^2 \\ x &= 0 + 4.9 \times 1.2^2 = 7.056 \end{aligned}$$

From diagram,

$$\begin{aligned} h &= H - x = 45.918 - 7.056 = 38.862 \\ h &= 39\ \ (\text{2 s.f.}) \end{aligned}$$

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Kinematics of a particle moving in a straight line
Exercise E, Question 18

Question:

Two cars A and B are moving in the same direction along a straight horizontal road. At time $t = 0$, they are side by side, passing a point O on the road. Car A travels at a constant speed of 30 m s^{-1}. Car B passes O with a speed of 20 m s^{-1}, and has constant acceleration of 4 m s^{-2}. Find

a the speed of B when it has travelled 78 m from O,

b the distance from O of A when B is 78 m from O,

c the time when B overtakes A.

Solution:

a

$$u = 20, a = 4, s = 78, v = \ ?$$
$$v^2 = u^2 + 2as$$
$$= 20^2 + 2 \times 4 \times 78 = 1024$$
$$v = \sqrt{1024} = 32$$

The speed of B when it has travelled 78 m is 32 m s^{-1}.

b

Find time for B to reach the point 78 m from O.

$$v = 32, u = 20, a = 4, t = \ ?$$
$$v = u + at$$
$$32 = 20 + 4t \quad \Rightarrow \quad t = \frac{32 - 20}{4} = 3$$

For A, distance = speed × time

$s = 30 \times 3 = 90$

The distance from O of A when B is 78 m from O is 90 m.

c At time t seconds, for A, $s = 30t$

$$\text{for } B, s = ut + \frac{1}{2}at^2 = 20t + 2t^2$$

On overtaking the distances are the same.

$$20t + t^2 = 30t$$
$$t^2 - 10t = t(t - 10) = 0$$

For overtaking $t > 0$, $t = 10$

B overtakes *A* 10 s after passing *O*.

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Kinematics of a particle moving in a straight line
Exercise E, Question 19

Question:

A car is being driven on a straight stretch of motorway at a constant speed of 34 m s $^{-1}$, when it passes a speed restriction sign *S* warning of road works ahead and requiring speeds to be reduced to 22 m s $^{-1}$. The driver continues at her speed for 2 s after passing *S*. She then reduces her speed to 22 m s $^{-1}$ with constant deceleration of 3 m s $^{-2}$, and continues at the lower speed.

a Draw a speed-time graph to illustrate the motion of the car after it passes *S*.

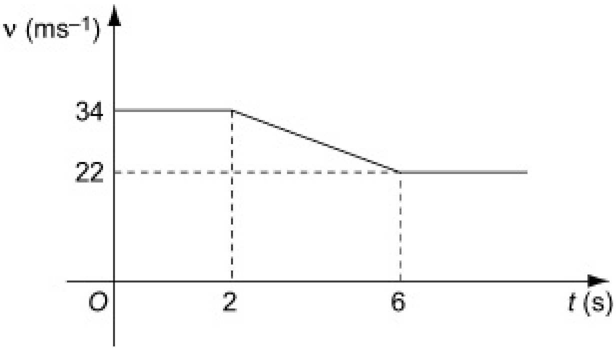
b Find the shortest distance before the road works that *S* should be placed on the road to ensure that a car driven in this way has had its speed reduced to 22 m s $^{-1}$ by the time it reaches the start of the road works.

Solution:

a

To find time decelerating.

$$u = 34, v = 22, a = -3, t = ?$$
$$v = u + at$$
$$22 = 34 - 3t \Rightarrow t = \frac{34 - 22}{3} = 4$$



b

$\boxed{\text{distance = rectangle + trapezium}}$

$$s = 34 \times 2 + \frac{1}{2}(22 + 34) \times 4$$
$$= 68 + 112 = 180$$

Distance required is 180 m.

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Kinematics of a particle moving in a straight line
Exercise E, Question 20

Question:

A train starts from rest at station *A* and accelerates uniformly at $3x$ m s^{-2} until it reaches a speed of 30 m s^{-1}. For the next
T seconds the train maintains this constant speed. The train then retards uniformly at x m s^{-2} until it comes to rest at a station *B*. The distance between the stations is 6 km and the time taken from *A* to *B* is 5 minutes.

a Sketch a speed-time graph to illustrate this journey.

b Show that $\frac{40}{x} + T = 300$.

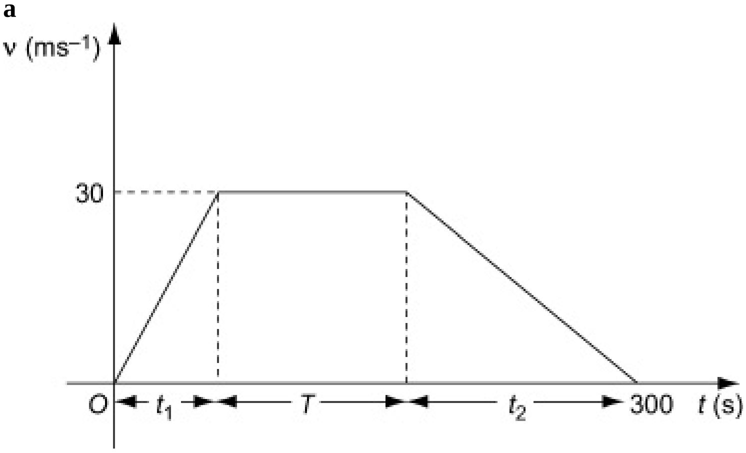
c Find the value of *T* and the value of *x*.

d Calculate the distance the train travels at constant speed.

e Calculate the time taken from leaving *A* until reaching the point half-way between the stations.

Solution:

a



b

Acceleration is the gradient of a line.

$$3x = \frac{30}{t_1} \Rightarrow t_1 = \frac{30}{3x} = \frac{10}{x}$$

$$-x = \frac{-30}{t_2} \Rightarrow t_2 = \frac{30}{x}$$

$$t_1 + T + t_2 = 300$$

$$\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300\text{, as required}$$

c $s = \frac{1}{2}(a+b)h$

$6000 = \frac{1}{2}(T + 300) \times 30 = 15T + 4500$

$T = \frac{6000 - 4500}{15} = 100$

Substitute into the result in part **b**.

$\frac{40}{x} + 100 = 300 \quad \Rightarrow \quad \frac{40}{x} = 200$

$x = \frac{40}{200} = 0.2$

d at constant speed, distance = speed × time = 30 × 100 = 3000 (m)

The distance travelled at a constant speed in 3 km.

e $t_1 = \frac{10}{x} = \frac{10}{0.2} = 50$

while accelerating, distance travelled is $\left(\frac{1}{2} \times 50 \times 30\right)$ m = 750 m

To reach half way, the train must travel 2250 m at a constant speed.

at constant speed, time = $\frac{\text{distance}}{\text{speed}} = \frac{2250}{30}$ s = 75 s

Time for train to reach half-way is (50 + 75) s = 125 s.

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Dynamics of a particle moving in a straight line
Exercise A, Question 1

Question:

Remember that g should be taken as 9.8 m s^{-2}.

Find the weight in newtons of a particle of mass 4 kg.

Solution:

$W = m \, g = 4 \times 9.8 = 39.2$

The weight of the particle is 39.2 N.

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Dynamics of a particle moving in a straight line
Exercise A, Question 2

Question:

Find the mass of a particle whose weight is 490 N.

Solution:

$$W = m\ g \text{ so } 490 = m \times 9.8$$
$$\Rightarrow m = \frac{490}{9.8} = 50$$

The mass of the particle is 50 kg.

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Dynamics of a particle moving in a straight line
Exercise A, Question 3

Question:

The weight of an astronaut on the Earth is 686 N. The acceleration due to gravity on the Moon is approximately 1.6 m s^{-2}. Find the weight of the astronaut when he is on the Moon.

Solution:

$$
\begin{aligned}
W_{\text{EARTH}} &= m \; g_{\text{EARTH}} \\
\text{so, } 686 &= m \times 9.8 \\
\Rightarrow m &= 70 \text{ i.e. the mass of the astronaut is 70 kg} \\
W_{\text{MOON}} &= m \; g_{\text{MOON}} \\
&= 70 \times 1.6 \\
&= 112
\end{aligned}
$$

The weight of the astronaut on the Moon is 112 N.

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Dynamics of a particle moving in a straight line
Exercise A, Question 4

Question:

Find the force required to accelerate a 1.2 kg mass at a rate of 3.5 m s^{-2}.

Solution:

$$\begin{aligned} F &= m\ a \\ &= 1.2 \times 3.5 \\ &= 4.2 \end{aligned}$$

So the force required is 4.2 N.

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Dynamics of a particle moving in a straight line
Exercise A, Question 5

Question:

Find the acceleration when a particle of mass 400 kg is acted on by a resultant force of 120 N.

Solution:

$$\begin{aligned} F &= m\ a \\ 120 &= 400a \\ a &= 0.3 \end{aligned}$$

The acceleration is 0.3 m s^{-2}.

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Dynamics of a particle moving in a straight line
Exercise A, Question 6

Question:

An object moving on a rough surface experiences a constant frictional force of 30 N which decelerates it at a rate of 1.2 m s $^{-2}$. Find the mass of the object.

Solution:

$$
\begin{aligned}
F &= m\ \ a \\
30 &= 1.2m \\
m &= 25
\end{aligned}
$$

The mass of the object is 25 kg.

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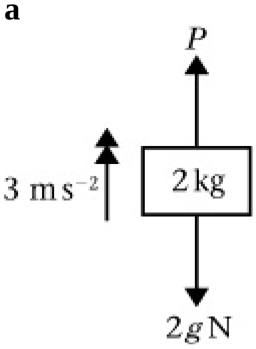
Dynamics of a particle moving in a straight line
Exercise A, Question 7

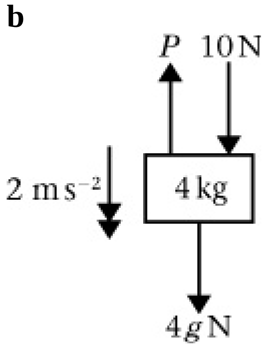
Question:

In each of the following scenarios, the forces acting on the body cause it to accelerate as shown. Find the magnitude of the unknown forces *P* and *Q*.

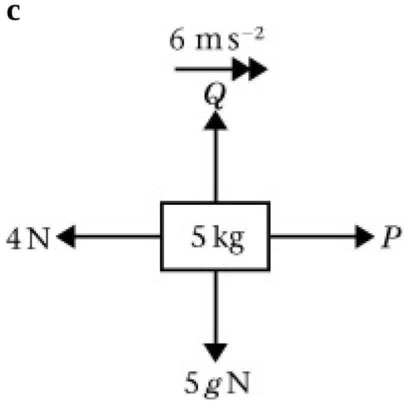
a



b



c



Solution:

a

$R(\uparrow), \quad P - 2g = 2 \times 3$

$P = 25.6$

The magnitude of P is 25.6 N.

Always resolve in the direction of the acceleration.

b

$R(\downarrow), \quad 4g + 10 - P = 4 \times 2$

$49.2 - P = 8$

$P = 41.2$

The magnitude of P is 41.2 N.

c

$R(\rightarrow), \quad P - 4 = 5 \times 6$

$P = 34$

The magnitude of P is 34 N.

$R(\uparrow), \quad Q - 5g = 5 \times 0$

$Q = 49$

No acceleration vertically.

The magnitude of Q is 49 N.

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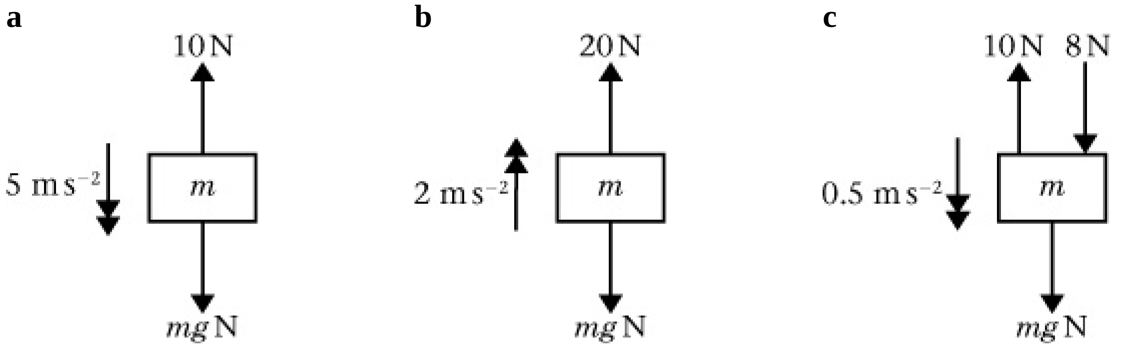
Dynamics of a particle moving in a straight line
Exercise A, Question 8

Question:

In each of the following situations, the forces acting on the body cause it to accelerate as shown. In each case find the mass of the body, *m*.



Solution:

a

$R(\downarrow)$, $mg - 10 = m \times 5$

$9.8m - 10 = 5m$

$4.8m = 10$

$m = 2.1 \quad (2 \text{ s.f.})$

The mass of the body is 2.1 kg (2 s.f.).

b

$R(\uparrow)$, $20 - mg = m \times 2$

$20 - 9.8m = 2m$

$20 = 11.8m$

$m = 1.7 \quad (2 \text{ s.f.})$

The mass of the body is 1.7 kg (2 s.f.).

c

$R(\downarrow)$, $mg + 8 - 10 = m \times 0.5$

$9.8m - 2 = 0.5m$

$9.3m = 2$

$m = 0.22 \quad (2 \text{ s.f.})$

The mass of the body is 0.22 kg (2 s.f.).

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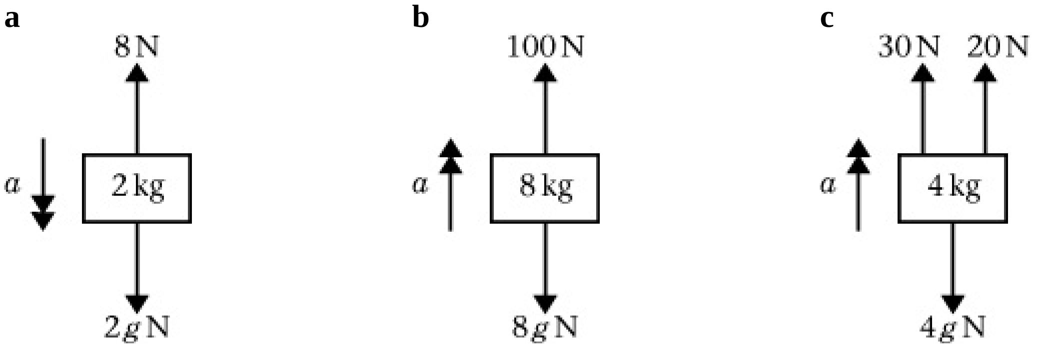
Dynamics of a particle moving in a straight line
Exercise A, Question 9

Question:

In each of the following situations, the forces acting on the body cause it to accelerate as shown with magnitude a m s^{-2}. In each case find the value of a.



Solution:

a

$R(\downarrow)$, $\quad 2g - 8 = 2a$

$19.6 - 8 = 2a$

$11.6 = 2a$

$5.8 = a$

The acceleration of the body is 5.8 m s^{-2}.

b

$R(\uparrow)$, $\quad 100 - 8g = 8a$

$100 - 78.4 = 8a$

$21.6 = 8a$

$2.7 = a$

The acceleration of the body is 2.7 m s^{-2}.

c

$R(\uparrow)$, $\quad 30 + 20 - 4g = 4a$

$50 - 39.2 = 4a$

$10.8 = 4a$

$2.7 = a$

The acceleration of the body is 2.7 m s^{-2}.

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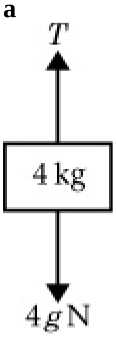
Dynamics of a particle moving in a straight line
Exercise A, Question 10

Question:

The diagram shows a block of mass 4 kg being lowered vertically by a rope.

a

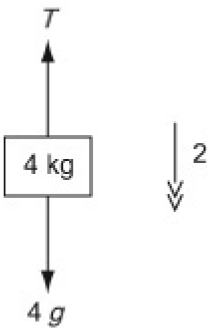


Find the tension in the rope when the block is lowered **a** with an acceleration of 2 m s $^{-2}$, **b** at a constant speed of 4 m s $^{-1}$,
c with a deceleration of 0.5 m s $^{-2}$.

Solution:

a

R (↓) ,	$4g - T$	$= 4 \times 2$
$39.2 - T$		$= 8$
T		$= 31.2$

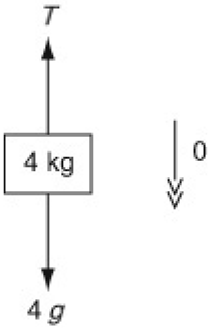


The tension in the rope is 31.2 N.

b

R (↓) ,	$4g - T$	$= 4 \times 0$
39.2		$= T$

Constant speed ⇒ zero acceleration.

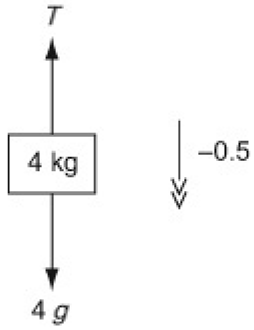


The tension in the rope is 39.2 N.

c

R (↓) , $4g - T$	$= 4 \times -0.5$
$39.2 - T$	$= -2$
T	$= 41.2$

Deceleration is a negative (downward) acceleration



The tension in the rope is 41.2 N.

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Dynamics of a particle moving in a straight line
Exercise B, Question 1

Question:

A ball of mass 200 g is attached to the upper end of a vertical light rod. Find the thrust in the rod when it raises the ball vertically with an acceleration of 1.5 m s $^{-2}$.

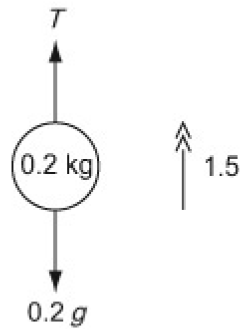
Solution:

R $(\uparrow)$, $T - 0.2g = 0.2 \times 1.5$

$T - 1.96 = 0.3$

$T = 2.26 = 2.3\ (2\text{ s.f.})$

The thrust in the rod is 2.3 N (2 s.f.).



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Dynamics of a particle moving in a straight line
Exercise B, Question 2

Question:

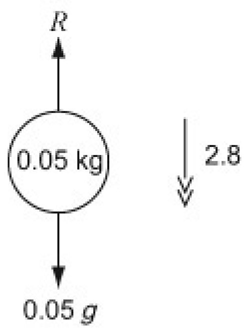
A small pebble of mass 50 g is dropped into a pond and falls vertically through it with an acceleration of 2.8 m s $^{-2}$. Assuming that the water produces a constant resistance, find its magnitude.

Solution:

R (↓) , $0.05g - R = 0.05 \times 2.8$

$R = 0.35$

The resistance of the water is 0.35 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 3

Question:

A lift of mass 500 kg is lowered or raised by means of a metal cable attached to its top. The lift contains passengers whose total mass is 300 kg. The lift starts from rest and accelerates at a constant rate, reaching a speed of 3 m s^{-1} after moving a distance of 5 m. Find

a the acceleration of the lift,

b the tension in the cable if the lift is moving vertically downwards,

c the tension in the cable if the lift is moving vertically upwards.

Solution:

a $u = 0$, $v = 3$, $s = 5$, $a = ?$

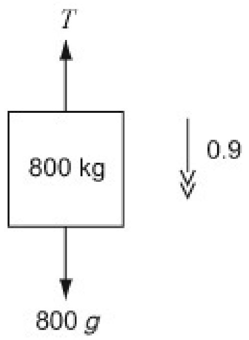
$$v^2 = u^2 + 2as$$
$$3^2 = 0^2 + 2a \times 5$$
$$9 = 10a$$
$$a = 0.9$$

The acceleration of the lift is 0.9 m s^{-2}.

b

$$R(\downarrow),\quad 800g - T = 800 \times 0.9$$
$$7840 - T = 720$$
$$T = 7120$$

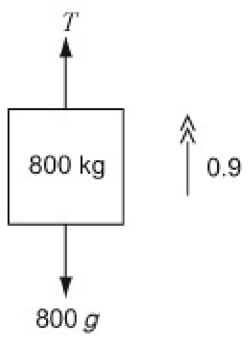
The tension in the cable is 7120 N.



c

$$R(\uparrow),\quad T - 800g = 800 \times 0.9$$
$$T - 7840 = 720$$
$$T = 8560$$

The tension in the cable is 8560 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 4

Question:

A block of mass 1.5 kg falls vertically from rest and hits the ground 16.6 m below after falling for 2 s. Assuming that the air resistance experienced by the block as it falls is constant, find its magnitude.

Solution:

$u = 0$, $s = 16.6$, $t = 2$, $a =$?

$s = ut + \frac{1}{2}at^2 \quad (\downarrow)$

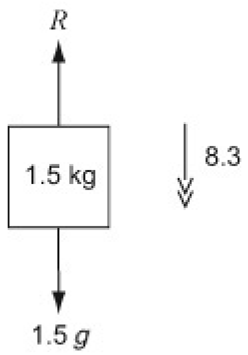
$16.6 = 0 + \frac{1}{2}a \times 2^2$

$a = 8.3$

R (↓) , $1.5g - R = 1.5 \times 8.3$

$R = 2.25$

The magnitude of the air resistance is 2.25 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 5

Question:

A trolley of mass 50 kg is pulled from rest in a straight line along a horizontal path by means of a horizontal rope attached to its front end. The trolley accelerates at a constant rate and after 2 s its speed is 1 m s^{-1}. As it moves, the trolley experiences a resistance to motion of magnitude 20 N. Find

a the acceleration of the trolley,

b the tension in the rope.

Solution:

a $u = 0$, $v = 1$, $t = 2$, $a =$?

$$v = u + at$$
$$1 = 0 + a \times 2$$
$$\Rightarrow a = 0.5$$

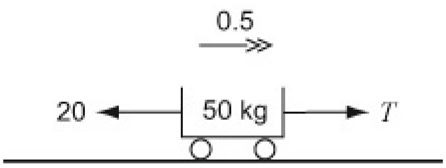
The acceleration of the trolley is 0.5 m s^{-2}.

b

R (→) , $T - 20 = 50 \times 0.5$

$T = 45$

The tension in the rope is 45 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 6

Question:

A trailer of mass 200 kg is attached to a car by a light tow-bar. The trailer is moving along a straight horizontal road and decelerates at a constant rate from a speed of 15 m s^{-1} to a speed of 5 m s^{-1} in a distance of 25 m. Assuming there is no resistance to the motion, find

a the deceleration of the trailer,

b the thrust in the tow-bar.

Solution:

a $u = 15$, $v = 5$, $s = 25$, $a = ?$

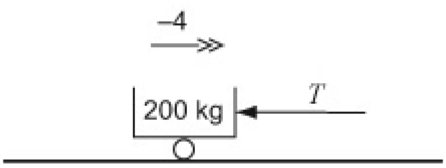
$$
\begin{aligned}
v^2 &= u^2 + 2as \quad (\rightarrow) \\
5^2 &= 15^2 + 2a \times 25 \\
25 &= 225 + 50a \\
-200 &= 50a \\
a &= -4
\end{aligned}
$$

The deceleration of the trailer is 4 m s^{-2}.

b

$$
\begin{aligned}
\mathrm{R}(\rightarrow),\quad -T &= 200 \times -4 \\
T &= 800
\end{aligned}
$$

The thrust in the tow-bar is 800 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 7

Question:

A woman of mass 60 kg is in a lift which is accelerating upwards at a rate of 2 m s^{-2}.

a Find the magnitude of the normal reaction of the floor of the lift on the woman.
The lift then moves at a constant speed and then finally decelerates to rest at 1.5 m s^{-2}.

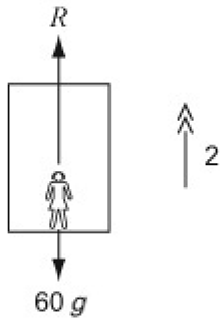
b Find the magnitude of the normal reaction of the floor of the lift on the woman during the period of deceleration.

c Hence explain why the woman will feel heavier during the period of acceleration and lighter during the period of deceleration.

Solution:

a

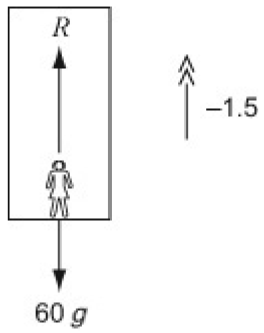
R (↑) , $R - 60g = 60 \times 2$
$R - 588 = 120$
$R = 708$



The normal reaction on the woman has magnitude 708 N.

b

R (↑) , $R - 60g = 60 \times -1.5$
$R - 588 = -90$
$R = 498$



The normal reaction on the woman has magnitude 498 N.

c The woman's sense of her own weight is the magnitude of the force that she feels from the floor i.e. the normal reaction. This is ‘usually’ 60g i.e. 588 N but when the lift is accelerating upwards it increases to 708 N i.e. she feels heavier and when the lift is decelerating upwards it decreases to 498 N i.e. she feels lighter.

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line

Exercise B, Question 8

Question:

The engine of a van of mass 400 kg cuts out when it is moving along a straight horizontal road with speed 16 m s^{-1}. The van comes to rest without the brakes being applied.

In a model of the situation it is assumed that the van is subject to a resistive force which has constant magnitude of 200 N.

a Find how long it takes the van to stop.

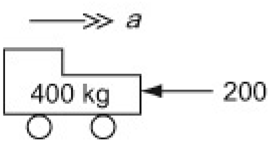
b Find how far the van travels before it stops.

c Comment on the suitability of the modelling assumption.

Solution:

a

$$\begin{aligned} R(\rightarrow), \quad -200 &= 400a \\ \Rightarrow a &= -0.5 \end{aligned}$$



$u = 16$, $v = 0$, $a = -0.5$, t ?

$$\begin{aligned} v &= u + at \quad (\rightarrow) \\ 0 &= 16 - 0.5t \\ 0.5t &= 16 \\ t &= 32 \end{aligned}$$

It takes 32 s for the van to stop.

b $u = 16$, $v = 0$, $a = -0.5$, s ?

$$\begin{aligned} v^2 &= u^2 + 2as \quad (\rightarrow) \\ 0^2 &= 16^2 + 2(-0.5)s \\ 0 &= 256 - s \\ s &= 256 \end{aligned}$$

The van travels 256 m before it stops.

c Air resistance is unlikely to be of constant magnitude. (It is usually a function of speed.)

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Dynamics of a particle moving in a straight line
Exercise B, Question 9

Question:

Albert and Bella are both standing in a lift. The mass of the lift is 250 kg. As the lift moves upward with constant acceleration, the floor of the lift exerts forces of magnitude 678 N and 452 N respectively on Albert and Bella. The tension in the cable which is pulling the lift upwards is 3955 N.

a Find the acceleration of the lift.

b Find the mass of Albert.

c Find the mass of Bella.

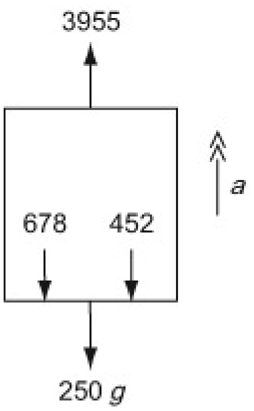
Solution:

a

$$\text{R}(\uparrow),\ 3955 - 678 - 452 - 250g = 250a$$
$$375 = 250a$$
$$a = 1.5$$

The acceleration of the lift upwards is 1.5 m s^{-2}.

Draw a diagram showing the forces acting an the LIFT only.

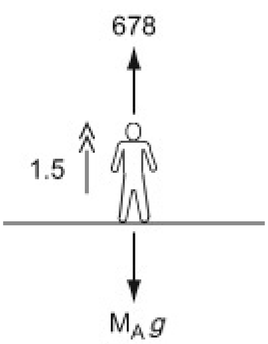


b

$$\text{R}(\uparrow),\ 678 - M_A g = M_A \times 1.5$$
$$678 = 11.3M_A$$
$$M_A = 60$$

The mass of Albert is 60 kg.

Draw a diagram showing the forces acting on ALBERT only.

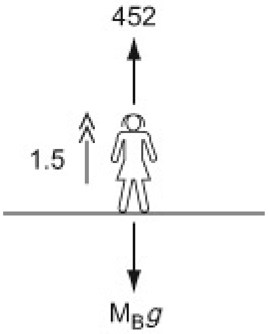


c

Draw a diagram showing the forces acting an

R (↑) , $452 - M_B g$	$= M_B \times 1.5$	BELLA only.
452	$= 11.3M_B$	
M_B	$= 40$	

The mass of Bella is 40 kg.



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Dynamics of a particle moving in a straight line
Exercise B, Question 10

Question:

A small stone of mass 400 g is projected vertically upwards from the bottom of a pond full of water with speed 10 m s^{-1}. As the stone moves through the water it experiences a constant resistance of magnitude 3 N. Assuming that the stone does not reach the surface of the pond, find

a the greatest height above the bottom of the pond that the stone reaches,

b the speed of the stone as it hits the bottom of the pond on its return,

c the total time taken for the stone to return to its initial position on the bottom of the pond.

Solution:

a

R (↑) , $-3 - 0.4g = 0.4a$

$a = -17.3$

$u = 10$, $v = 0$, $a = -17.3$, $s =$?

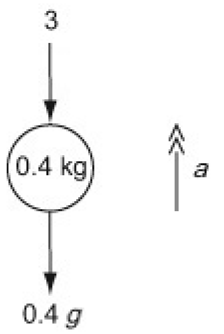
$v^2 = u^2 + 2as$ (↑)

$0 = 10^2 + 2(-17.3)s$

$0 = 100 - 34.6s$

$s = 2.89\ldots = 2.9$ (2 s.f.)

The stone rises to a height of 2.9 m (2 s.f.) above the bottom of the pond.



b

R (↓) , $0.4g - 3 = 0.4a$

$0.92 = 0.4a$

$a = 2.3$

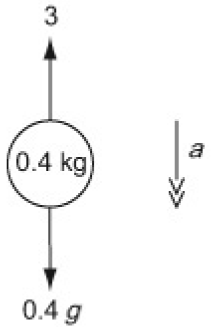
$u = 0$, $s = \left(\frac{100}{34.6}\right)$, $a = 2.3$, $v =$?

$v^2 = u^2 + 2as$ (↓)

$v^2 = 0^2 + 2 \times 2.3 \times \left(\frac{100}{34.6}\right)$

$v = 3.646.. = 3.6$ (2 s.f.)

The stone hits the bottom of the pond with speed 3.6 m s^{-1} (2 s.f.).



c $u = 10$, $v = 0$, $a = -17.3$, $t =$?

$v = u + at \quad (\uparrow)$

$0 = 10 - 17.3t$,

$t_1 = \frac{10}{17.3} = 0.57803 \ldots$

$u = 0$, $a = 2.3$, $s = \left(\frac{100}{34.6}\right)$, $t = ?$

$s = ut + \frac{1}{2}at^2 \quad (\downarrow)$

$\frac{100}{34.6} = 0 + \frac{1}{2} \times 2.3t_2^2$

$t_2^2 = \cdot \frac{2 \times 100}{2.3 \times 34.6} = 2.51319$

$t_2 = 1.585$

$t_1 + t_2 = 0.57803 + 1.585 = 2.16$

The total time is 2.16 s (3 s.f.).

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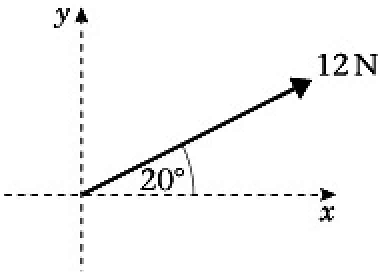
Dynamics of a particle moving in a straight line
Exercise C, Question 1

Question:

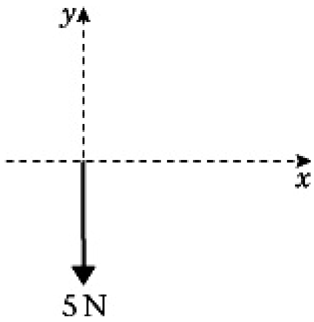
Find the component of each force that acts in

i the *x*-direction, **ii** the *y*-direction.

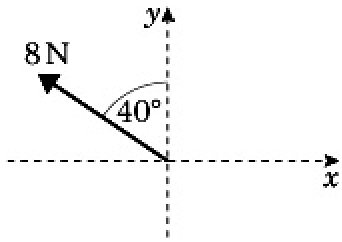
a



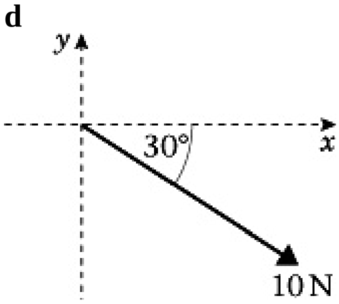
b



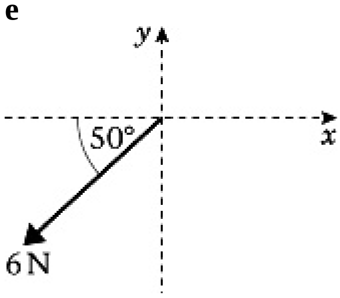
c



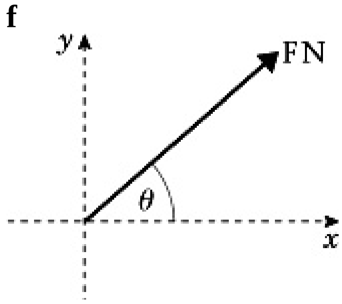
d



e



f



Solution:

a

i $12\cos 20° = 11.3$ N (3 s.f.)

ii $12\cos 70° = 12\sin 20° = 4.10$ N (3 s.f.)

b

i $5\cos 90° = 0$ N

ii $-5\cos 0° = -5$ N

(or $5\cos 180°$)

c

i $-8\cos 50° = -5.14$ N (3 s.f.)

(or $8\cos 130°$)

ii $8\cos 40° = 6.13$ N (3 s.f.)

d

i $10\cos 30° = 8.66$ N (3 s.f.)

ii $-10\cos 60° = -5$ N

(or $10\cos 120°$)

e

i $-6\cos 50° = -3.86$ N (3 s.f.)

(or $6\cos 130°$)

ii $-6\cos 40° = -4.60$ N (3 s.f.)

(or $6\cos 140°$)

f

i $F\cos\theta$ N

ii $F\cos(90° - \theta) = F\sin\theta$ N

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Edexcel AS and A Level Modular Mathematics

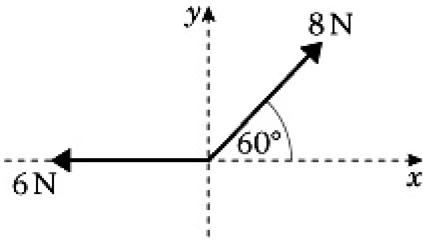
Dynamics of a particle moving in a straight line
Exercise C, Question 2

Question:

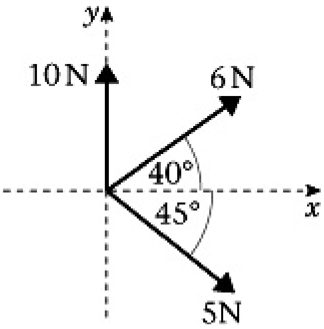
For each of the following systems of forces, find the sum of the components in

i the *x*-direction, **ii** the *y*-direction.

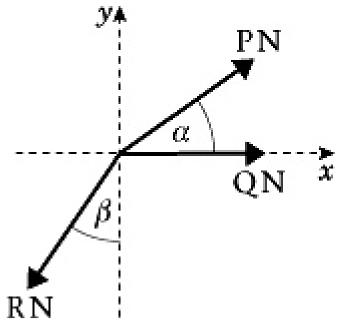
a



b



c



Solution:

a

i $8\cos 60° - 6 = -2$ N

ii $8\cos 30° = 6.93$ N (3 s.f.)

b

i $6\cos 40° + 5\cos 45° + (10\cos 90°) = 8.13$ N (3 s.f.)

ii $10(\cos 0°) + 6\cos 50° - 5\cos 45° = 10.3$ N (3 s.f.)

c

i $P\cos\alpha + Q(\cos 0°) - R\cos(90° - \beta)$

$= P\cos\alpha + Q - R\sin\beta$

ii $P\cos(90° - \alpha) - R\cos\beta = P\sin\alpha - R\cos\beta$

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Dynamics of a particle moving in a straight line
Exercise D, Question 1

Question:

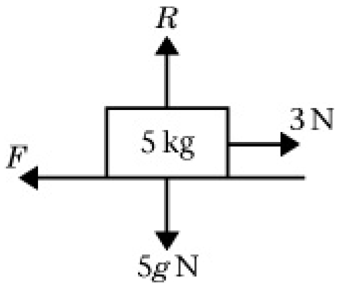
Each of the following diagrams shows a body of mass 5 kg lying initially at rest on rough horizontal ground. The coefficient of friction between the body and the ground is $\frac{1}{7}$. In each diagram *R* is the normal reaction from the ground on the body and *F* is the friction force exerted on the body by the ground. Any other forces applied to the body are as shown on the diagram. In each case

i find the magnitude of *F*,

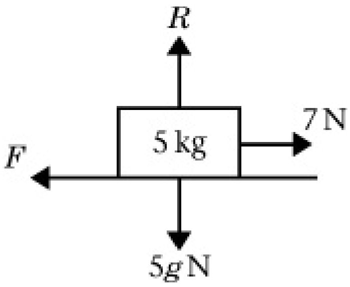
ii state whether the body will remain at rest or accelerate from rest along the ground,

iii find, when appropriate, the magnitude of this acceleration.

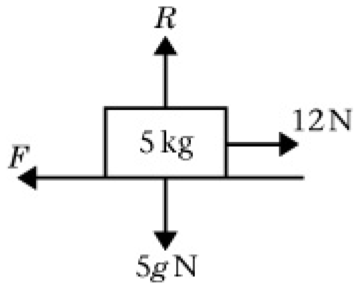
a



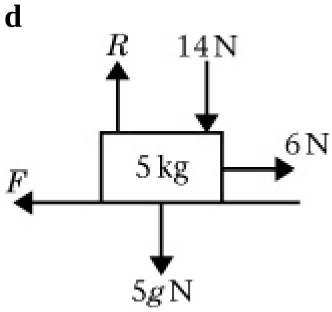
b



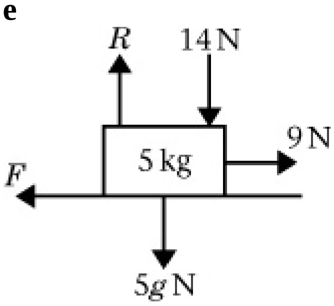
c



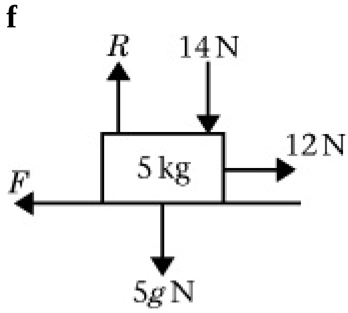
d



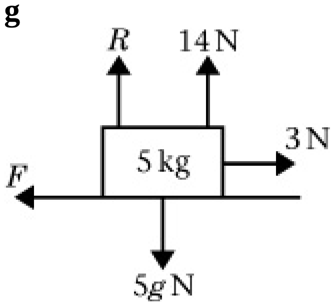
e



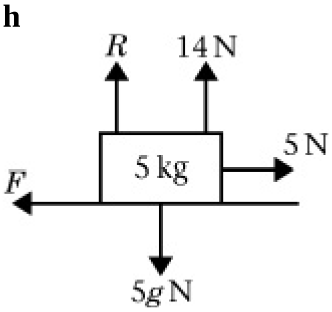
f



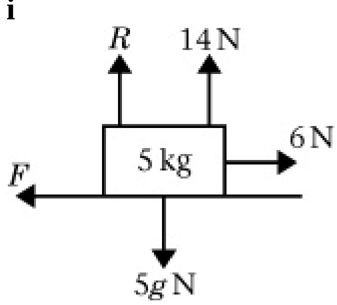
g



h



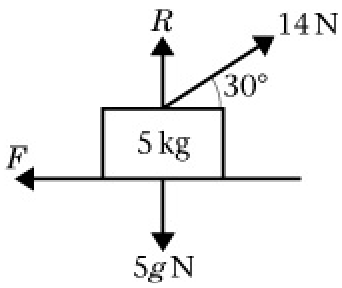
i

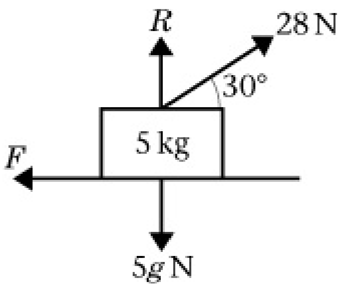


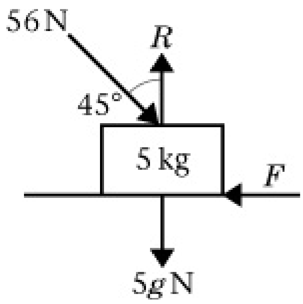
j

k

l







Solution:

a

i

$$\begin{aligned} R(\uparrow)\ R - 5g &= 0 \\ R &= 5g = 49\text{ N} \end{aligned}$$

$\therefore F_{MAX} = \frac{1}{7} \times 49 = 7\text{ N} \ \therefore F = 3\text{ N}$

ii $\therefore F = 3$ N and body remains at rest

b

i $F_{MAX} = 7\text{ N} \ \therefore F = 7\text{ N}$

ii $F = 7$ N and body remains at rest (in limiting equilibrium)

c

i $F_{MAX} = 7\text{ N} \ \therefore F = 7\text{ N}$

ii $F = 7$ N and body accelerates

iii

$$\begin{aligned} R(\rightarrow),12 - 7 &= 5a \\ a &= 1\text{ m s}^{-2} \end{aligned}$$

Body accelerates at 1 m s^{-2}

d

i

$$\begin{aligned} R(\uparrow),R - 14 - 5g &= 0 \\ R &= 63\text{ N} \end{aligned}$$

$\therefore F_{MAX} = \frac{1}{7} \times 63 = 9\text{ N} \ \therefore F = 6\text{ N}$

ii $F = 6$ N and body remains at rest

e

i $F = 9$ N

ii $F = 9$ N and body remains at rest in limiting equilibrium

f

i $F = 9$ N

ii $F = 9$ N and body accelerates

iii

$$\begin{aligned} \text{R}(\rightarrow), 12 - 9 &= 5a \\ a &= 0.6 \text{ m s}^{-2} \end{aligned}$$

Body accelerates at 0.6 m s^{-2}

g

i

$$\begin{aligned} \text{R}(\uparrow), R + 14 - 5g &= 0 \\ R &= 35 \text{ N} \end{aligned}$$

$\therefore F_{MAX} = \frac{1}{7} \times 35 = 5 \text{ N} \therefore F = 3 \text{ N}$

ii $F = 3$ N and body remains at rest

h

i $F = 5$ N

ii $F = 5$ N and body remains at rest in limiting equilibrium

i

i $F = 5$ N

ii $F = 5$ N and body accelerates

iii

$$\begin{aligned} \text{R}(\rightarrow), 6 - 5 &= 5a \\ a &= 0.2 \text{ m s}^{-2} \end{aligned}$$

Body accelerates at 0.2 m s^{-2}

j

i

$$\begin{aligned} \text{R}(\uparrow), R + 14\cos 60° - 5g &= 0 \\ R &= 42 \text{ N} \end{aligned}$$

$\therefore F_{MAX} = \frac{1}{7} \times 42 = 6 \text{ N}$

R ($\rightarrow$) ,$14\cos 30° - F = 5a$

Since $14\cos 30° > 6$

$\therefore F = 6$ N

ii $F = 6$ N and body accelerates

iii

$$\begin{aligned} \text{R}(\rightarrow),\ 14\cos 30° - 6 &= 5a \\ a &= 1.22 \text{ m s}^{-2} \ (3 \text{ s.f.}) \end{aligned}$$

Body accelerates at 1.22 m s^{-2} (3 s.f.)

k

i

$$\begin{aligned} \text{R}(\uparrow),\ R + 28\cos 60° - 5g &= 0 \\ R &= 35 \text{ N} \\ \therefore F_{\text{MAX}} = \frac{1}{7} \times 35 &= 5 \text{ N} \end{aligned}$$

R ($\rightarrow$) ,$28\cos 30° - F = 5a$

Since $28\cos 30° > 5$

$\therefore F = 5$ N

ii $F = 5$ N and body accelerates

iii

$$\begin{aligned} \text{R}(\rightarrow),\ 28\cos 30° - 5 &= 5a \\ a &= 3.85 \text{ m s}^{-2} \ (3 \text{ s.f.}) \end{aligned}$$

Body accelerates at 3.85 m s^{-2} (3 s.f.)

l

i R ($\uparrow$) ,$R - 56\cos 45° - 5g = 0$

$\therefore R = 88.6$ N (3 s.f.)

$\therefore F_{\text{MAX}} = \frac{1}{7} \times 88.6 = 12.657$ N

R ($\rightarrow$) ,$56\cos 45° - F$

Since $56\cos 45° > 12.657$ N

$F = F_{\text{MAX}} = 12.657$ or 12.7 N (3 s.f.)

ii body accelerates

iii $56\cos(45°) = 39.5979$

$5a = 39.598 - 12.657 = 26.941$

$a = 5.3882$

So the acceleration is 5.39 m s^{-2} (3 s.f.).

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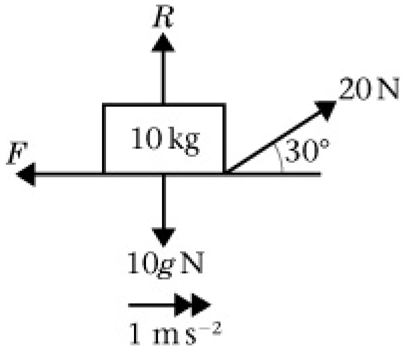
Dynamics of a particle moving in a straight line
Exercise D, Question 2

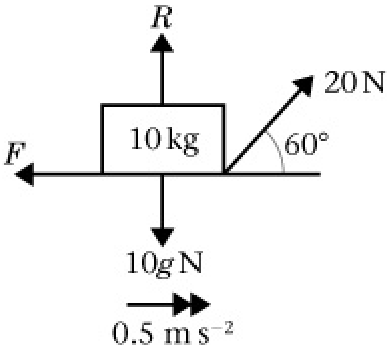
Question:

In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane. *R* is the normal reaction and *F* is the friction force. Find the coefficient of friction in each case.

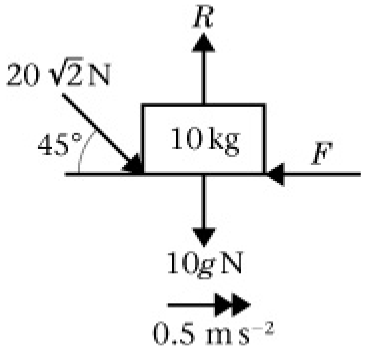
a



b



c



Solution:

a

R (↑) ,$R + 20\cos 60° - 10g = 0$
$R = 88$ N

R (→) ,$20\cos 30° - \mu \times 88 = 10 \times 1$
$\mu = 0.083$ (2 s.f.)

b

R (↑) ,$R + 20\cos 30° - 10g = 0$
$R = 80.679...$ N

R (→) ,$20\cos 60° - \mu \times 80.679 = 10 \times 0.5$
$\mu = 0.062$ (2 s.f.)

c

R (↑) ,$R - 20\sqrt{2}\cos 45° - 10g = 0$
$R = 118$ N

R (→) ,$20\sqrt{2}\cos 45° - \mu \times 118 = 10 \times 0.5$
$\mu = 0.13$ (2 s.f.)

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Dynamics of a particle moving in a straight line
Exercise E, Question 1

Question:

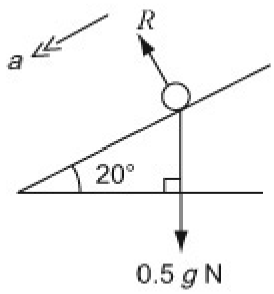
A particle of mass 0.5 kg is placed on a smooth inclined plane. Given that the plane makes an angle of 20° with the horizontal, find the acceleration of the particle.

Solution:

a

R (↙) , $0.5g \cos 70°$ $= 0.5a$

a $= 3.35$ (3 s.f.)



The acceleration of the particle is 3.35 m s^{-2} (3 s.f.) .

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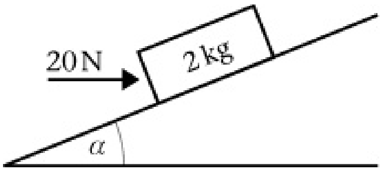
Dynamics of a particle moving in a straight line
Exercise E, Question 2

Question:

The diagram shows a box of mass 2 kg being pushed up a smooth plane by a horizontal force of magnitude 20 N. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.



Find

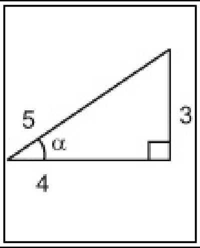
a the normal reaction between the box and the plane,

b the acceleration of the box up the plane.

Solution:

a

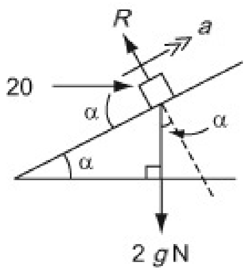
$$\begin{aligned} &R(\nwarrow), \quad R - 20\cos(90° - \alpha) - 2g\cos\alpha = 0 \\ &R = 20\sin\alpha + 19.6\cos\alpha \\ &\quad = 12 + 15.68 \\ &\quad = 27.7 \text{ N } (3 \text{ s.f.}) \end{aligned}$$



$\tan \alpha = \frac{3}{4}$

$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$



The normal reaction is 27.7 N (3 s.f.).

b

$$\begin{aligned} R(\nearrow), \quad 20\cos\alpha - 2g\cos(90° - \alpha) &= 2a \\ 20\cos\alpha - 2g\sin\alpha &= 2a \\ a &= 2.12 \text{ m s}^{-2} \end{aligned}$$

The acceleration of the box is 2.12 m s^{-2}.

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Dynamics of a particle moving in a straight line
Exercise E, Question 3

Question:

A boy of mass 40 kg slides from rest down a straight slide of length 5 m. The slide is inclined to the horizontal at an angle of 20°. The coefficient of friction between the boy and the slide is 0.1. By modelling the boy as a particle, find

a the acceleration of the boy,

b the speed of the boy at the bottom of the slide.

Solution:

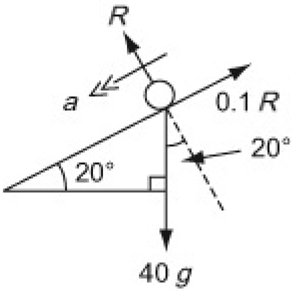
a

$$\text{R}(\swarrow),\quad 40g\cos 70^\circ - 0.1R = 40a$$
$$\text{R}(\nwarrow),\quad R - 40g\cos 20^\circ = 0$$
$$R = 368.36$$

Substituting,

$$392\cos 70^\circ - 36.836 = 40a$$
$$a = 2.43\ (3\text{ s.f.})$$

The acceleration of the boy is 2.43 m s^{-2} (3 s.f.).



b $u = 0$, $a = 2.43$, $s = 5$, $u = ?$

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2 \times 2.43 \times 5 = 24.3$$
$$u = 4.93\text{ m s}^{-1}\ (3\text{ s.f.})$$

The speed of the boy is 4.93 m s^{-1} (3 s.f.).

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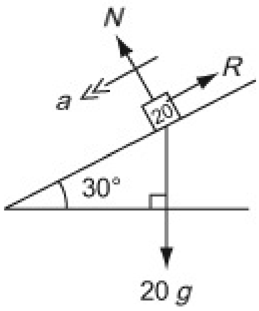
Dynamics of a particle moving in a straight line
Exercise E, Question 4

Question:

A block of mass 20 kg is released from rest at the top of a rough slope. The slope is inclined to the horizontal at an angle of 30°. After 6 s the speed of the block is 21 m s^{-1}. As the block slides down the slope it is subject to a constant resistance of magnitude *R* N. Find the value of *R*.

Solution:



$u = 0$, $v = 21$, $t = 6$, $a = ?$

$v = u + at$

$21 = 0 + 6a$

$a = 3.5 \text{ m s}^{-2}$

R (↙) , $20g \cos 60° - R = 20 \times 3.5$

$98 - R = 70$

$R = 28$ N

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Dynamics of a particle moving in a straight line
Exercise E, Question 5

Question:

A book of mass 2 kg slides down a rough plane inclined at 20° to the horizontal. The acceleration of the book is 1.5 m s $^{-2}$. Find the coefficient of friction between the book and the plane.

Solution:

R (↖) , $R - 2g \cos 20° = 0$

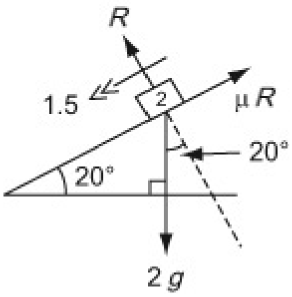
$R = 2g \cos 20°$

R (↙) ,

$2g \cos 70° - \mu \times 2g \cos 20° = 2 \times 1.5$

$\mu = \dfrac{2g \cos 70° - 3}{2g \cos 20°} = 0.201$ (3 s.f.)

The coefficient of friction is 0.20 (2 s.f.).



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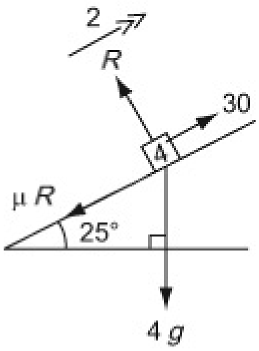
Dynamics of a particle moving in a straight line
Exercise E, Question 6

Question:

A block of mass 4 kg is pulled up a rough slope, inclined at 25° to the horizontal, by means of a rope. The rope lies along the line of greatest slope. The tension in the rope is 30 N. Given that the acceleration of the block is 2 m s $^{-2}$ find the coefficient of friction between the block and the plane.

Solution:



R (↖) , $R - 4g \cos 25° = 0$

$R = 4g \cos 25°$

R (↗) , $30 - 4g \cos 65° - \mu 4g \cos 25° = 4 \times 2$

$\dfrac{22 - 4g \cos 65°}{4g \cos 25°} = \mu$

$0.15 \ (2 \text{ s.f.}) = \mu$

The coefficient of friction is 0.15 (2 s.f.).

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Dynamics of a particle moving in a straight line
Exercise E, Question 7

Question:

A parcel of mass 10 kg is released from rest on a rough plane which is inclined at 25° to the horizontal.

a Find the normal reaction between the parcel and the plane.

Two seconds after being released the parcel has moved 4 m down the plane.

b Find the coefficient of friction between the parcel and the plane.

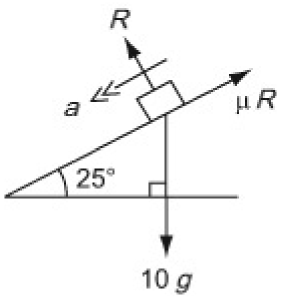
Solution:

a

R (↖) , $R - 10g \cos 25° = 0$

$R = 98 \cos 25° = 88.8$ N (3 s.f.)

The normal reaction is 88.8 N (3 s.f.).



b

$u = 0$, $s = 4$, $t = 2$, $a = ?$

$s = ut + \frac{1}{2}at^2$

$4 = 0 + \frac{1}{2}a \times 2^2$

$a = 2 \text{ m s}^{-2}$

R (↙) , $10g \cos 65° - \mu R = 10 \times 2$

$\mu \times 98 \cos 25° = 10g \cos 65° - 20$

$\mu = \frac{98 \cos 65° - 20}{98 \cos 25°}$

$= 0.241$ (3 s.f.)

The coefficient of friction is 0.24 (2 s.f.).

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Dynamics of a particle moving in a straight line
Exercise E, Question 8

Question:

A particle P is projected up a rough plane which is inclined at an angle α to the horizontal, where tan $\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{3}$. The particle is projected from the point A with speed 20 m $s^{-1}$and comes to instantaneous rest at the point B.

a Show that while P is moving up the plane its deceleration is $\frac{13g}{15}$.

b Find, to three significant figures, the distance AB.

c Find, to three significant figures, the time taken for P to move from A to B.

d Find the speed of P when it returns to A.

Solution:

a

Let mass of particle be m.

$R(\nwarrow)$, $R - mg\cos\alpha = 0$

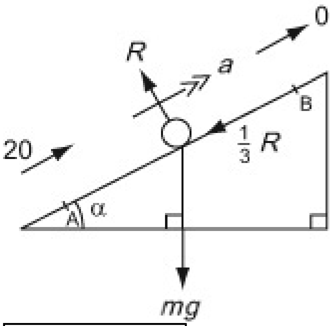
$R = \frac{4mg}{5}$

$R(\nearrow)$, $-mg\sin\alpha - \frac{1}{3}R = ma$

$-\frac{3\cancel{m}g}{5} - \frac{1}{3}\times\frac{4\cancel{m}g}{5} = \cancel{m}a$

$-\frac{13g}{15} = a$

The deceleration is $\frac{13g}{15}$.



$\tan\alpha$	=	$\frac{3}{4}$
$\sin\alpha$	=	$\frac{3}{5}$
$\cos\alpha$	=	$\frac{4}{5}$

b

$u = 20$, $v = 0$, $a = -\frac{13g}{15}$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 20^2 - \frac{26g}{15}s \Rightarrow s = \frac{6000}{26g} = 23.5$ m (3 s.f.)

$AB = 23.5$ m (3 s.f.)

c

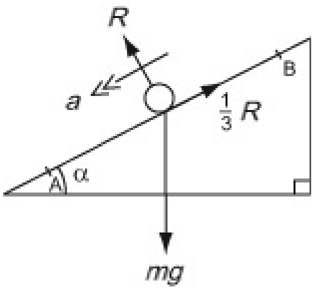
$u = 20$, $v = 0$, $a = \frac{-13g}{15}$, $t =$?

$v = u + a\ t$

$0 = 20 - \frac{13g\ t}{15}$

$t = \frac{300}{13g} = 2.35$ s (3 s.f.)

d



$R = \frac{4\ m\ g}{5}$ as before

R (↙) , $m\ g\ \sin\ \alpha - \frac{1}{3}R = m\ a$

$\frac{3 m\ g}{5} - \frac{1}{3} - \frac{4 m\ g}{5} = m\ a$

$\frac{g}{3} = a$

$u = 0$, $a = \frac{g}{3}$, $s = \frac{6000}{26g}$, $v =$?

$v^2 = u^2 + 2a\ s$

$v^2 = 0 + \frac{2g}{3} \times \frac{6000}{26g} = \frac{4000}{26}$

$u = 12.4$ m s^{-1} (3 s.f.)

The speed of the particle as it passes A on the way down is 12.4 m s^{-1} (3 s.f.).

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Dynamics of a particle moving in a straight line
Exercise F, Question 1

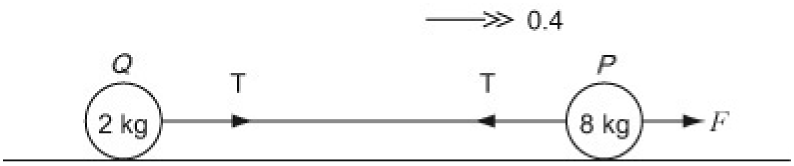
Question:

Two particles P and Q of mass 8 kg and 2 kg respectively, are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of magnitude F is applied to P in a direction away from Q and when the string is taut the particles move with acceleration 0.4 m s^{-2}.

a Find the value of F.

b Find the tension in the string.

Solution:



For whole system:

a

$$\begin{aligned} R(\rightarrow): \quad F &= (2+8)\times 0.4 \\ &= 4 \end{aligned}$$

Hence F is 4 N.

b For Q:

$$\begin{aligned} R(\rightarrow): \quad T &= 2\times 0.4 \\ &= 0.8 \end{aligned}$$

The tension in the string is 0.8 N.

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise F, Question 2

Question:

Two bricks P and Q, each of mass 5 kg, are connected by a light inextensible string. Brick P is held at rest and Q hangs freely, vertically below P. A force of 200 N is then applied vertically upwards to P causing it to accelerate at 1.2 m s^{-2}. Assuming there is a resistance to the motion of each of the bricks of magnitude R N, find

a the value of R,

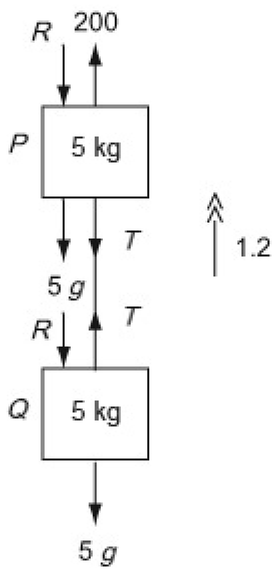
b the tension in the string connecting the bricks.

Solution:

a

For whole system
R (↑) ,

$$200 - 2R - 10g = 10 \times 1.2$$
$$200 - 2R - 98 = 12$$
$$2R = 90$$
$$R = 45$$



b For Q only:

R (↑) , $T - R - 5g = 5 \times 1.2$
$$T - 45 - 49 = 6$$
$$T = 100$$

∴ The tension in the string is 100 N.

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Dynamics of a particle moving in a straight line
Exercise F, Question 3

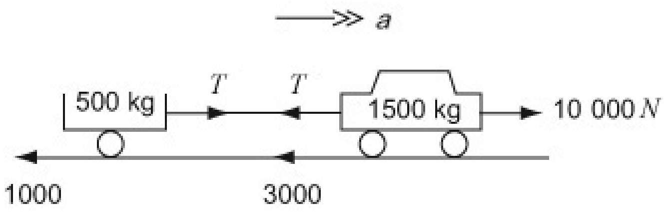
Question:

A car of mass 1500 kg is towing a trailer of mass 500 kg along a straight horizontal road. The car and the trailer are connected by a light inextensible tow-bar. The engine of the car exerts a driving force of magnitude 10 000 N and the car and the trailer experience resistances of magnitudes 3000 N and 1000 N respectively.

a Find the acceleration of the car.

b Find the tension in the tow-bar.

Solution:



a For whole system:

$$\begin{aligned} R(\rightarrow), \quad 10\,000 - 1000 - 3000 &= 2000a \\ a &= 3 \end{aligned}$$

The acceleration of the car is 3 m s^{-2}.

b For trailer:

$$\begin{aligned} R(\rightarrow), \quad T - 1000 &= 500 \times 3 \\ T &= 2500 \end{aligned}$$

The tension in the tow-bar is 2500 N.

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Dynamics of a particle moving in a straight line
Exercise F, Question 4

Question:

Two particles *A* and *B* of mass 4 kg and 3 kg respectively are connected by a light inextensible string which passes over a small smooth fixed pulley. The particles are released from rest with the string taut.

a Find the tension in the string.

When *A* has travelled a distance of 2 m it strikes the ground and immediately comes to rest.

b Assuming that *B* does not hit the pulley find the greatest height that *B* reaches above its initial position.

Solution:

a

For A : R ($\downarrow$) , $4g - T = 4a$ (1)

For B : R ($\uparrow$) , $T - 3g = 3a$ (2)

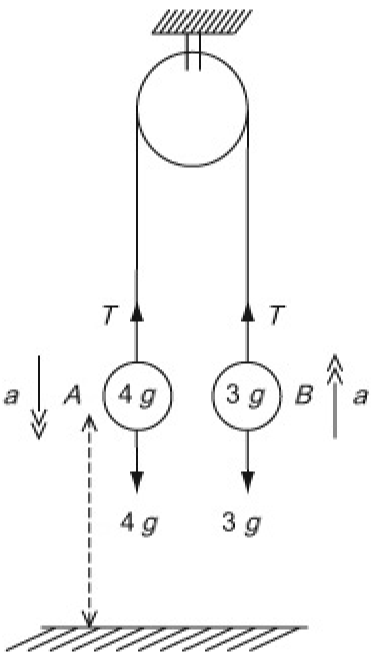
(1) + (2) : $4g - 3g = 7a$

$\frac{1}{7}g = a$

i.e. $a = 1.4 \text{ m s}^{-2}$

$\therefore T = 3g + \frac{3g}{7}$

$= 33.6$ N (3 s.f.)



The tension in the string is 33.6 N (3 s.f.).

b For *A*: ($\downarrow$) $u = 0$, $s = 2$, $a = \frac{5}{7}$, $v =$?

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + \left(2 \times \frac{5}{7} \times 2 \right)$$

$$= \frac{4g}{7}$$

For *B*: ($\uparrow$) $u^2 = \frac{4g}{7}$, $v = 0$, $a = -g$, $s =$?

$$v^2 = u^2 + 2\,a\,\,s$$

$$0 = \frac{4g}{7} - 2g\,s \Rightarrow s = \frac{2}{7}$$

$\therefore$ Height above initial position is 2 $\frac{2}{7}$ m.

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise F, Question 5

Question:

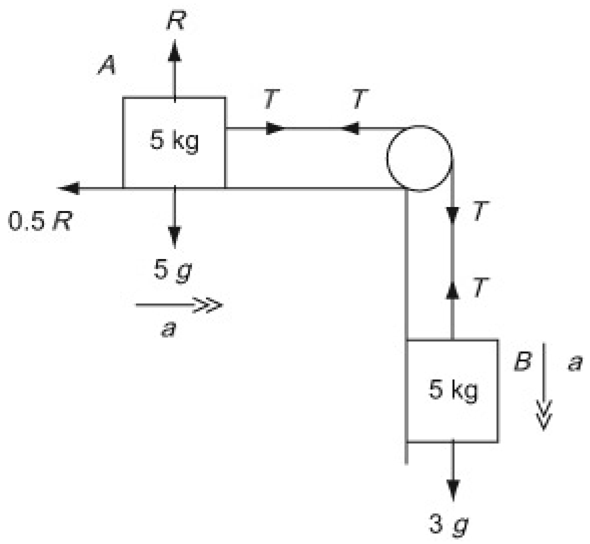
Two particles *A* and *B* of mass 5 kg and 3 kg respectively are connected by a light inextensible string. Particle *A* lies on a rough horizontal table and the string passes over a small smooth pulley which is fixed at the edge of the table. Particle *B* hangs freely. The coefficient of friction between *A* and the table is 0.5. The system is released from rest. Find

a the acceleration of the system,

b the tension in the string,

c the magnitude of the force exerted on the pulley by the string.

Solution:



a For *A*:

$$R(\uparrow),\ R - 5g = 0$$
$$R = 49$$

$$R(\rightarrow),\quad T - 0.5R = 5a$$
$$\text{i.e. } T - 24.5 = 5a \qquad (1)$$

$$\text{For } B:\quad R(\downarrow),\quad 3g - T = 3a$$
$$29.4 - T = 3a \qquad (2)$$

$$(1) + (2):\quad 29.4 - 24.5 = 8a$$
$$4.9 = 8a$$
$$0.6125 = a$$

The acceleration of the system is $0.613\ \text{m s}^{-2}$ (3 s.f.) or $0.61\ \text{m s}^{-2}$ (2 s.f.).

b

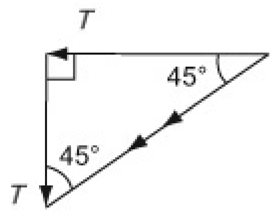
$$T - 24.5 = 5 \times 0.6125$$
$$T = 27.5625$$

The tension in the string is 27.6 N (3 s.f.) or 28 N (2 s.f.).

c

By Pythagoras,

$$F^2 = T^2 + T^2 = 2T^2$$
$$F = T\sqrt{2}$$
$$= 38.979 \ldots$$



The magnitude of the force exerted on the pulley is 39 N (2 s.f.) or 39.0 N (3 s.f.).

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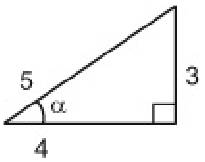
Dynamics of a particle moving in a straight line
Exercise F, Question 6

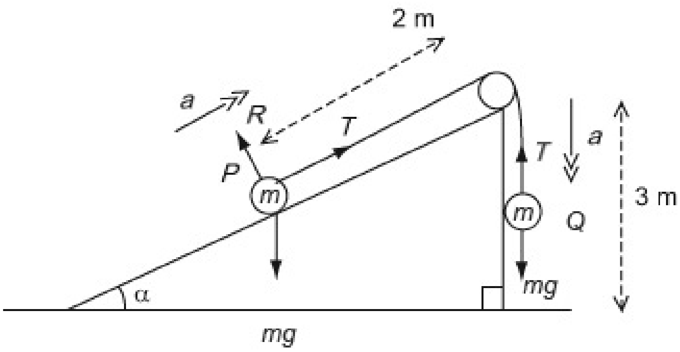
Question:

Two particles P and Q of equal mass are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a smooth inclined plane. The plane is inclined to the horizontal at an angle α where $\tan \alpha = 0.75$. Particle P is held at rest on the inclined plane at a distance of 2 m from the pulley and Q hangs freely on the edge of the plane at a distance of 3 m above the ground with the string vertical and taut. Particle P is released. Find the speed with which it hits the pulley.

Solution:

$\tan \alpha = \frac{3}{4}$





$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$

For P : R (↗) , $T - mg\sin\alpha = ma$

$$T - \frac{3mg}{5} = ma \qquad (1)$$

For Q : R (↓) , $mg - T = ma \qquad (2)$

(1) + (2) : $mg - \frac{3mg}{5} = 2ma$

$$\frac{g}{5} = a$$

For P: $u = 0$, $a = \frac{g}{5}$, $s = 2$, $v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + \frac{2g}{5} \times 2$

$v = \sqrt{\frac{4g}{5}} = 2.8 \text{ m s}^{-1}$

P hits the pulley with speed 2.8 m s^{-1}.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise F, Question 7

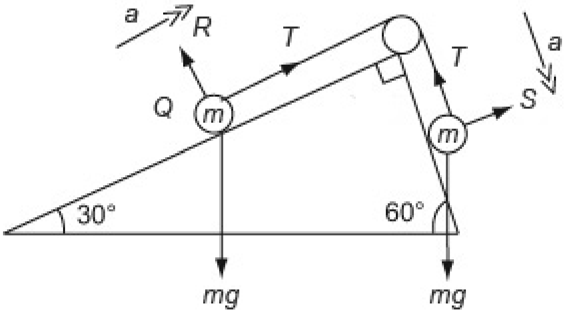
Question:

Two particles P and Q of equal mass are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed wedge. One face of the wedge is smooth and inclined to the horizontal at an angle of 30° and the other face of the wedge is rough and inclined to the horizontal at an angle of 60°. Particle P lies on the rough face and particle Q lies on the smooth face with the string connecting them taut. The coefficient of friction between P and the rough face is 0.5.

a Find the acceleration of the system.

b Find the tension in the string.

Solution:



a

For Q : R ($\rightarrow$) , $T - mg\cos 60° = ma$ (1)

For P : R ($\nearrow$) , $S = mg\cos 60°$

R ($\searrow$) , $mg\cos 30° - \frac{1}{2}S - T = ma$

$mg\cos 30° - \frac{1}{2}mg\cos 60° - T = ma$ (2)

(1) + (2) : $\cancel{m}g\frac{\sqrt{3}}{2} - \frac{3\cancel{m}g}{4} = 2\cancel{m}a$

$\boxed{\cos 30° = \frac{\sqrt{3}}{2} \quad \cos 60° = \frac{1}{2}}$

$\frac{g}{8}(2\sqrt{3} - 3) = a$

$\therefore$ The acceleration of the system is 0.569 m s^{-2} (3 s.f.)
or 0.57 m s^{-2} (2 s.f.)

b From (1),

$$T = \frac{1}{2}mg + \frac{mg}{8}(2\sqrt{3} - 3)$$

$$= \frac{mg}{8}(1 + 2\sqrt{3})$$

The tension in the string is $\frac{mg}{8}\left(1+2\sqrt{3}\right) = 0.56mg$.

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Dynamics of a particle moving in a straight line
Exercise F, Question 8

Question:

A van of mass 900 kg is towing a trailer of mass 500 kg up a straight road which is inclined to the horizontal at an angle α where $\tan \alpha = 0.75$. The van and the trailer are connected by a light inextensible tow-bar. The engine of the van exerts a driving force of magnitude 12 kN and the van and the trailer experience resistances to motion of magnitudes 1600 N and 600 N respectively.

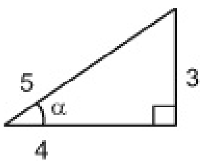
a Find the acceleration of the van.

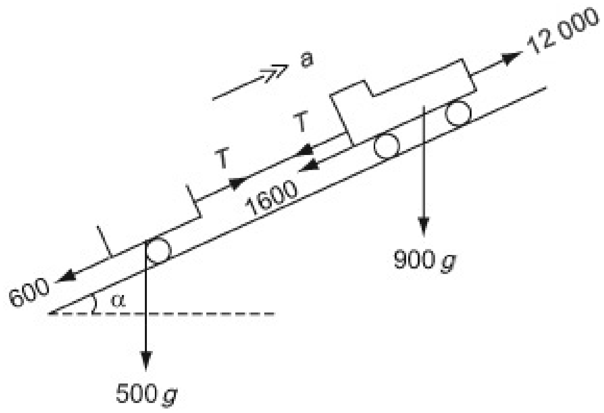
b Find the tension in the tow-bar.

Solution:

a

$\tan \alpha = 0.75 = \frac{3}{4}$





$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$

For whole system:

$$
\begin{aligned}
R(\nearrow),\ 12\,000 - 1600 - 600 - 1400g\sin \alpha &= 1400a \\
9800 - 1400g\sin \alpha &= 1400a \\
7 - 5.88 &= a \\
1.12 &= a
\end{aligned}
$$

The acceleration of the van is 1.12 m s^{-2}.

b For trailer:

$$
\begin{aligned}
R(\nearrow),\quad T - 600 - 500g\sin \alpha &= 500 \times 1.12 \\
T - 600 - 2940 &= 560 \\
T &= 4100
\end{aligned}
$$

The tension in the tow-bar is 4100 N.

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Dynamics of a particle moving in a straight line
Exercise F, Question 9

Question:

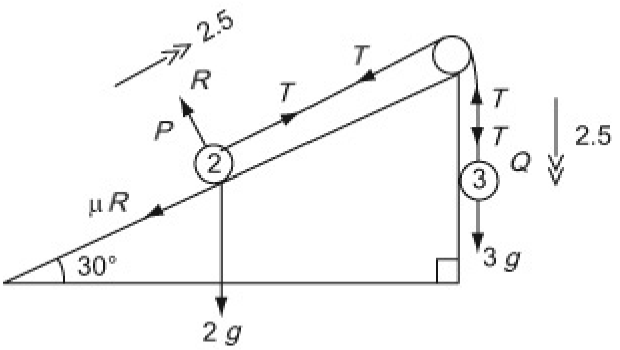
Two particles P and Q of mass 2 kg and 3 kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of 30°. Particle P is held at rest on the inclined plane and Q hangs freely on the edge of the plane with the string vertical and taut. Particle P is released and it accelerates up the plane at 2.5 m s^{-2}. Find

a the tension in the string,

b the coefficient of friction between P and the plane,

c the force exerted by the string on the pulley.

Solution:



a For P:

$$R(\nwarrow),\quad R - 2g\cos 30° = 0$$
$$R = g\sqrt{3}$$
$$R(\nearrow),\quad T - \mu g\sqrt{3} - 2g\cos 60° = 2 \times 2.5$$
$$T - \mu g\sqrt{3} - g = 5 \qquad (1)$$

$$\boxed{\cos 30° = \frac{\sqrt{3}}{2}}$$
$$\boxed{\cos 60° = \frac{1}{2}}$$

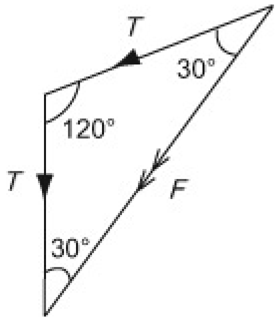
$$\text{For } Q: R(\downarrow),\ 3g - T = 3 \times 2.5$$
$$3g - T = 7.5 \qquad (2)$$

$\therefore T = 21.9$
The tension is 21.9 N.

b
$$(1) + (2) \quad 2g - \mu g\sqrt{3} = 12.5$$
$$\mu g\sqrt{3} = 7.1$$
$$\mu = \frac{7.1}{g\sqrt{3}} = 0.418 \ (3 \text{ s.f.}) \text{ or } 0.42 \ (2 \text{ s.f.})$$

The coefficient of friction is 0.42 (2 s.f.).

c



$$\begin{aligned} F &= 2T\cos 30° \\ &= 43.8\cos 30° \\ &= 37.9\text{ N (3 s.f.) or } 38\text{ N (2 s.f.)} \end{aligned}$$

The force exerted by the string on the pulley is 38 N (2 s.f.).

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Dynamics of a particle moving in a straight line
Exercise F, Question 10

Question:

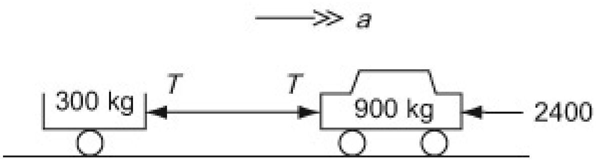
A car of mass 900 kg is towing a trailer of mass 300 kg along a straight horizontal road. The car and the trailer are connected by a light inextensible tow-bar and when the speed of the car is 20 m s^{-1}the brakes are applied. This produces a braking force of 2400 N. Find

a the deceleration of the car,

b the magnitude of the force in the tow-bar,

c the distance travelled by the car before it stops.

Solution:



a For whole system:

$$\begin{aligned} R(\rightarrow), \quad -2400 &= 1200a \\ a &= -2 \end{aligned}$$

The deceleration of the car is 2 m s^{-2}.

b For trailer:

$$\begin{aligned} R(\rightarrow), \quad -T &= 300 \times -2 \\ \Rightarrow T &= 600 \end{aligned}$$

The thrust in the tow-bar is 600 N.

c $u = 20$, $a = -2$, $v = 0$, $s = ?$

$$\begin{aligned} v^2 &= u^2 + 2as \ (\rightarrow), \\ 0^2 &= 20^2 + 2(-2)s \\ 0 &= 400 - 4s \\ s &= 100 \end{aligned}$$

The car stops in 100 m.

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Dynamics of a particle moving in a straight line
Exercise G, Question 1

Question:

A ball of mass 0.5 kg is at rest when it is struck by a bat and receives an impulse of 15 N s. Find its speed immediately after it is struck.

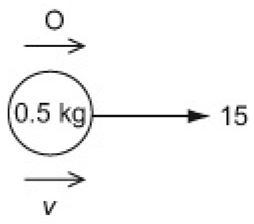
Solution:

$(\rightarrow)$:

$15 = 0.5v$

$30 = v$

Its initial speed is 30 m s^{-1}.



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Dynamics of a particle moving in a straight line
Exercise G, Question 2

Question:

A ball of mass 0.3 kg moving along a horizontal surface hits a fixed vertical wall at right angles with speed 3.5 m s^{-1}. The ball rebounds at right angles to the wall. Given that the magnitude of the impulse exerted on the ball by the wall is 1.8 N s, find the speed of the ball just after it has hit the wall.

Solution:

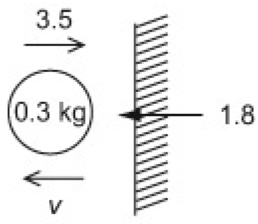
$(\leftarrow)$:

$1.8 = 0.3(v - -3.5)$

$6 = v + 3.5$

$2.5 = v$

The ball rebounds with speed 2.5 m s^{-1}.



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Dynamics of a particle moving in a straight line
Exercise G, Question 3

Question:

A ball of mass 0.2 kg is dropped from a height of 2.5 m above horizontal ground. After hitting the ground it rises to a height of 1.8 m above the ground. Find the magnitude of the impulse received by the ball from the ground.

Solution:

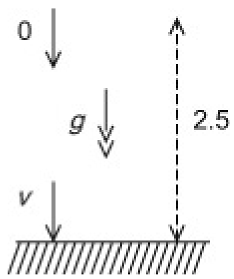
$u = 0$, $a = g$, $s = 2.5$, $v =$?

$v^2 = u^2 + 2as$ (↓)

$v^2 = 0^2 + 2 \times 9.8 \times 2.5$

$= 49$

$v = 7$ (not -7)



(↑) : $v = 0$, $a = -g$, $s = 1.8$, $u =$?

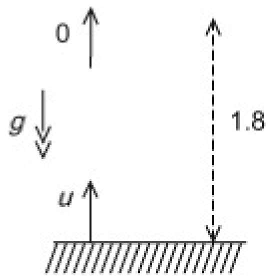
$v^2 = u^2 + 2as$

$0^2 = u^2 + 2(-9.8) \times 1.8$

$u^2 = 35.28$

$u = 5.94$ (not -5.94)

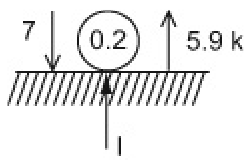
Take downwards as positive.



(↑) : $I = 0.2(5.94 - 7)$

$= -2.588$

The magnitude of the impulse on the ball is 2.59 N s (3 s.f.).



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Dynamics of a particle moving in a straight line
Exercise G, Question 4

Question:

A ball of mass 0.2 kg, moving along a horizontal surface, hits a fixed vertical wall at right angles. The ball rebounds at right angles to the wall with speed 3.5 m s $^{-1}$. Given that the magnitude of the impulse exerted on the ball by the wall is 2 N s, find the speed of the ball just before it hit the wall.

Solution:

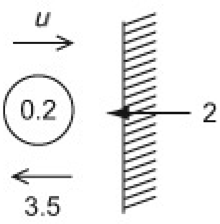
(←) :

$2 = 0.2 (3.5 - - u)$

$10 = 3.5 + u$

$u = 6.5$

The speed of the ball before it hits the wall is 6.5 m s $^{-1}$.



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Dynamics of a particle moving in a straight line
Exercise G, Question 5

Question:

A toy car of mass 0.2 kg is pushed from rest along a smooth horizontal floor by a horizontal force of magnitude 0.4 N for 1.5 s. Find its speed at the end of the 1.5 s.

Solution:

$$\begin{aligned} Ft &= mv - mu \\ 0.4 \times 1.5 &= 0.2(v - 0) \\ 0.6 &= 0.2v \\ 3 &= v \end{aligned}$$

The speed of the toy car is 3 m s $^{-1}$.

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Dynamics of a particle moving in a straight line
Exercise H, Question 1

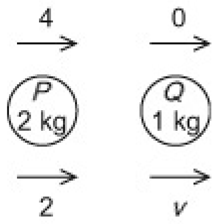
Question:

A particle P of mass 2 kg is moving on a smooth horizontal plane with speed 4 m s^{-1}. It collides with a second particle Q of mass 1 kg which is at rest. After the collision P has speed 2 m s^{-1}and it continues to move in the same direction. Find the speed of Q after the collision.

Solution:

Conservation of Momentum ($\rightarrow$)

$$(2 \times 4) + (1 \times 0) = (2 \times 2) + (1 \times v)$$
$$8 = 4 + v$$
$$4 = v$$



The speed of Q is 4 m s^{-1}.

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Dynamics of a particle moving in a straight line
Exercise H, Question 2

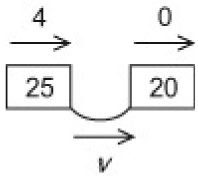
Question:

A railway truck of mass 25 tonnes moving at 4 m s^{-1} collides with a stationary truck of mass 20 tonnes. As a result of the collision the trucks couple together. Find the common speed of the trucks after the collision.

Solution:

Conservation of Momentum ($\rightarrow$)

$$(4 \times 25) + (20 \times 0) = 45v$$
$$100 = 45v$$
$$\frac{20}{9} = v$$



The common speed of the trucks is $2\frac{2}{9}$ m s^{-1}.

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Dynamics of a particle moving in a straight line
Exercise H, Question 3

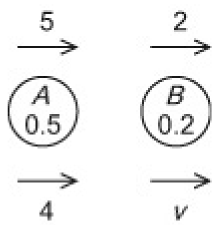
Question:

Particles *A* and *B* have mass 0.5 kg and 0.2 kg respectively. They are moving with speeds 5 m s^{-1} and 2 m s^{-1} respectively in the same direction along the same straight line on a smooth horizontal surface when they collide. After the collision *A* continues to move in the same direction with speed 4 m s^{-1}. Find the speed of *B* after the collision.

Solution:

Conservation of Momentum ($\rightarrow$)

$(0.5 \times 5) + (0.2 \times 2)$	$= (0.5 \times 4) + (0.2 \times v)$
$2.5 + 0.4$	$= 2.0 + 0.2v$
0.9	$= 0.2v$
4.5	$= v$



The speed of *B* after the collision is 4.5 m s^{-1}.

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Dynamics of a particle moving in a straight line
Exercise H, Question 4

Question:

A particle of mass 2 kg is moving on a smooth horizontal plane with speed 4 m s^{-1}. It collides with a second particle of mass 1 kg which is at rest. After the collision the particles join together.

a Find the common speed of the particles after the collision.

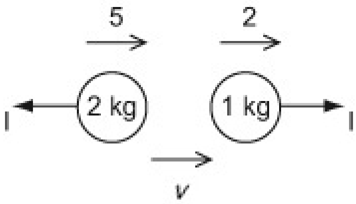
b Find the magnitude of the impulse in the collision.

Solution:

a

Conservation of Momentum $(\rightarrow)$

$$(2 \times 4) + (1 \times 0) = 3v$$
$$8 = 3v$$
$$\frac{8}{3} = v$$



The common speed of the particles is $2\frac{2}{3}$ m s^{-1}.

b For 1 kg: $\left(\rightarrow \right)$ $I = 1 \times v = 2\frac{2}{3}$ m s^{-1}

The impulse in the collision is $2\frac{2}{3}$ m s^{-1} N s.

[For 2 kg: $(\leftarrow)$ $I = 2(-v - -4)$

$$= 2\left(-2\frac{2}{3} + 4\right)$$

$$= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$$]

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise H, Question 5

Question:

Two particles *A* and *B* of mass 2 kg and 5 kg respectively are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speeds of *A* and *B* are 6 m s^{-1} and 4 m s^{-1} respectively. After the collision the direction of motion of *A* is reversed and its speed is 1.5 m s^{-1}. Find

a the speed and direction of *B* after the collision,

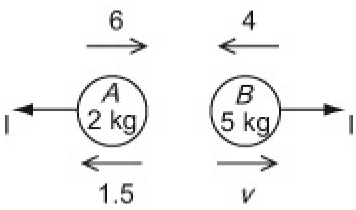
b the magnitude of the impulse given by *A* to *B* in the collision.

Solution:

a

Conservation of Momentum $(\rightarrow)$

$$\begin{aligned} (2 \times 6) + (5 \times -4) &= (2 \times -1.5) + 5v \\ 12 - 20 &= -3 + 5v \\ -5 &= 5v \\ v &= -1 \end{aligned}$$



The speed of *B* is 1 m s^{-1} and its direction of motion is unchanged by the collision.

b

$$\begin{aligned} \text{For } A \ (\leftarrow): \ I &= 2(1.5 - -6) \\ &= 2 \times 7.5 \\ &= 15 \end{aligned}$$

$$\begin{aligned} [\text{ or for } B \ (\rightarrow): \ I &= 5(v - -4) \\ &= 5(-1 + 4) \\ &= 15 \ ] \end{aligned}$$

The magnitude of the impulse given to *B* is 15 N s.

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Dynamics of a particle moving in a straight line
Exercise H, Question 6

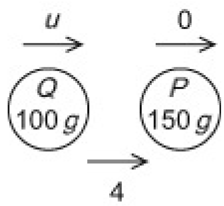
Question:

A particle P of mass 150 g is at rest on a smooth horizontal plane. A second particle Q of mass 100 g is projected along the plane with speed u m s^{-1}and collides directly with P. On impact the particles join together and move on with speed 4 m s^{-1}. Find the value of u.

Solution:

Conservation of Momentum ($\rightarrow$)

$$100u + (150 \times 0) = 250 \times 4$$
$$100u = 1000$$
$$u = 10$$



The value of u is 10.

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Dynamics of a particle moving in a straight line
Exercise H, Question 7

Question:

A particle A of mass $4m$ is moving along a smooth horizontal surface with speed $2u$. It collides with another particle B of mass $3m$ which is moving with the same speed along the same straight line but in the opposite direction. Given that A is brought to rest by the collision, find

a the speed of B after the collision and state its direction of motion,

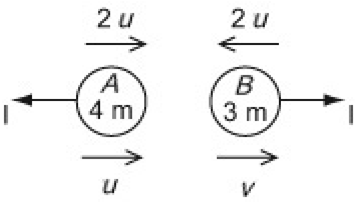
b the magnitude of the impulse given by A to B in the collision.

Solution:

a

Conservation of Momentum ($\rightarrow$)

$$(4m \times 2u) + (3m \times -2u) = (4m \times 0) + 3mv$$
$$8mu - 6mu = 3mv$$
$$\frac{2u}{3} = v$$



The speed of B after the collision is $\frac{2u}{3}$ and its direction of motion is reversed by the collision.

b

For A ($\leftarrow$): $I = 4m (0 - -2u)$
$= 8mu$

[or For B ($\rightarrow$): $I = 3m (v - -2u)$
$= 3m \left(\frac{2u}{3} + 2u \right)$
$= 2mu + 6mu = 8mu$]

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Dynamics of a particle moving in a straight line
Exercise H, Question 8

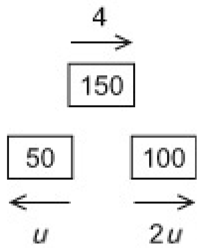
Question:

An explosive charge of mass 150 g is designed to split into two parts, one with mass 100 g and the other with mass 50 g. When the charge is moving at 4 m s^{-1} it splits and the larger part continues to move in the same direction whilst the smaller part moves in the opposite direction. Given that the speed of the larger part is twice the speed of the smaller part, find the speeds of the two parts.

Solution:

Conservation of Momentum ($\rightarrow$)

$$
\begin{aligned}
(150 \times 4) &= (100 \times 2u) + (50 \times -u) \\
600 &= 200u - 50u \\
600 &= 150u \\
4 &= u
\end{aligned}
$$



The larger has speed 8 m s^{-1} and the smaller part has speed 4 m s^{-1}.

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Dynamics of a particle moving in a straight line
Exercise H, Question 9

Question:

Two particles P and Q of mass m and km respectively are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speeds of P and Q are $3u$ and u respectively. After the collision the direction of motion of both particles is reversed and the speed of each particle is halved.

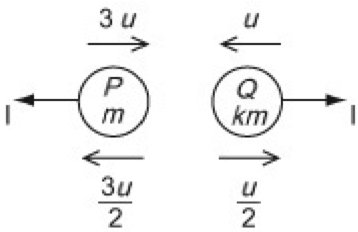
a Find the value of k.

b Find, in terms of m and u, the magnitude of the impulse given by P to Q in the collision.

Solution:

a

Conservation of Momentum ($\rightarrow$)



$$\left(m \times 3u \right) + \left(k\ m \times -u \right) = \left(m \times \frac{3u}{2} \right) + \left(k\ m \times \frac{u}{2} \right)$$

$$3mu - kmu = -3\frac{mu}{2} + \frac{kmu}{2}$$

$$6 - 2k = -3 + k$$

$$9 = 3k$$

$$3 = k$$

The value of k is 3.

b

For P ($\leftarrow$): $I = m\left(\frac{3u}{2} - -3u \right) = \frac{9mu}{2}$

[or For Q: ($\rightarrow$) $I = k\ m\left(\frac{u}{2} - -u \right) = 3m \times \frac{3u}{2} = \frac{9mu}{2}$]

The magnitude of the impulse is $\frac{9mu}{2}$.

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Dynamics of a particle moving in a straight line
Exercise H, Question 10

Question:

Two particles A and B of mass 4 kg and 2 kg respectively are connected by a light inextensible string. The particles are at rest on a smooth horizontal plane with the string slack. Particle A is projected directly away from B with speed u m s^{-1}. When the string goes taut the impulse transmitted through the string has magnitude 6 N s. Find

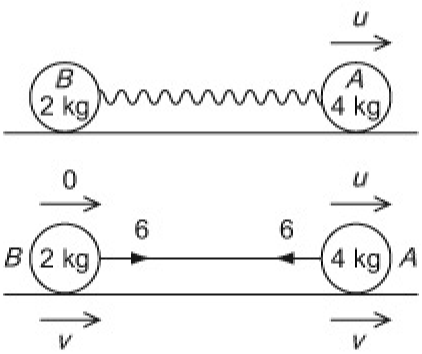
a the common speed of the particles just after the string goes taut,

b the value of u.

Solution:

a

For B $(\rightarrow)$:
$6 = 2(v - 0)$
$3 = v$



The common speed is 3 m s^{-1}.

b Conservation of Momentum $(\rightarrow)$

$4u = 2v + 4v = 6 \times 3 = 18$
$u = 4.5$

[or For A : $(\rightarrow)$ $-6 = 4(3 - u)$
$-1.5 = 3 - u$
$u = 4.5$]

The value of u is 4.5.

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Dynamics of a particle moving in a straight line
Exercise H, Question 11

Question:

Two particles P and Q of mass 3 kg and 2 kg respectively are moving along the same straight line on a smooth horizontal surface. The particles collide. After the collision both the particles are moving in the same direction, the speed of P is 1 m s^{-1} and the speed of Q is 1.5 m s^{-1}. The magnitude of the impulse of P on Q is 9 N s. Find

a the speed and direction of P before the collision,

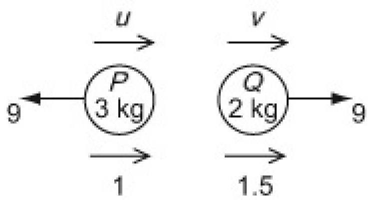
b the speed and direction of Q before the collision.

Solution:

a

For P: $(\leftarrow)$

$$\begin{aligned} 9 &= 3(-1 - -u) \\ 9 &= 3(-1 + u) \\ 3 &= -1 + u \\ 4 &= u \end{aligned}$$



The speed of P before the collision is 4 m s^{-1} and it was moving in the same direction as it was after the collision.

b For Q: $(\rightarrow)$

$$\begin{aligned} 9 &= 2(1.5 - v) \\ 9 &= 3 - 2v \\ 2v &= -6 \\ v &= -3 \end{aligned}$$

[or Conservation of Momentum $(\rightarrow)$

$$\begin{aligned} 3u + 2v &= (3 \times 1) + (2 \times 1.5) \\ 12 + 2v &= 3 + 3 = 6 \\ 2v &= -6 \\ v &= -3\,] \end{aligned}$$

The speed of Q before the collision was 3 m s^{-1} and it was moving in the opposite direction to its direction after the collision.

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Dynamics of a particle moving in a straight line
Exercise H, Question 12

Question:

Two particles A and B are moving in the same direction along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speed of B is 1.5 m s^{-1}. After the collision the direction of motion of both particles is unchanged, the speed of A is 2.5 m s^{-1}and the speed of B is 3 m s^{-1}. Given that the mass of A is three times the mass of B,

a find the speed of A before the collision.

Given that the magnitude of the impulse on A in the collision is 3 N s.

b find the mass of A.

Solution:

a

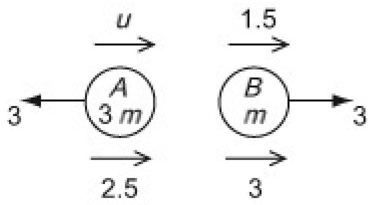
Conservation of Momentum $(\rightarrow)$

$$3m\ u + 1.5m = (3m \times 2.5) + (m \times 3)$$

$$3\cancel{m}\ u + 1.5\cancel{m} = 7.5\cancel{m} + 3\cancel{m}$$

$$3u = 9$$

$$u = 3$$



The speed of A before the collision is 3 m s^{-1}.

b For B: $(\rightarrow)$

$$3 = m(3 - 1.5)$$

$$2 = m$$

[or For A: $\leftarrow$

$$3 = 3m(-2.5 - -u)$$

$$3 = 3m(-2.5 + 3)$$

$$1 = 0.5m$$

$$2 = m\ ]$$

The mass of A is 6 kg.

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Dynamics of a particle moving in a straight line
Exercise I, Question 1

Question:

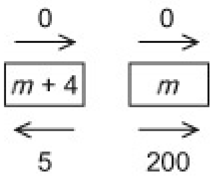
A bullet is fired by a gun which is 4 kg heavier than the bullet. Immediately after the bullet is fired, it is moving with speed 200 m s^{-1} and the gun recoils in the opposite direction with speed 5 m s^{-1}. Find

a the mass of the bullet,

b the mass of the gun.

Solution:

a



Draw a diagram showing velocities, before and after, with arrows.

CLM ($\rightarrow$) : $O = 200m - 5(m + 4)$

$20 = 195m$

$0.103 = m$

Momentum is conserved, solving for m.

Mass of bullet is 0.103 kg.

b Mass of gun is 4.103 kg.

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Dynamics of a particle moving in a straight line

Exercise I, Question 2

Question:

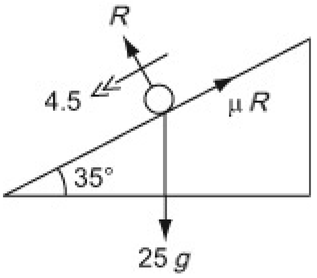
A child of mass 25 kg moves from rest down a slide which is inclined to the horizontal at an angle of 35°. When the child has moved a distance of 4 m, her speed is 6 m s^{-1}. By modelling the child as a particle, find the coefficient of friction between the child and the slide.

Solution:

$v^2 = u^2 + 2as$ — Choose the appropriate equation.

$6^2 = 0^2 + 2a \times 4$

$a = 4.5$ — Solving for a



$R(\nwarrow)\quad R - 25g\cos 35° = 0$ — We need R to find the friction.

$R = 25g\cos 35°$

$R(\swarrow)\quad 25g\cos 55° - \mu R = 25 \times 4.5$ — Resolve in the direction of the acceleration. Solve for μ.

$25g\cos 55° - \mu \times 25g\cos 35° = 112.5$

$$\mu = \frac{25g\cos 55° - 112.5}{25g\cos 35°}$$ — Answer to 2 s.f. since $g = 9.8$ has been used.

$= 0.14$ (2 s.f.)

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Dynamics of a particle moving in a straight line
Exercise I, Question 3

Question:

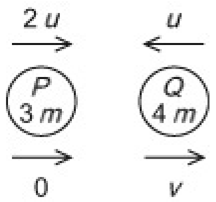
A particle *P* of mass 3*m* is moving along a straight line with constant speed 2*u*. It collides with another particle *Q* of mass 4*m* which is moving with speed *u* along the same line but in the opposite direction. As a result of the collision *P* is brought to rest.

a Find the speed of *Q* after the collision and state its direction of motion.

b Find the magnitude of the impulse exerted by *Q* on *P* in the collision.

Solution:

a



A diagram showing all the velocities, before and after, with arrows.

CLM (→) :

$$6mu - 4mu = 4mv$$

$$\frac{1}{2}u = v$$

Conserving momentum

b I = $(3m \times 2u - 0) = 6mu$ impulse *6 mu*

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Dynamics of a particle moving in a straight line
Exercise I, Question 4

Question:

A small box of mass 2 kg is projected with speed 2 m s^{-1} up a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α where tan α = 0.75. The coefficient of friction between the box and the plane is 0.4. The box is projected from the point *P* on the plane.

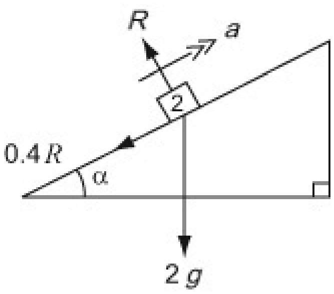
a Find the distance that the box travels up the plane before coming to rest.

b Show that the box will slide back down the plane.

c Find the speed of the box when it reaches the point *P*.

Solution:

a



R (↖) $R = 2g \cos \alpha = \frac{8g}{5}$

$\boxed{R \text{ is needed to find the friction.}}$

R (↗)

$$-\frac{2R}{5} - 2g\sin\alpha = 2a$$

$\boxed{\text{Note the signs.}}$

$$-\frac{16g}{25} - \frac{6s}{5} = 2a$$

$$-\frac{23s}{25} = a$$

$\boxed{\text{Solving for } a.}$

$v^2 = u^2 + 2as$ (↗)

$0 = 2^2 - 2 \times \frac{23s}{25} \times s$

$$s = \frac{50}{23g} = 0.22\ m \ (2 \text{ s.f.})$$

$\boxed{2 \text{ s.f. since } g = 9 \text{ has been used.}}$

b Wt component down plane $= 2g \times \frac{3}{5} = \frac{6g}{5}$

Limiting friction $= \frac{2}{5} \times R = \frac{16g}{25}$

Hence, net force down the plane $= \frac{6g}{5} - \frac{16g}{25}$

$= \frac{14s}{25}$ so it will slide back down

c

R (↙) $\frac{14g}{25} = 2a \Rightarrow a =$

$\frac{7g}{25}$

$v^2 = u^2 + 2as$ (↙) :

$v^2 = 2 \times \frac{7g}{25} \times \frac{50}{23g} = \frac{28}{23}$

2 s.f since g = 9.8 has been used.

v $= 1.1 \text{ m s}^{-1}$ (2 s.f.)

Finding the acceleration. 2 s.f since g = 9.8 has been used.

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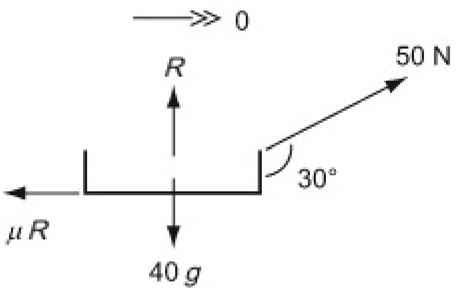
Dynamics of a particle moving in a straight line
Exercise I, Question 5

Question:

Peter is pulling Paul, who is on a toboggan, along a rough horizontal snow surface using a rope which makes an angle of 30° with the horizontal. Paul and the toboggan have a total mass of 40 kg and the toboggan is moving in a straight line with constant speed. The rope is modelled as a light inextensible string. Given that the tension in the rope is 50 N, find the coefficient of friction between the toboggan and the snow.

Solution:



R (↑) , $R + 50\cos 60° - 40g = 0$
$R = 367$ N

No vertical acceleration.

R (→) , $50\cos 30° - \mu \times 367 = 0$
$\Rightarrow \mu = 0.12$ (2 s.f.)

No horizontal acceleration as moving with constant speed.

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Dynamics of a particle moving in a straight line
Exercise I, Question 6

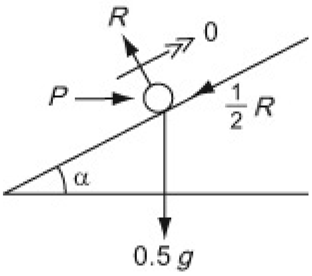
Question:

A particle of mass 0.5 kg is pushed up a line of greatest slope of a rough plane by a horizontal force of magnitude *P* N. The plane is inclined to the horizontal at an angle α where tan α = 0.75 and the coefficient of friction between *P* and the plane is 0.5. The particle moves with constant speed. Find

a the magnitude of the normal reaction between the particle and the plane

b the value of *P*.

Solution:



a

$$R(\uparrow),\quad R\cos\alpha - \frac{1}{2}R\sin\alpha - 0.5g = 0$$

$$R = g = 9.8\text{ N}$$

Resolving at right angles to *P* gives a quick solution.

b

$$R(\rightarrow),\quad P - R\sin\alpha - \frac{1}{2}R\cos\alpha = 0$$

$$\Rightarrow P = \frac{3}{5}R + \frac{2R}{5} = R = 9.8\text{ N}$$

Alternatively, resolve up the plane.

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Dynamics of a particle moving in a straight line
Exercise I, Question 7

Question:

A pile driver consists of a pile of mass 200 kg which is knocked into the ground by dropping a driver of mass 1000 kg onto it. The driver is released from rest at a point which is 10 m vertically above the pile. Immediately after the driver impacts with the pile it can be assumed that they both move off with the same speed. By modelling the pile and the driver as particles,

a find the speed of the driver immediately before it hits the pile,

b find the common speed of the pile and driver immediately after the impact. The ground provides a constant resistance to the motion of the pile driver of magnitude 120,000 N.

c Find the distance that the pile driver is driven into the ground before coming to rest.

Solution:

a

$v^2 = u^2 + 2as$

$v^2 = 2 \times 10 \times 9.8$

$v = 14 \text{ m s}^{-1}$

Motion under gravity.

b

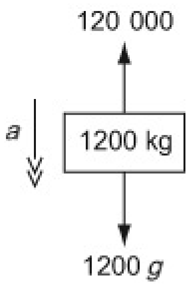
$CLM\ (\downarrow)$

$1000 \times 14 = 1200v$

$v = \frac{35}{3} \text{ m s}^{-1}$

Momentum is conserved.

c



R (↓) , first find the deceleration.

$1200g - 120\,000 = 1200a$

$a = -90.2$

$v^2 = u^2 + 20g\ (\downarrow)$

$0^2 = \left(\frac{35}{3}\right)^2 - 2 \times 90.2 \times s$

$s = 0.75\ m\ (2 \text{ s.f.})$

Using u = answer from part **b**.

2 s.f. as $g = 9.8$ has been used.

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Dynamics of a particle moving in a straight line
Exercise I, Question 8

Question:

Particles P and Q of masses $2m$ and m respectively are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. They both hang at a distance of 2 m above horizontal ground. The system is released from rest.

a Find the magnitude of the acceleration of the system.

b Find the speed of P as it hits the ground.

Given that particle Q does not reach the pulley,

c find the greatest height that Q reaches above the ground.

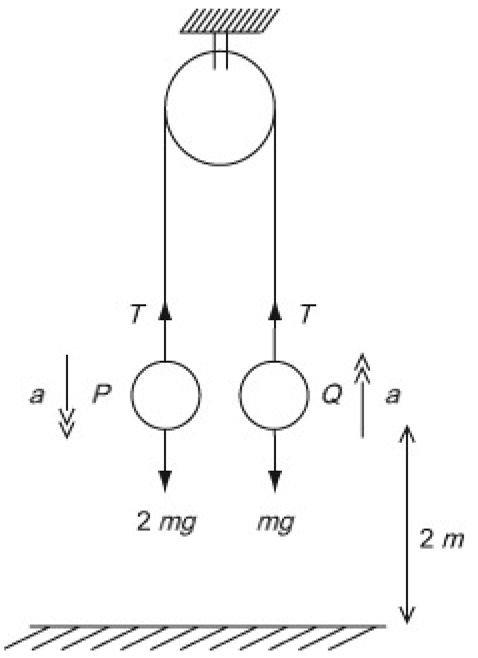
d State how you have used in your calculation,

(i) the fact that the string is inextensible,

(ii) the fact that the pulley is smooth.

Solution:

a



The diagram should show all the forces and the accelerations.

For P : R (↓) , $2mg - T = 2ma$
For Q : R (↑) , $T - mg = ma$
ADD , $mg = 3ma$
$a = \frac{1}{3}g$

Resolve in the direction of the acceleration for each mass.

b

For P $v^2 = u^2 + 2a\ g$ | The acceleration is constant.

$v^2 = 2 \times \frac{5}{3} \times 2$

$v = \sqrt{\frac{4g}{3}}$

$= 3.6 \text{ m s}^{-1}$ (2 s.f.)

c

For Q: R (↑) , $-\boxed{m}\ g = \boxed{m}\ a$ | Q now moves under gravity as the string is now slack.

$v^2 = u^2 + 2a\ s$ (↑) ,

$0 = \frac{4g}{3} - 2gs$

$s = \frac{2}{3}$ m | Note that g cancels.

$\therefore$ Height above $= 2\frac{2}{3}$ m

d

(i) In extensible string $\Rightarrow$ acceleration of both masses in equal. | Learn this.

(ii) Smooth pulley $\Rightarrow$ same tension in string either side of the pulley. | Learn this.

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Dynamics of a particle moving in a straight line
Exercise I, Question 9

Question:

The diagram shows two blocks *A* and *B*, of masses 20 kg and 30 kg respectively, inside a cage of mass 10 kg. Block *A* is on top of block *B*. The blocks are being lowered to the ground using a rope which is attached to the cage. The rope is modelled as a light inextensible string. Given that the blocks are moving vertically downwards with acceleration 0.8 m s^{-2}, find

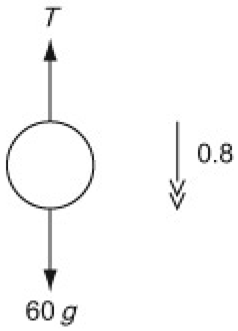
a the tension in the rope,

b the magnitude of the force that block *B* exerts on block *A*,

c the normal reaction between block *B* and the floor of the cage.

Solution:

a

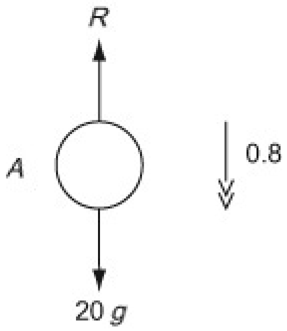


R (↓) , for whole system

$$60g - T = 60 \times 0.8$$
$$T = 588 - 48$$
$$= 540 \text{ N}$$

Note that the reactions between *A* and *B*, and between *B* and the cage cancel.

b
R (↓) , for *A*



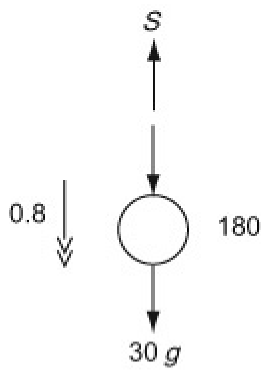
$$20g - R = 20 \times 0.8$$
$$R = 196 - 16$$
$$= 180 \text{ N}$$

Include only the forces acting on *A*.

c

R (↓) for *B*

Include only the forces acting on *B*.



$$
\begin{aligned}
30g + 180 - S &= 30 \times 0.8 \\
474 - S &= 24 \\
S &= 450 \text{ N}
\end{aligned}
$$

An alternative would be to consider the cage only.

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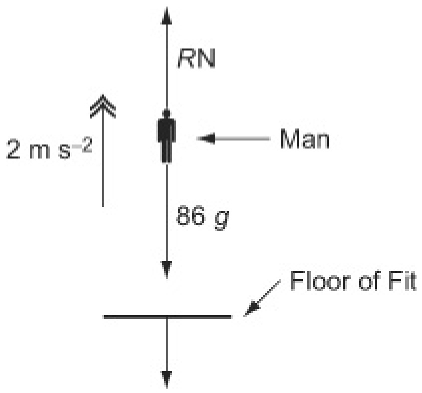
Dynamics of a particle moving in a straight line
Exercise I, Question 10

Question:

A man, of mass 86 kg, is standing in a lift which is moving upwards with constant acceleration 2 m s $^{-2}$. Find the magnitude and direction of the force that the man is exerting on the floor of the lift.

Solution:



By Newton's Third Law the action of the man on the floor and the reaction of the floor on the man are equal in magnitude, here labelled R, and in opposite directions.

$F = ma$

For the man

R (↑) $R - 86g = 86 \times 2$

$R = 86 \times 9.8 + 86 \times 2$

$= 1014.8 \approx 1000$

As the numerical value 9.8 has been used for g you should give your answer to 2 significant figures. You should not give any answer to an accuracy greater than the data you have used to calculate that answer.

The reaction on the man on the floor is of equal magnitude to the action of the floor on the man and in the opposite direction.
The force that the man exerts on the floor of the lift is of magnitude 1000 N (2 s.f.) and acts vertically downwards.

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Dynamics of a particle moving in a straight line
Exercise I, Question 11

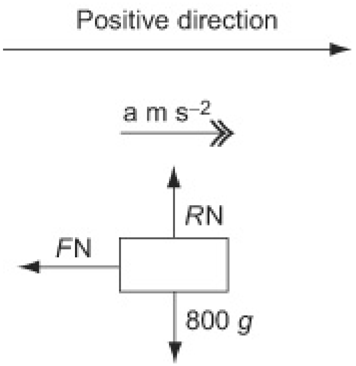
Question:

A car, of mass 800 kg and travelling along a straight horizontal road. A constant retarding force of F N reduces the speed of the car from 18 m s $^{-1}$ to 12 m s $^{-1}$ in 2.4 s. Calculate

a the value of F,

b the distance moved by the car in these 2.4 s.

Solution:



a

$u = 18$, $v = 12$, $t = 2.4$, $a =$?

$v = u + at$

$12 = 18 + 2.4a$

$a = \frac{12 - 18}{2.4} = -2.5$

You are going to have to use $F = ma$ to find F. So the first step of your solution must be to find a.

$F = ma$

$-F = 800 \times -2.5 = -2000 \Rightarrow F = 2000$

The retarding force is slowing the car down and is in the negative direction. So, in the positive direction, the force is $-F$.

b

$u = 18$, $v = 12$, $t = 2.4$, $s =$?

$s = \left(\frac{u + v}{2} \right) t$

$= \left(\frac{18 + 12}{2} \right) \times 2.4 = 15 \times 2.5 = 36$

You could use the value of a you found in part a and another formula. Unless it causes you extra work, it is safer to use the data in the question.

The distance moved by the car is 36 m.

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Dynamics of a particle moving in a straight line
Exercise I, Question 12

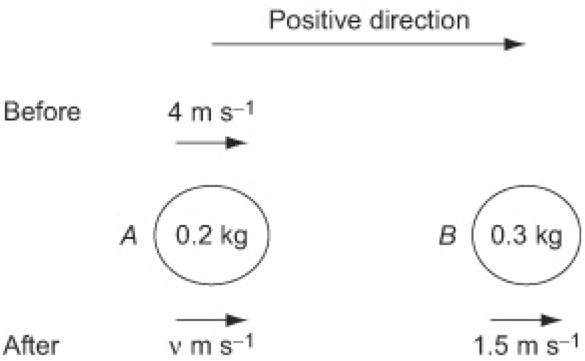
Question:

Two particles A and B, of mass 0.2 kg and 0.3 kg respectively, are free to move in a smooth horizontal groove. Initially B is at rest and A is moving toward B with a speed of 4 m s^{-1}. After the impact the speed of B is 1.5 m s^{-1}. Find

a the speed of A after the impact,

b the magnitude of the impulse of B on A during the impact.

Solution:



a

Conservation of linear momentum

$0.2 \times 4 = 0.2 \times v + 0.3 \times 1.5$

$0.8 = 0.2v + 0.45$

$v = \frac{0.8 - 0.45}{0.2} = 1.75$

A full formula for the conservation of momentum is $m_A u_A + m_B u_B = m_A v_A + m_B v_B$. In this case the velocity of B is zero.

The speed of A after the impact is 1.75 m s^{-1}.

b

Consider the impulse of A

$I = mv - mu$

$= 0.2 \times 1.75 - 0.2 \times 4$

$= 0.35 - 0.8 = -0.45$

The magnitude of the impulse of B on A during the impact is 0.45 N s.

It is a common mistake to mix up the particles. The impulses on the two particles are equal and opposite. Finding the magnitude of the impulse, you can consider either particle – either would give the same magnitude. However, you must work on only one single particle. Here you can work on A or B but not both.

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Dynamics of a particle moving in a straight line
Exercise I, Question 13

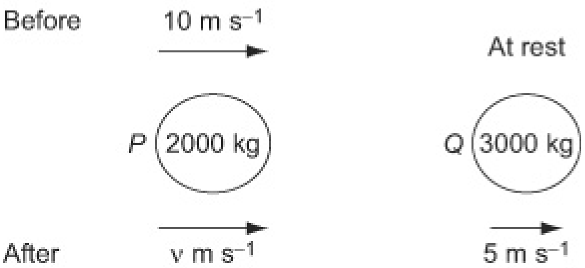
Question:

A railway truck P of mass 2000 kg is moving along a straight horizontal track with speed 10 m s^{-1}. The truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision Q moves with speed 5 m s^{-1}. Calculate

a the speed of P immediately after the collision

b the magnitude of the impulse exerted by P on Q during the collision.

Solution:



a Conservation of linear momentum

$$2000 \times 10 = 2000 \times v + 3000 \times 5$$
$$20\,000 = 2000v + 15\,000$$
$$v = \frac{20\,000 - 15\,000}{2000} = 2.5$$

The speed of P immediately after the collision is 2.5 m s^{-1}.

b

For Q,
$$I = mv - mu$$
$$I = 3000 \times 5 - 3000 \times 0 = 15\,000$$

To find the magnitude of the impulse you could consider **either** the change in momentum of P **or** the change of momentum of Q. You must not mix them up.

The magnitude of the impulse of P on Q is 15 000 N s.

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Dynamics of a particle moving in a straight line
Exercise I, Question 14

Question:

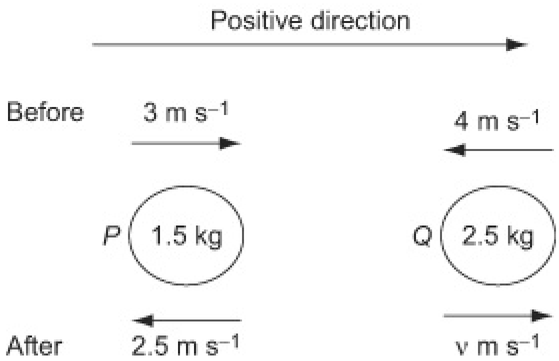
A particle P of mass 1.5 kg is moving along a straight horizontal line with speed 3 m s^{-1}. Another particle Q of mass 2.5 kg is moving, in the opposite direction, along the same straight line with speed 4 m s^{-1}. The particles collide. Immediately after the collision the direction of motion of P is reversed and its speed is 2.5 m s^{-1}.

a Calculate the speed of Q immediately after the impact.

b State whether or not the direction of motion of Q is changed by the collision.

c Calculate the magnitude of the impulse exerted by Q on P, giving the units of your answer.

Solution:



You do not know which direction Q will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction. After you have worked out v, the sign of v will tell you the direction Q is moving in.

a Conservation of linear momentum

$1.5 \times 3 + 2.5 \times (-4) = 1.5 \times (-2.5) + 2.5 \times v$

$4.5 - 10 = -3.75 + 2.5v$

$2.5v = 4.5 - 10 + 3.75 = -1.75$

$v = -\frac{1.75}{2.5} = -0.7$

The sign of v is negative, so Q is moving in the negative direction. It was moving in the negative direction before the impact and so its direction has not changed.

The speed of Q immediately after the impact is 0.7 m s^{-1}.

b The direction of Q is unchanged.

c For P, $I = mv - mu$

$I = 1.5 \times (-2.5) - 1.5 \times 3 = 8.25$

The magnitude of the impulse exerted by Q on P is 8.25 N s.

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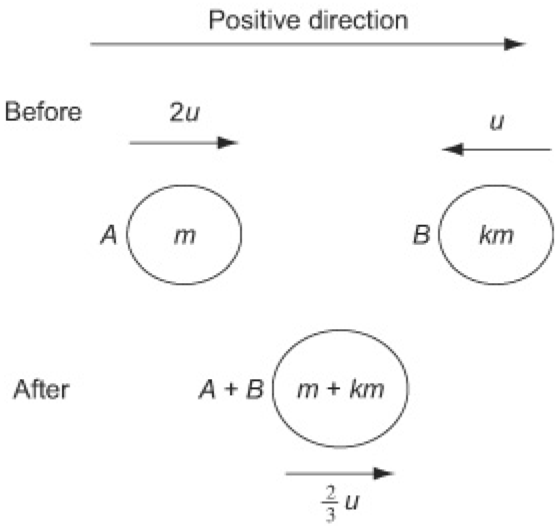
Dynamics of a particle moving in a straight line
Exercise I, Question 15

Question:

A particle A of mass m is moving with speed $2u$ in a straight line on a smooth horizontal table. It collides with another particle B of mass km which is moving in the same straight line on the table with speed u in the opposite direction to A.
In the collision, the particles form a single particle which moves with speed $\frac{2}{3}u$ in the original direction of A's motion.

Find the value of k.

Solution:



After the collision A (of mass m) and B (of mass km) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, $m + km$.

Conservation of linear momentum

$$m \times 2u + km \times (- u) = (m + km) \times \frac{2}{3}u$$

The total linear momentum before impact must equal the total linear momentum after impact. Particle B is moving in the negative direction before the collision and so it has a negative linear momentum.

$$2mu - kmu = \frac{2}{3}mu + \frac{2}{3}kmu$$

m and u are common factors on both sides of the equation and can be cancelled.

$$2mu - \frac{2}{3}mu = kmu + \frac{2}{3}kmu$$

$$\frac{4}{3}\cancel{m}\cancel{u} = \frac{5}{3}k\cancel{m}\cancel{u}$$

$$k = \frac{4}{3} \times \frac{3}{5} = \frac{4}{5}$$

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise I, Question 16

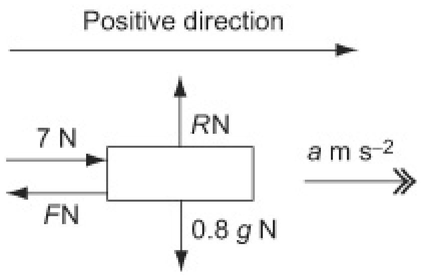
Question:

A block of mass 0.8 kg is pushed along a rough horizontal floor by a constant horizontal force of magnitude 7 N. The speed of the block increases from 2 m s $^{-1}$ to 4 m s $^{-1}$ in a distance of 4.8 m. Calculate

a the magnitude of the acceleration of the block,

b the coefficient of friction between the block and the floor.

Solution:



You should begin by drawing a diagram which shows all of the forces acting on the block and the acceleration of the block.

a

$u = 2$, $v = 4$, $s = 4.8$, $a =$?

$v^2 = u^2 + 2as$

$4^2 = 2^2 + 9.6a$

$a = \frac{16-4}{9.6} = 1.25$

The magnitude of the acceleration of the block is 1.25 m s $^{-1}$.

b

R (↑) $R - 0.8g = 0$

$R = 0.8g$

$F = \mu R = \mu 0.8g$

$F = ma$

R (→) $7 - F = 0.8 \times 1.5$

$F = 7 - 0.8 \times 1.5 = 7 - 1 = 6$

Newton's Second Law is a relation between vector quantities and so you must be careful with the directions of forces. In this case, the force of 7 N pushing the block is in the positive direction and the friction force of F N is in the negative direction.

From $*$

$\mu 0.8g = 6$

$\mu = \frac{6}{0.8g} = \frac{6}{0.8 \times 9.8} = 0.765 \ldots$

As a numerical approximation for g has been used, you should correct your final answer to 2 significant figures.

The coefficient of friction is 0.77 (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

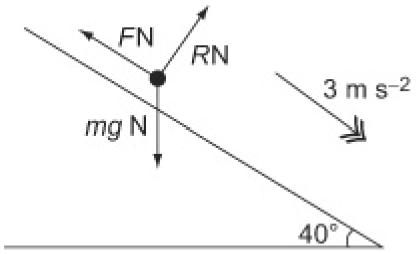
Dynamics of a particle moving in a straight line
Exercise I, Question 17

Question:

A particle is sliding with acceleration 3 m s^{-2} down a line of greatest slope of a fixed plane. The plane is inclined at 40° to the horizontal.

Calculate the coefficient of friction between the particle and the plane.

Solution:



You are given no value for mass of the particle and you will need to have an expression for the weight of the particle. Let the mass of the particle be m kg, then the weight of the stone is mg N.

R (↗) $R - mg \cos 40^\circ = 0$

$R = mg \cos 40^\circ$

You could multiply 9.8 by $\cos 40^\circ$ using your calculator here but, in general, it is advisable to do all of the calculation, in one go, at the end of the question. This avoids rounding errors.

Friction is limiting

$F = \mu R \quad = \mu mg \cos 40^\circ$

R (↘) $mg \sin 40^\circ - F \quad = ma$

$\cancel{m}g \sin 40^\circ - \mu \cancel{m}g \cos 40^\circ \quad = \cancel{m}3$

$$\mu = \frac{g \sin 40^\circ - 3}{g \cos 40^\circ} = \frac{9.8 \sin 40^\circ - 3}{9.8 \cos 40^\circ} = 0.4394 \ldots$$

The coefficient of friction between the particle and the plane is 0.44, (2 s.f.).

As a numerical value of g has been given, you should give your answer to 2 significant figures unless the question instructs you otherwise.

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Dynamics of a particle moving in a straight line
Exercise I, Question 18

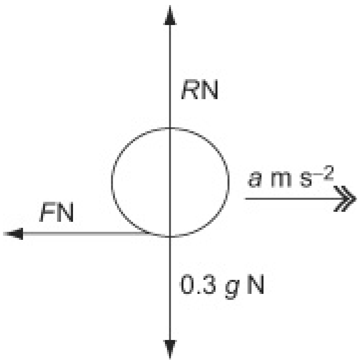
Question:

A pebble of mass 0.3 kg slides in a straight line on the surface of a rough horizontal concrete path. Its initial speed is 12.6 m s $^{-1}$. The coefficient of friction between the pebble and the path is $\frac{3}{7}$.

a Find the frictional force retarding the pebble.

b Find the total distance covered by the pebble before it comes to rest.

Solution:



a

R (↑) $R - 0.3g = 0 \Rightarrow R = 0.3g$

Friction is limiting

$F = \mu R = \frac{3}{7} \times 0.3 \times 9.8 = 1.26$

The frictional force retarding the pebble is 1:3 N (2 s.f.).

Although the answer was correctly given to 2 significant figures in part **a**, in working out the deceleration in part **b**, you should use the value of 1.26 for the friction force. All working should be carried out to at least 3 figures.

b

$F = ma$

R (→) $-1.26 = 0.3a$

$a = -\frac{1.26}{0.3} = -4.2$

The question has not asked you to work out the deceleration but you could not find out *s* without working out *a* first. You have to see this kind of step for yourself.

$u = 12.6$, $v = 0$, $a = -4.2$, $s = ?$

$v^2 = u^2 + 2as$

$0^2 = 12.6^2 - 8.4s$

$s = \frac{12.6^2}{8.4} = 18.9$

The total distance covered by the pebble before it comes to rest is 19 m (2 s.f.).

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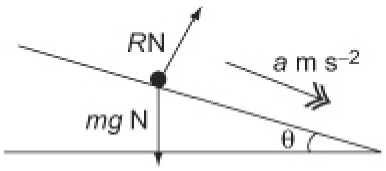
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise I, Question 19

Question:

A particle moves down a line of greatest slope of a smooth plane inclined at an angle θ to the horizontal. The particle starts from rest and travels 3.5 m in time 2 s. Find the value of sin θ.

Solution:



$u = 0$, $s = 3.5$, $t = 2$, $a = ?$

$s = ut + \frac{1}{2}at^2$

$3.5 = 0 + \frac{1}{2}a \times 2^2 = 2a$

$a = \frac{3.5}{2} = 1.75$

R (↗) $\quad \cancel{m}g \sin\theta = \cancel{m}a$

$\sin\theta = \frac{1.75}{9.8} = \frac{5}{28}$

You are given no value for mass of the particle and you will need to have an expression for the weight of the particle. Let the mass of the particle be *m* kg, then the weight of the stone is *mg* N. As often happens, the *m* can later be removed from the working, using the usual processes of algebra.

There is no friction in this question and the only force acting on the particle parallel to the plane is the component of the weight of the particle. There is an exact answer here which can be left. However a decimal answer, $\sin\theta \approx 0.18$, would be acceptable.

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Dynamics of a particle moving in a straight line
Exercise I, Question 20

Question:

A man of mass 80 kg stands in a lift. The lift has mass 60 kg and is being raised vertically by a cable attached to the top of the lift. Given that the lift with the man inside is rising with a constant acceleration of 0.6 m s^{-2}, find, to two significant figures,

a the magnitude of the force exerted by the lift on the man,

b the magnitude of the force exerted by the cable on the lift.

The lift starts from rest and, 5 s after starting to rise, the coupling between the cable and the lift suddenly snaps. There is an emergency cable attached to the lift but this only becomes taut when the lift is at the level of its initial position. After the coupling snaps, the lift moves freely under gravity until it is suddenly brought to rest in its initial position by the emergency cable. By modelling the lift with the man inside as a particle moving freely under gravity,

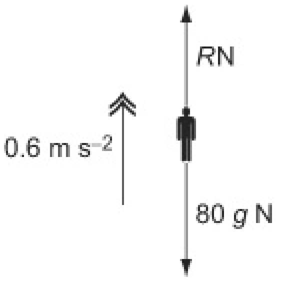
c find, to two significant figures, the magnitude of the impulse exerted by the emergency cable on the lift when it brings the lift to rest.

The model used in calculating the value required in part **c** ignores any effect of air resistance.

d State, with a reason, whether the answer obtained in **c** is higher or lower than the answer which would be obtained using a model which did incorporate the effect of air resistance.

Solution:

a



R N is the normal reaction of the lift on the man. The only forces on the man are this reaction and his weight. The cable has no direct contact with the man and you should not include this in your equation.

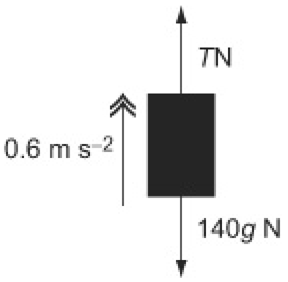
R (↑) $R - 80g = 80 \times 0.6$

$R = 80 \times 9.8 + 80 \times 0.6 = 832$

The magnitude of the force exerted by the lift on the man is 830 N (2 s.f.).

b

Combining the man and the lift, and modelling them as a single particle of mass 140 kg



There is a reaction of the lift on the man and an equal and opposite reaction of the man on the lift. When the lift and the man are combined, these cancel out and you need not consider them when writing down the equation of motion of the man and lift combined.

$\boldsymbol{F} = m\boldsymbol{a}$

R (↑) $T - 140g = 140 \times 0.6$

$T = 140 \times 9.8 + 140 \times 0.6 = 1456$

The magnitude of the force exerted by the cable on the lift is 1500 N (2 s.f.)

c

To find the speed of the lift when the cable breaks, take the **upward** direction as positive.

$u = 0$, $t = 5$, $a = 0.6$, $v = ?$

$v = u + at = 0 + 0.6 \times 5 = 3$

To find the distance travelled before the cable breaks, take the **upward** direction as positive.

$u = 0$, $t = 5$, $a = 0.6$, $s = ?$

After the cable breaks, take **downwards** as positive.

$u = -3$, $a = 9.8$, $s = 75$, $v = ?$

$v^2 = u^2 + 2as = 3^2 + 2 \times 9.8 \times 7.5 = 156$

$v = \sqrt{156}$

$I = mv - mu$

$I = 0 - 140\sqrt{156} = -1748 \ldots$

The magnitude of the impulse exerted by the emergency cable on the lift when it brings the lift to rest is 1700 N s (2 s.f.).

The emergency cable reduces the lift to rest. To find the impulse, you need to find the speed with which the lift returns to its original position. After the lift breaks, the lift falls freely under gravity. Its acceleration is no longer 0.6 m s $^{-2}$ upwards but 9.8 m s $^{-2}$ downwards.

d

Air resistance would reduce the speed of the lift as it falls and so the impulse would be reduced.

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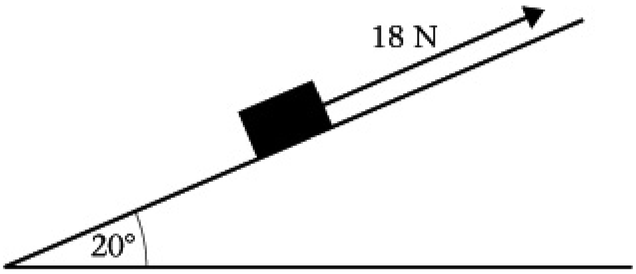
Dynamics of a particle moving in a straight line
Exercise I, Question 21

Question:

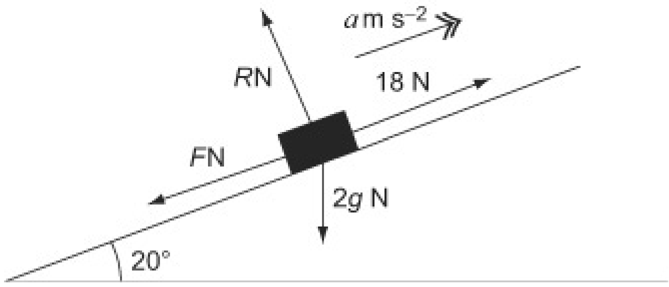
A box of mass 2 kg is pulled up a rough plane face by means of a light rope. The plane is inclined at an angle of 20 ° to the horizontal, as shown in the figure. The rope is parallel to a line of greatest slope of the plane. The tension in the rope is 18 N. The coefficient of friction between the box and the plane is 0.6. By modelling the box as a particle, find



a the normal reaction of the plane on the box,

b the acceleration of the box.

Solution:



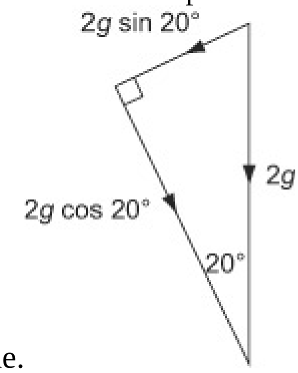
a

R (↗) $R - 2g\cos 20° = 0$

$R = 2g\cos 20° = 18.417\,97\ldots$

The normal reaction of the plane on the box is 18 N (2 s.f.).

The weight $2g$ N has components perpendicular and parallel to the



plane.

b

Friction is limiting

There are three forces acting on the box in the direction parallel to the plane. The

$F = \mu R$	$= 0.6 \times 18.417\,97\ldots$ $= 11.0507\ldots$	tension in the rope acting up the plane, and the friction force and the component of the weight acting down the plane. The **F** in $F = ma$ is the vector sum of these three forces.
R (↘) 18	$- F - 2g\sin 20° = 2a$	
18	$- 11.0507\ldots - 6.70359\ldots = 2a$	
$2a$	$= 0.2456\ldots \Rightarrow a = 0.1228\ldots$	

The acceleration of the box is 0.12 m s $^{-1}$ (2 s.f.).

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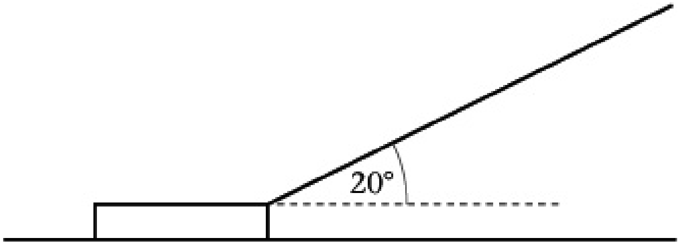
Dynamics of a particle moving in a straight line
Exercise I, Question 22

Question:

A sledge has mass 30 kg. The sledge is pulled in a straight line along horizontal ground by means of a rope. The rope makes an angle 20 ° with the horizontal, as shown in the figure. The coefficient of friction between the sledge and the ground is 0.2. The sledge is modelled as a particle and the rope as a light inextensible string. The tension in the rope is 150 N. Find, to three significant figures,



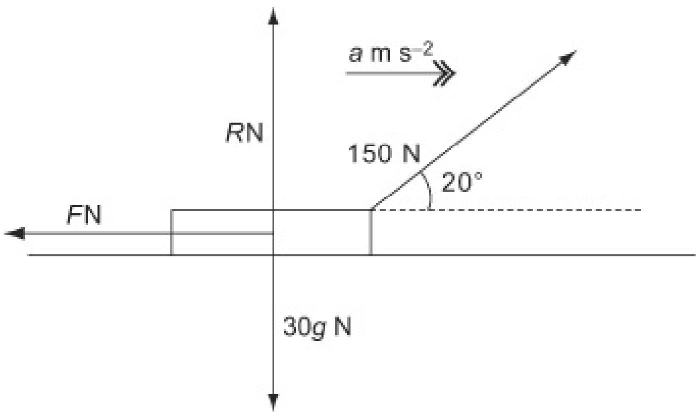
a the normal reaction of the ground on the sledge,

b the acceleration of the sledge.

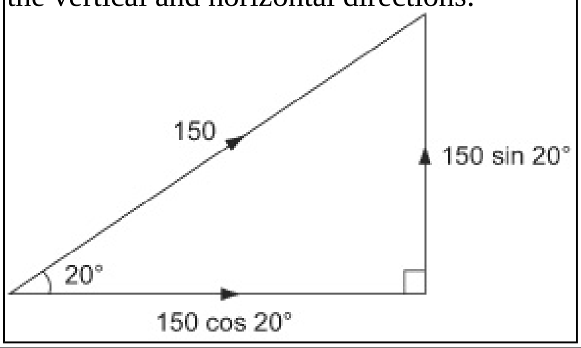
When the sledge is moving at 12 m s $^{-1}$, the rope is released from the sledge.

c Find, to three significant figures, the distance travelled by the sledge from the moment when the rope is released to the moment when the sledge comes to rest.

Solution:



The tension in the rope has components in the vertical and horizontal directions.



a

$R(\uparrow)\quad R + 150\sin 20^\circ - 30g = 0$

$R = 30g - 150\sin 20^\circ$

$= 30 \times 9.8 - 150\sin 20^\circ = 242.69\ldots$

The normal reaction of the ground on the sledge is 243 N (3 s.f.).

This question asks you to give answers to 3 significant figures. Usually when a numerical value of g is used you correct to 2 significant figures, but in all A level papers, you must follow the instructions given in a particular question. From time to time the conditions in a question may vary.

b

Friction is limiting

$F = \mu R = 0.2 \times 242.69 \ldots = 48.539 \ldots$

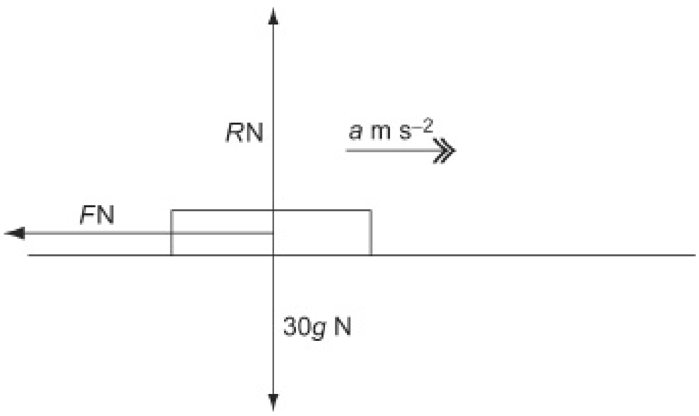
$R(\rightarrow)$

$150 \cos 20^\circ - 48.539 \ldots = 30a$

$a = \dfrac{150 \cos 20^\circ - 48.539 \ldots}{30} = 3.080 \ldots$

The acceleration of the sledge is 3.08 m s $^{-1}$ (3 s.f.).

c



$R(\uparrow) \quad R - 30g = 0 \Rightarrow R = 30g$

Friction is limiting

$F = \mu R = 0.2 \times 30g = 6g$

$R(\rightarrow) \quad -F = 30a$

$a = \dfrac{-6g}{30} = -1.96$

$u = 12,\ v = 0,\ a = -1.96,\ s = \ ?$

$v^2 = u^2 + 2as$

$0^2 = 12^2 - 2 \times 1.96 \times s$

$s = \dfrac{12^2}{2 \times 1.96} = 36.734 \ldots$

The distance travelled by the sledge is 36.7 m (3 s.f.).

The only force acting in a horizontal direction is the friction force. However, with the removal of the rope, this has changed. The friction force depends on the normal reaction and that is now $30g \approx 294$ N. It was about 243 N in part **a**. It has increased with the removal of the rope. Assuming the friction and the normal reaction are unchanged is a common error. You must start again, draw a fresh diagram and work through the question again, resolving in both directions.

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise I, Question 23

Question:

A metal stake of mass 2 kg is driven vertically into the ground by a blow from a sledgehammer of mass 10 kg. The sledgehammer falls vertically on to the stake, its speed just before impact being 9 m s $^{-1}$. In a model of the situation it is assumed that, after impact, the stake and the sledgehammer stay in contact and move together before coming to rest.

a Find the speed of the stake immediately after impact.

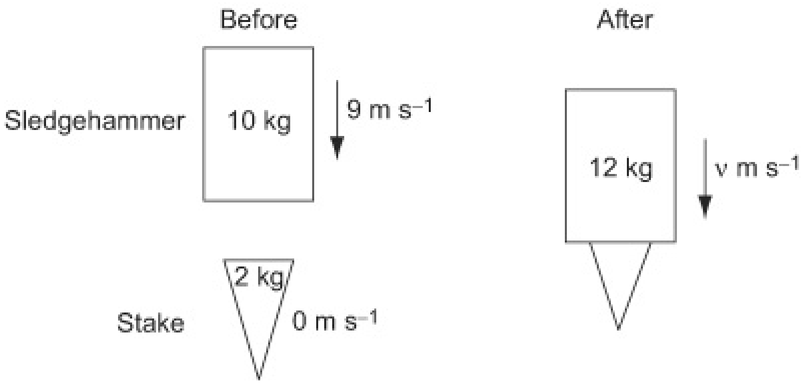
The stake moves 3 cm into the ground before coming to rest. Assuming in this model that the ground exerts a constant resistive force of magnitude *R* newtons as the stake is driven down,

b find the value of *R*.

c State one way in which this model might be refined to be more realistic.

Solution:

a



After impact, the sledgehammer and the metal stake move together. You model the sledgehammer and stake as a single particle of mass 12 kg.

Conservation of linear momentum

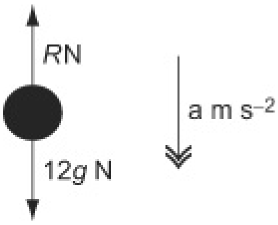
$10 \times 9 + 2 \times 0 = 12 \times v$

$v = \frac{90}{12} = 7.5$

The speed of the stake immediately after impact is 7.5 m s $^{-1}$.

The model given in the question assumes that the stake and sledgehammer stay in contact and move together after impact, before coming to rest. Although the question only refers to the stake, you must consider the stake and the sledgehammer as moving together, with the same velocity and the same acceleration, throughout the motion after the impact.

b



$u = 7.5$, $v = 0$, $s = 0.03$, $a =$?

$v^2 = u^2 + 2as$

$0^2 = 7.5^2 + 2 \times a \times 0.03$

$a = -\frac{7.5^2}{0.06} = -937.5$

$F = ma$

$12g - R = 12 \times (-937.5)$

$R = 12 \times 9.8 + 12 \times 937.5 = 11367.6$

$= 11\ 000$ (2 s.f.) .

c The resistance (R) could be modelled as varying with speed.

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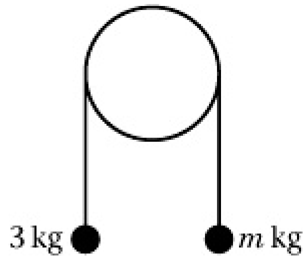
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise I, Question 24

Question:

The particles have mass 3 kg and m kg, where $m < 3$. They are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The particles are held in position with the string taut and the hanging parts of the string vertical, as shown in the figure. The particles are then released from rest. The initial acceleration of each particle has magnitude $\frac{3}{7}g$.



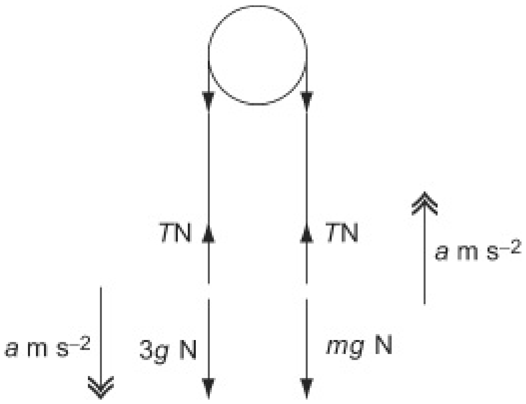
Find

a the tension in the string immediately after the particles are released,

b the value of m.

Solution:

a



As $m < 3$, the 3 kg mass will move downwards and the m kg mass will move downwards.

For the 3 kg mass

R (↓) $F = ma$

$3g - T = 3 \times \frac{3}{7}g$

$T = 3g - \frac{9}{7}g = \frac{12}{7}g$

Newton's second law is a relation between vectors and the forces must be given their correct sign. For the 3 kg mass, the weight is in the same direction as the acceleration and the tension in the string is in the opposite direction.

The tension in the string is $\frac{12}{7}g$ N.

The answer can be left in this exact form and, in this question, leaving the tension in this form leads to g cancelling in part **b**.

b

For the m kg mass

For the m kg mass, the tension is in the

R ($\uparrow$) $F = ma$

$T - mg = m \times \frac{3}{7}g$

Using the answer to part **a**

$\frac{12}{7}g - mg = \frac{3}{7}mg$

$\frac{12}{7} = \frac{10}{7}m \Rightarrow m = 1.2$

same direction as the acceleration and the weight is in the opposite direction.

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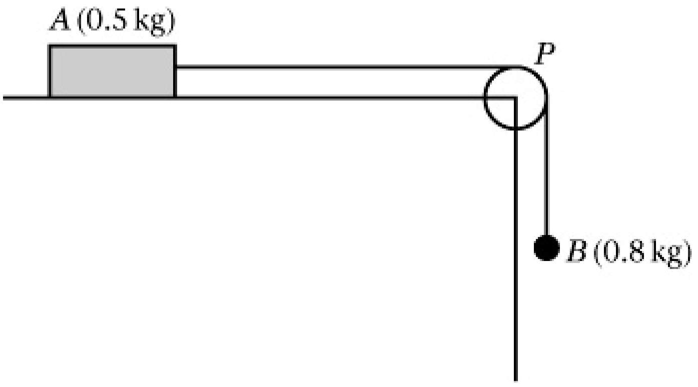
Dynamics of a particle moving in a straight line

Exercise I, Question 25

Question:



A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in the figure. The coefficient of friction between A and the table is μ. The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Modelling A and B as particles, calculate

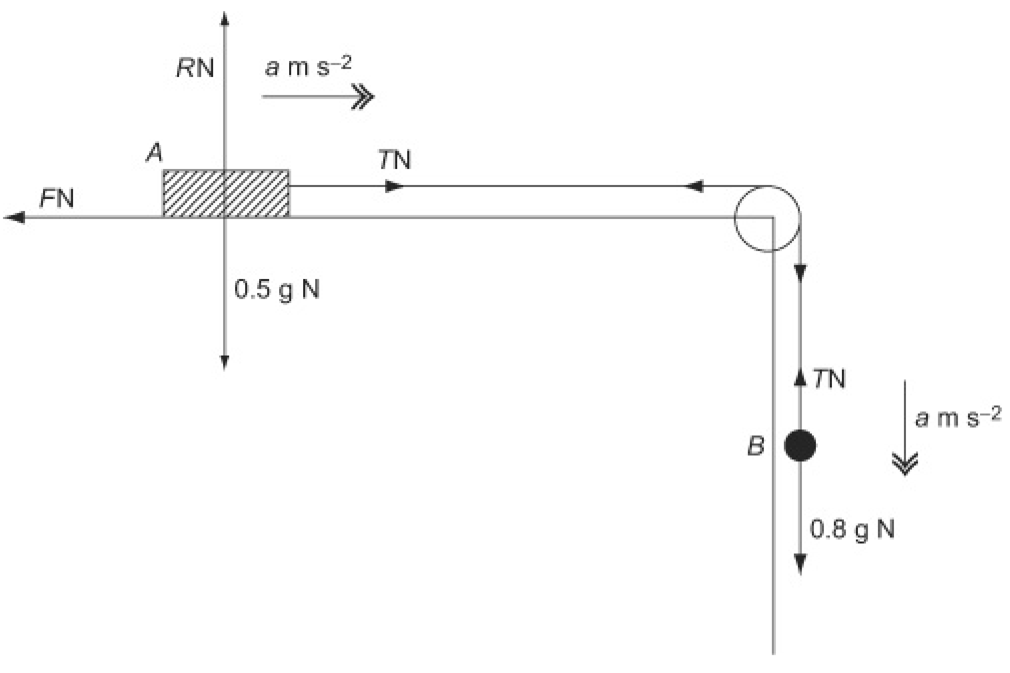
a the acceleration of B,

b the tension in the string,

c the value of μ.

d State how in your calculations you have used the information that the string is inextensible.

Solution:



a For B

$$u = 0,\ s = 0.4,\ t = 0.5,\ a = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$0.4 = 0 + \frac{1}{2}a \times 0.5^2 = \frac{1}{8}a$$
$$a = 8 \times 0.4 = 3.2$$

The acceleration of B is 3.2 ms^{-2}.

b For B

$$F = ma$$
$$0.8g - T = 0.8 \times 3.2$$

$$T = 0.8 \times 9.8 - 0.8 \times 3.2 = 5.28$$

As the numerical value $g = 9.8$ has been used, you should correct your answer to 2 significant figures.

The tension in the string is 5.3 N (2 s.f.).

c For A

R (↑) $R - 0.5g = 0 \Rightarrow R = 0.5g$

Friction is limiting

$$F = \mu R = \mu 0.5g$$
R (→) $T - F = ma$
$$5.28 - \mu 0.5g = 0.5 \times 3.2$$
$$\mu = \frac{5.28 - 0.5 \times 3.2}{0.5 \times 9.8} = 0.751\ \ldots$$
$$= 0.75 \quad (2 \text{ s.f.})$$

It is a common error to include the weight $0.5g$ in this equation. The weight acts vertically downwards and has no component in the horizontal direction, which is the direction you are resolving in. The weight does, however, affect the friction force.

Although you, correctly, gave the answer for the tension to two significant figures in part **b**, all working should be given to at least 3 significant figures, so you should use $T = 5.28$ here.

d The information that the string is inextensible has been used when, in part **c**, the acceleration of A has been taken equal to the acceleration of B obtained in part **a**.

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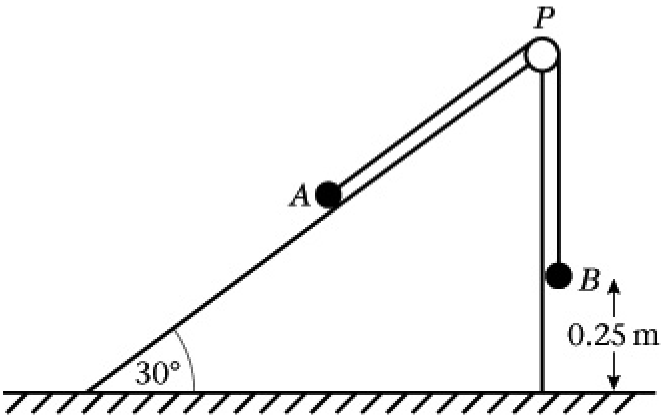
Dynamics of a particle moving in a straight line
Exercise I, Question 26

Question:



Two particles *A* and *B*, of mass *m*kg and 3 kg respectively, are connected by a light inextensible string. The particle *A* is held resting on a smooth fixed plane inclined at 30 ° to the horizontal. The string passes over a smooth pulley *P* fixed at the top of the plane. The portion *AP* of the string lies along a line of greatest slope of the plane and *B* hangs freely from the pulley, as shown in the figure. The system is released from rest with *B* at a height of 0.25 m above horizontal ground. Immediately after release, *B* descends with an acceleration of $\frac{2}{5}g$. Given that *A* does not reach *P*, calculate

a the tension in the string while *B* is descending,

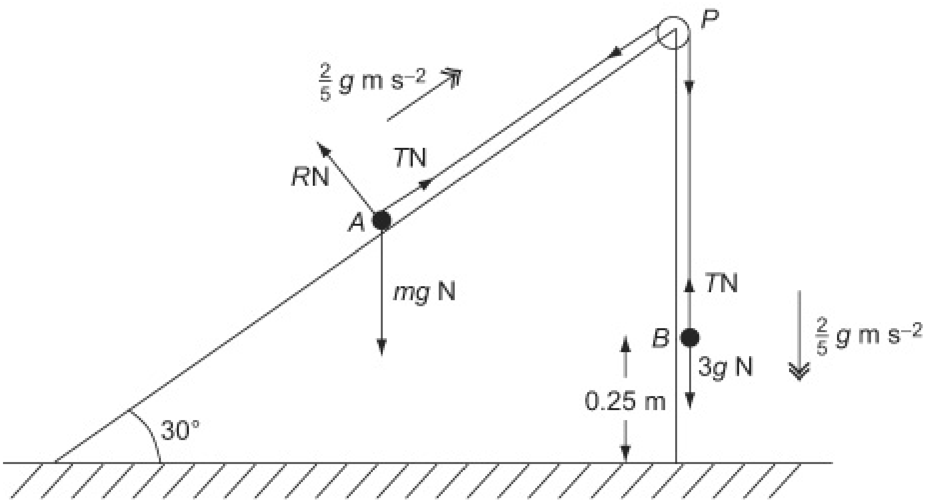
b the value of *m*.

The particle *B* strikes the ground and does not rebound. Find

c the magnitude of the impulse exerted by *B* on the ground,

d the time between the instant when *B* strikes the ground and the instant when *A* reaches its highest point.

Solution:



a

For B $F = ma$

$3g - T = 3 \times \frac{2}{5}g$

$T = 3g - \frac{6}{5}g = \frac{9}{5}g = 17.64$

The tension in the string while B is descending is 18 N, to 2 significant figures.

b

For A $F = ma$

R (↗) $T - mg\sin 30^\circ = m \times \frac{2}{5}g$

$\frac{9}{5}\cancel{g} - \frac{1}{2}m\cancel{g} = \frac{2}{5}m\cancel{g}$

$\left(\frac{1}{2} + \frac{2}{5}\right)m = \frac{9}{10}m = \frac{9}{5} \Rightarrow m = 2$

$\sin 30^\circ = \frac{1}{2}$ is the exact value.

c To find the speed of B immediately before it strikes the ground

$u = 0$, $a = \frac{2}{5}g$, $s = 0.25$, $v = ?$

$v^2 = u^2 + 2as = 0^2 + 2 \times \frac{2}{5}g \times 0.25 = 1.96$

$v = \sqrt{1.96} = 1.4$

$I = mv - mu$

$I = 3 \times 0 - 3 \times 1.4 = -4.2$

The magnitude of the impulse exerted by B on the ground is 4.2 N s.

As the particle does not rebound, the velocity of B, after it strikes the ground, is zero.

The impulse I calculated here is the impulse exerted on B by the ground – it is upwards.The impulse asked for is equal to I in magnitude but is in the opposite direction.

d

After B strikes the ground, for A

R (↘) $-\cancel{m}g\sin 30^\circ = \cancel{m}a$

$a = -\frac{1}{2}g$

$u = 1.4$, $v = 0$, $a = -\frac{1}{2}g$, $t = ?$

$v = u + at$

$0 = 1.4 - \frac{1}{2}gt \Rightarrow t = \frac{2.8}{9.8} = \frac{28}{98} = \frac{2}{7}$

After B strikes the ground, there is no tension in the string and the only force acting on B parallel to the plane is the component of its weight acting down the plane.

The approximate answer, 0.28 s, would also be acceptable.

The time between the instants is $\frac{2}{7}$ s.

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Statics of a particle
Exercise A, Question 1

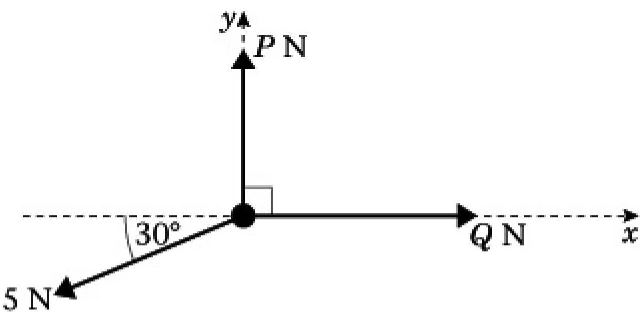
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



Solution:

a $Q - 5\cos 30° = 0$
b $P - 5\sin 30° = 0$
c $Q = 5\cos 30° = \frac{5\sqrt{3}}{2} = 4.33\text{ N (3 s.f.)}$

$P = 5\sin 30° = 2.5\text{ N}$

Give exact answers using $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$ or give decimal answers using your calculator.

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Statics of a particle
Exercise A, Question 2

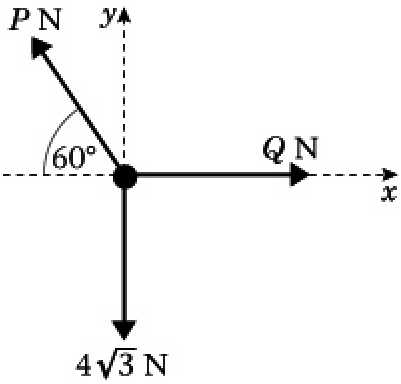
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



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Statics of a particle
Exercise A, Question 3

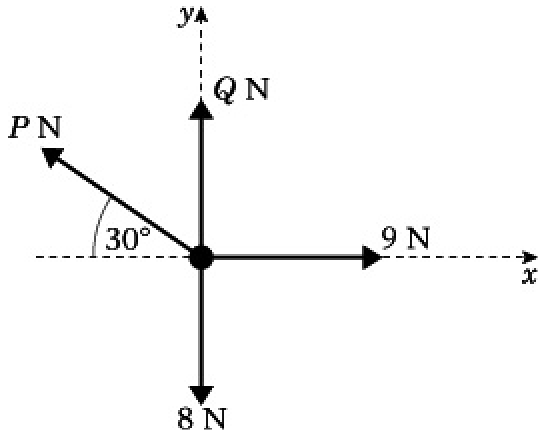
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $9 - P\cos 30° = 0$

b $Q + P\sin 30° - 8 = 0$

c From part **a**,

$$P = \frac{9}{\cos 30°} = 9 \div \frac{\sqrt{3}}{2}$$
$$= 9 \times \frac{2}{\sqrt{3}}$$
$$= \frac{18}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{18\sqrt{3}}{3}$$
$$= 6\sqrt{3}$$
$$= 10.4 \text{ N} \quad (3 \text{ s.f.})$$

Use part **a** to find *P*, then substitute into **b** to find **a**.

$\cos 30° = \frac{\sqrt{3}}{2}$

Substitute into part **b**

$$Q + 6\sqrt{3}\sin 30° - 8 = 0$$

$$\boxed{\sin 30° = \frac{1}{2}}$$

$$\therefore Q = 8 - 6\sqrt{3} \times \frac{1}{2}$$
$$= 8 - 3\sqrt{3}$$
$$= 2.80 \text{ N } (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 4

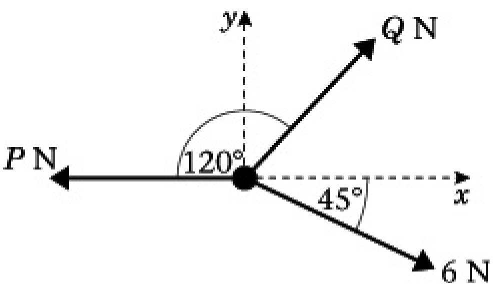
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $Q\cos 60° + 6\cos 45° - P = 0$
b $Q\sin 60° - 6\sin 45° = 0$
c use part **b** to give

$$Q = \frac{6\sin 45°}{\sin 60°}$$
$$= 6 \times \frac{1}{\sqrt{2}} \div \frac{\sqrt{3}}{2}$$
$$= 6 \times \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$$
$$= \frac{12}{\sqrt{6}}$$
$$= \frac{12\sqrt{6}}{\sqrt{6}\sqrt{6}}$$
$$= 2\sqrt{6} = 4.90 \text{ N} \ (3 \text{ s.f.})$$

Use angles on **a** straight line to find Q makes on angle of 60° with the x-axis.

$$\sin 45° = \frac{1}{\sqrt{2}} \quad \cos 60° = \frac{1}{2} \quad \sin 60° = \frac{\sqrt{3}}{2}$$

Substitute into part **a** to give:

$$2\sqrt{6} \times \frac{1}{2} + 6 \times \frac{1}{\sqrt{2}} - P = 0$$

$$\therefore P = \sqrt{6} + \frac{6}{\sqrt{2}}$$
$$= \sqrt{6} + \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}}$$
$$= \sqrt{6} + 3\sqrt{2}$$
$$= 6.69 \text{ N} \ \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 5

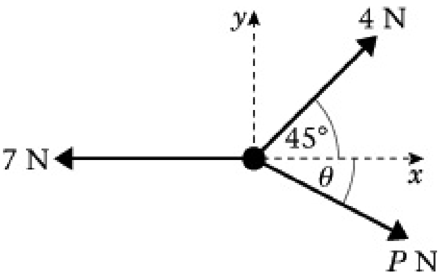
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $4\cos 45° + P\cos\theta - 7 = 0$

b $4\sin 45° - P\sin\theta = 0$

c $P\cos\theta = 7 - 4\cos 45°$ (from **a**) (1)

$P\sin\theta = 4\sin 45°$ (from **b**) (2)

Divide equation (2) by equation (1)
Then

$$\frac{P\sin\theta}{P\cos\theta} = \frac{4\sin 45°}{7 - 4\cos 45°}$$

$$\therefore \tan\theta = \frac{2.828}{4.172}$$

$$= 0.678$$

$$\therefore \theta = 34.1° \ (3 \text{ s.f.})$$

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate P from the equations after resolving.

Substitute this value for θ into equation (2)

Then

$$P = \frac{2.828}{\sin 34.1°}$$

$$P = 5.04 \text{ N} \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 6

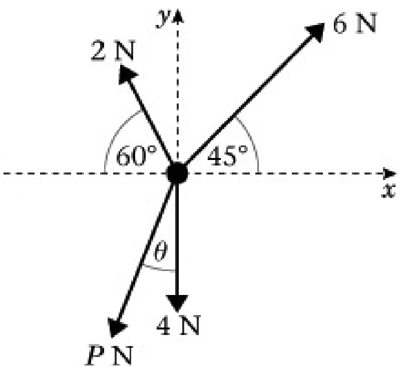
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $6\cos 45° - 2\cos 60° - P\sin\theta = 0$

b $6\sin 45° + 2\sin 60° - P\cos\theta - 4 = 0$

c $P\sin\theta = 6\cos 45° - 2\cos 60°$ (1) [from **a**]

$P\cos\theta = 6\sin 45° + 2\sin 60° - 4$ (2) [from **b**]

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{6\cos 45° - 2\cos 60°}{6\sin 45° + 2\sin 60° - 4}$$
$$\therefore \tan\theta = \frac{3.24264}{1.97469..}$$
$$= 1.642$$
$$\therefore \theta = 58.7° \ (3\text{ s.f.})$$

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate P from the equations after resolving.

Substitute θ back into equation (1)

$$P\sin\theta = 6\cos 45° - 2\cos 60°$$
$$\therefore P = \frac{3.24264}{\sin 58.65°}$$
$$P = 3.80\text{ N} \ (3\text{ s.f.})$$

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Statics of a particle
Exercise A, Question 7

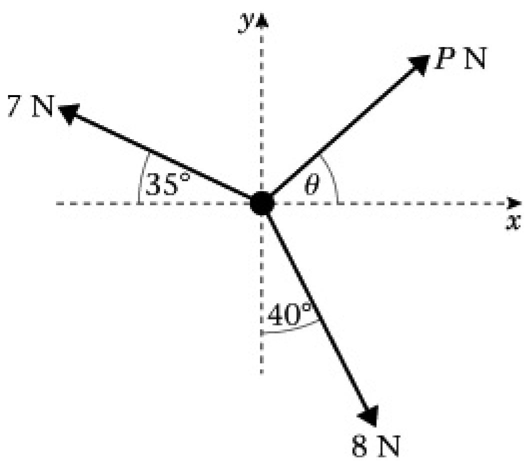
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



Solution:

a $P\cos\theta + 8\sin 40° - 7\cos 35° = 0$

b $P\sin\theta + 7\sin 35° - 8\cos 40° = 0$

c From **a** $P\cos\theta = 7\cos 35° - 8\sin 40° = 0.5918$ (1)

From **b** $P\sin\theta = 8\cos 40° - 7\sin 35° = 2.113$ (2)

Divide equation (2) by equation (1)

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 40° - 7\sin 35°}{7\cos 35° - 8\sin 40°}$$

$$\therefore \tan\theta = \frac{2.113}{0.5918}$$

$$= 3.57$$

$$\therefore \theta = 74.4° \text{ (3 s.f.)} \quad (\text{allow } 74.3°)$$

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate *P* from the equations obtained in **a** and **b**.

Substitute θ into equation (1)

$$\therefore P \cos 74.4° = 0.5918$$

$$\therefore P = \frac{0.5918}{\cos 74.3569°}$$

$$= 2.19 \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 8

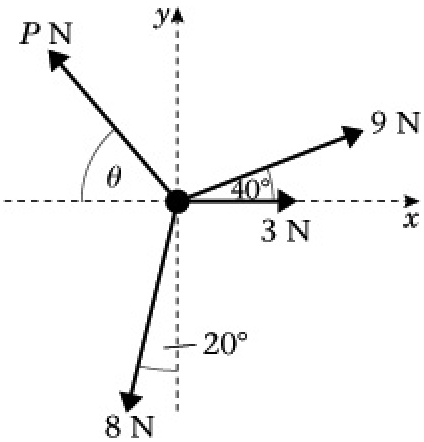
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $9\cos 40° + 3 - P\cos\theta - 8\sin 20° = 0$

b $P\sin\theta + 9\sin 40° - 8\cos 20° = 0$

c From **a**: $P\cos\theta = 9\cos 40° + 3 - 8\sin 20°$ (1)

From **b**: $P\sin\theta = 8\cos 20° - 9\sin 40°$ (2)

Divide equation (2) by equation (1)

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 20° - 9\sin 40°}{9\cos 40° + 3 - 8\sin 20°}$$

$$\therefore \tan\theta = \frac{1.732}{7.158}$$

$$= 0.242$$

$$\therefore \theta = 13.6° \ (3 \text{ s.f.})$$

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate P from the equations obtained in parts **a** and **b**.

Substitute into equation (2)

$$P\cos 13.6° = 9\cos 40° + 3 - 8\sin 20°$$
$$= 7.158$$
$$\therefore P = \frac{7.158}{\cos 13.6°}$$
$$= 7.36 \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 9

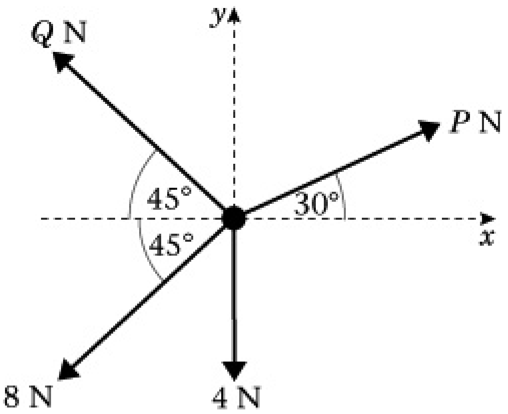
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $P\cos 30° - Q\cos 45° - 8\cos 45° = 0$

b $P\sin 30° + Q\sin 45° - 8\sin 45° - 4 = 0$

c From **a** $P\frac{\sqrt{3}}{2} - \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}}$ (1)

From **b** $\frac{P}{2} + \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}} + 4$ (2)

These are simultaneous equations.
Add (1) and (2)

$$\frac{P\sqrt{3}}{2} + \frac{P}{2} - \frac{Q}{\sqrt{2}} + \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}} + \frac{8}{\sqrt{2}} + 4$$

$$\therefore P\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) + 0 = \frac{16}{\sqrt{2}} + 4$$

$$\therefore P = \left(\frac{16\sqrt{2}}{\sqrt{2}\sqrt{2}} + 4\right) \div \left(\frac{\sqrt{3}+1}{2}\right)$$

$$= \left(8\sqrt{2} + 4\right) \times \frac{2}{\sqrt{3}+1}$$

$$= 8\,\frac{(2\sqrt{2}+1)}{\sqrt{3}+1} = 11.21 = 11.2\ (3\text{ s.f.})$$

Substitute into equation (2)

You will have two equations in two unknown forces *P* and *Q*, so should use simultaneous equations to solve them. Give your answers to 3 s.f.

$\cos 30° = \frac{\sqrt{3}}{2}\ \cos 45° = \frac{1}{\sqrt{2}}\ \sin 45° = \frac{1}{\sqrt{2}}$

$Q = 8 + 4\sqrt{2} - \frac{P}{2}\sqrt{2} = 5.73$ (3 s.f.) .

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Statics of a particle

Exercise A, Question 10

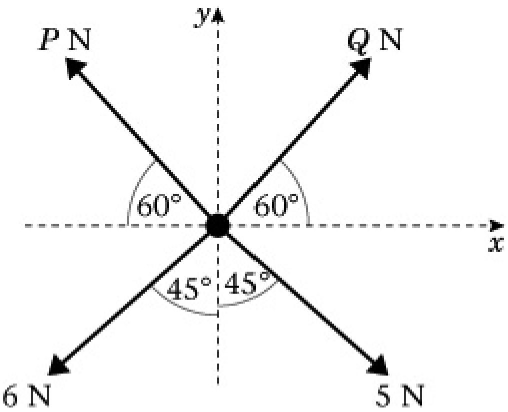
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $Q\cos 60° - P\cos 60° + 5\sin 45° - 6\sin 45° = 0$

b $P\sin 60° + Q\sin 60° - 5\cos 45° - 6\cos 45° = 0$

c From **a** $\frac{Q}{2} - \frac{P}{2} = \frac{6}{\sqrt{2}} - \frac{5}{\sqrt{2}}$

$\therefore Q - P = \sqrt{2}$ (1)

From **b** $\frac{P\sqrt{3}}{2} + \frac{Q\sqrt{3}}{2} = \frac{5}{\sqrt{2}} + \frac{6}{\sqrt{2}}$

$\therefore P\sqrt{3} + Q\sqrt{3} = 11\sqrt{2}$ (2)

Use simultaneous equations to solve the equations in parts **a** and **b**.

$\cos 60° = \frac{1}{2}$ $\sin 45° = \frac{1}{\sqrt{2}}$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 45° = \frac{1}{\sqrt{2}}$

Multiply equation (1) by $\sqrt{3}$ and add to equation (2).

$$\therefore 2Q\sqrt{3} = \sqrt{3}\sqrt{2} + 11\sqrt{2}$$

$$\therefore Q = \frac{\sqrt{2}}{2} + \frac{11\sqrt{2}}{2\sqrt{3}}$$

$$= 5.198 \text{ N } (4 \text{ s.f.})$$

Substitute into equation (1)

$$\therefore P = Q - \sqrt{2}$$
$$= 3.784 \text{ N} \quad (\text{ 4 s.f. })$$

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Statics of a particle
Exercise A, Question 11

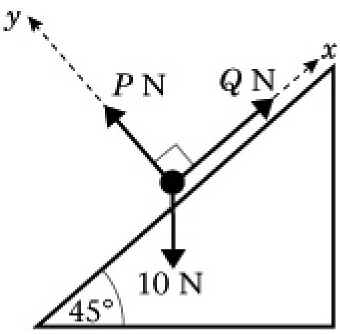
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



Solution:

a R (↗)
$Q - 10\sin 45° = 0$

Resolve along the plane and perpendicular to the plane.

b R (↖)
$P - 10 \cos 45° = 0$

c From **b**,

$P = 10 \cos 45°$
$= 5\sqrt{2}$
$= 7.07$ N (3 s.f.)

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

From **a**,

$Q = 10\sin 45°$
$= 5\sqrt{2}$
$= 7.07$ N (3 s.f.)

$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

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Statics of a particle

Exercise A, Question 12

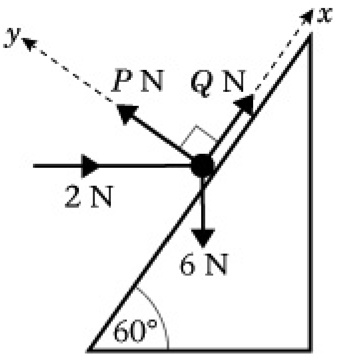
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $Q + 2\cos 60° - 6\sin 60° = 0$

b $P - 2\sin 60° - 6\cos 60° = 0$

c From part **b**

$$P = 2\sin 60° + 6\cos 60°$$
$$P = 4.73 \ (3 \text{ s.f.})$$

From part **a**

$$Q = 6\sin 60° - 2\cos 60°$$
$$Q = 4.20 \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 13

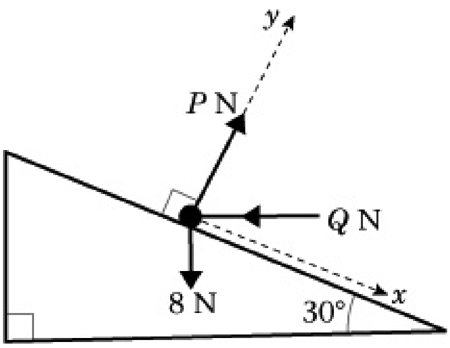
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $8\sin 30° - Q\cos 30° = 0$
b $P - Q\sin 30° - 8\cos 30° = 0.$

Find Q from the equation written down in part **a**, then substitute into the equation obtained in part **b** to find P.

c From **a**

$$Q = \frac{8\sin 30°}{\cos 30°}$$
$$= 8\tan 30°$$
$$= \frac{8\sqrt{3}}{3}$$
$$= 4.62\text{ N } (3\text{ s.f.})$$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Substitute into **b**

$$P = Q\sin 30° + 8\cos 30°$$
$$= \frac{8\sqrt{3}}{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2}$$
$$= \frac{4\sqrt{3}}{3} + 4\sqrt{3}$$
$$= \frac{16\sqrt{3}}{3}$$
$$= 9.24\text{ N } (3\text{ s.f.})$$

$\sin 30° = \frac{1}{2}\ \cos 30° = \frac{\sqrt{3}}{2}$

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Statics of a particle
Exercise A, Question 14

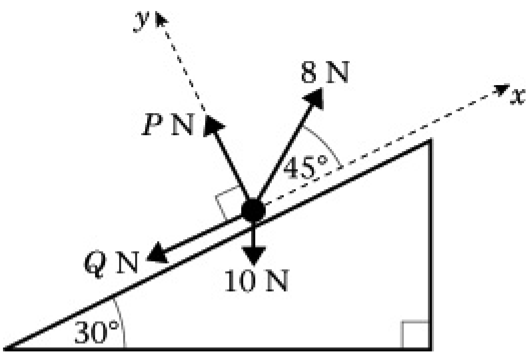
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $8\cos 45° - 10\sin 30° - Q = 0$

b $P + 8\sin 45° - 10\cos 30° = 0$

c From part **b**,

$$\begin{aligned} P &= 10\cos 30° - 8\sin 45° \\ &= 5\sqrt{3} - 4\sqrt{2} \\ &= 3.00\text{ N } (3\text{ s.f.}) \end{aligned}$$

From part **a**,

$$\begin{aligned} Q &= 8\cos 45° - 10\sin 30° \\ &= 4\sqrt{2} - 5 \\ &= 0.657\text{ N } (3\text{ s.f.}) \end{aligned}$$

You may give your answers as exact answers using surds as $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$ and $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ or you may give answers to 3 significant figures, using a calculator.

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Statics of a particle
Exercise A, Question 15

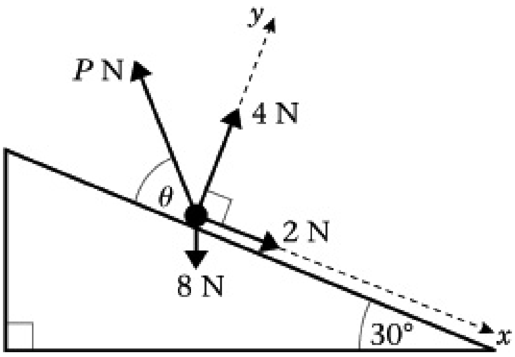
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $2 + 8\sin 30° - P\cos\theta = 0$

b $4 - 8\cos 30° + P\sin\theta = 0$

c From **a** $P\cos\theta = 2 + 8\sin 30°$ (1)

From **b** $P\sin\theta = 8\cos 30° - 4$ (2)

Divide equation (2) by equation (1)

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 30° - 4}{2 + 8\sin 30°}$$

$$\text{i.e. } \tan\theta = \frac{4\sqrt{3} - 4}{6} = 0.488$$

$$\therefore \theta = 26.0° \ (3 \text{ s.f.})$$

Substitute into equation (1)

$$P\cos 26.0° = 2 + 8\sin 30°$$

$$\therefore P = \frac{6}{\cos 26.0°}$$

$$\therefore P = 6.68 \text{ N} \ (3 \text{ s.f.})$$

Eliminate *P* from your equations by using $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$.

$\cos 30° = \frac{\sqrt{3}}{2}$ $\sin 30° = \frac{1}{2}$

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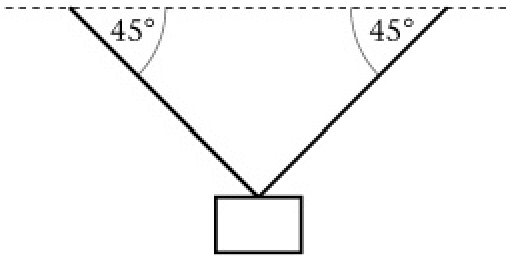
Statics of a particle
Exercise B, Question 1

Question:

A picture of mass 5 kg is suspended by two light inextensible strings, each inclined at 45 ° to the horizontal as shown. By modelling the picture as a particle find the tension in the strings when the system is in equilibrium.



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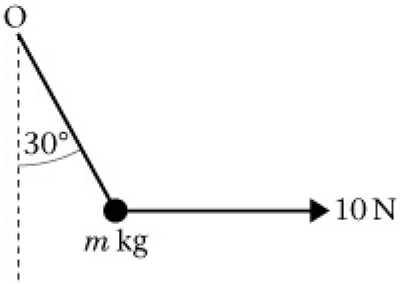
Statics of a particle

Exercise B, Question 2

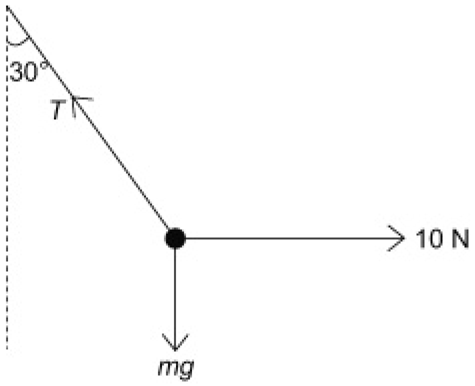
Question:

A particle of mass m kg is suspended by a single light inextensible string. The string is inclined at an angle of 30 ° to the vertical and the other end of the string is attached to a fixed point O. Equilibrium is maintained by a horizontal force of magnitude 10 N which acts on the particle, as shown in the figure.



Find **a** the tension in the string, **b** the value of m.

Solution:



Draw a diagram showing the forces acting on the particle; i.e.: the tension in the string, the weight mg and the force 10 N.

a Let the tension in the string be TN.

The weight is $m g$.

R(←)

$T \sin 30° - 10 = 0$

$$\therefore T = \frac{10}{\sin 30°}$$

$$T = 20 \text{ N}$$

b R(↑)

$T \cos 30° - m g = 0$

As $T = 20$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$mg = 20\cos 30°$$

$$\therefore m = \frac{20\cos 30°}{g}$$

$$= \frac{10\sqrt{3}}{g}$$

$$= 1.8 \ (2 \text{ s.f.})$$

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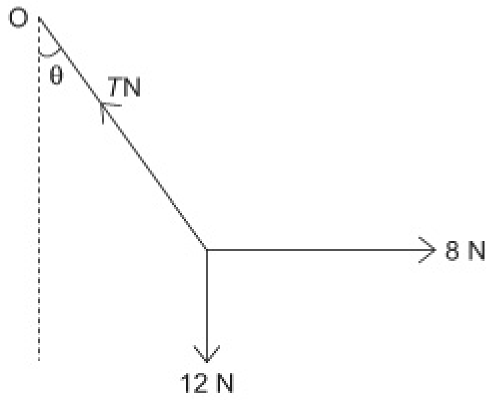
Statics of a particle
Exercise B, Question 3

Question:

A particle of weight 12 N is suspended by a light inextensible string from a fixed point O. A horizontal force of 8 N is applied to the particle and the particle remains in equilibrium with the string at an angle θ to the vertical.

Find **a** the angle θ, **b** the tension in the string.

Solution:



Let the tension in the string be TN.

>

Resolve horizontally and vertically then divide one equation by the other to eliminate the tension T.

R(→)

$$8 - T\sin\theta = 0$$
$$\therefore T\sin\theta = 8 \qquad (1)$$

R(↑)

$$T\cos\theta - 12 = 0$$
$$\therefore T\cos\theta = 12 \qquad (2)$$

Divide equation (1) by equation (2)

$$\frac{T\sin\theta}{T\cos\theta} = \frac{8}{12}$$
$$\therefore \tan\theta = \frac{2}{3}$$
$$\therefore \theta = 33.7° \quad (3 \text{ s.f.})$$

Substitute into equation (1)

$$T \sin 33.7° = 8$$

$$\therefore T = \frac{8}{\sin 33.7°} = 14.4 \ (3 \text{ s.f.})$$

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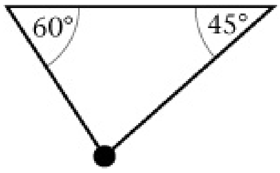
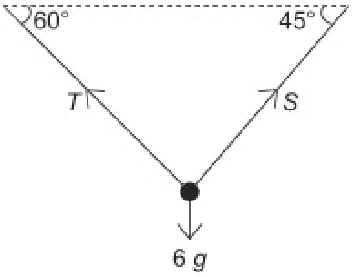
Statics of a particle
Exercise B, Question 4

Question:

A particle of mass 6 kg hangs in equilibrium, suspended by two light inextensible strings, inclined at 60 ° and 45 ° to the horizontal, as shown. Find the tension in each of the strings.



Solution:



Let the tension in the strings be TN and SN as shown in the figure.

R(←)

$T\cos 60° - S\cos 45° = 0$

$\therefore \frac{T}{2} - \frac{S}{\sqrt{2}} = 0$

$\therefore T = S\sqrt{2}$ (1)

$\boxed{\text{Resolve in two directions and obtain simultaneous equations.}}$

R(↑)

$T\sin 60° + S\sin 45° - 6g = 0$ (2)

$\boxed{\cos 60° = \frac{1}{2} \cos 45° = \frac{1}{\sqrt{2}}}$

Substitute $T = S\sqrt{2}$ into equation (2)

$\boxed{\text{sun } 60° = \frac{\sqrt{3}}{2} \text{sun } 45° = \frac{1}{\sqrt{2}}}$

$\therefore S(\sqrt{2}\sin 60° + \sin 45°) = 6g$

$$\therefore S = \frac{6g}{(\sqrt{2}\sin 60° + \sin 45°)}$$

$$= \frac{6g\sqrt{2}}{(\sqrt{3}+1)}$$

$$= 3g\sqrt{2}(\sqrt{3}-1)$$

$$= 30 \ (2 \text{ s.f.})$$

and $T = 6g(\sqrt{3}-1) = 43 \ (2 \text{ s.f.})$

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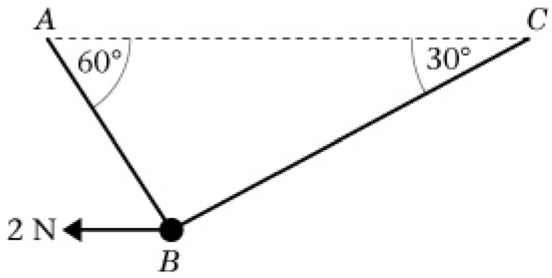
Statics of a particle
Exercise B, Question 5

Question:

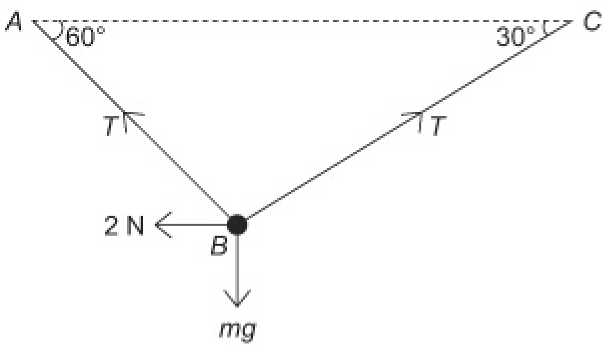
A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 2 N acting parallel to CA. The sections of string make angles of 60 ° and 30 ° with the horizontal.



Find **a** the tension in the string,

b the mass of the bead.

Solution:



a

Let the tension in the string be T and the mass of the bead be m.

The tension is the same throughout the string. Resolve horizantally first to find T.

R(→)

$$T\cos 30° - T\cos 60° - 2 = 0$$
$$\therefore T(\cos 30° - \cos 60°) = 2$$
$$\therefore T = \frac{2}{\cos 30° - \cos 60°}$$
$$= \frac{4}{\sqrt{3}-1}$$
$$= \frac{4(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= \frac{4(\sqrt{3}+1)}{2}$$
$$= 2(\sqrt{3}+1) = 5.46\text{ N } (3\text{ s.f.})$$

$\cos 30° = \frac{\sqrt{3}}{2} \cos 60° = \frac{1}{2}$

b R(↑)

$T\sin 60° + T\sin 30° - mg = 0$

$\therefore mg = T(\sin 60° + \sin 30°)$

$m = \frac{2}{g}(\sqrt{3}+1)\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \frac{4+2\sqrt{3}}{g} = 0.76 \text{ kg} \quad (2 \text{ s.f.})$

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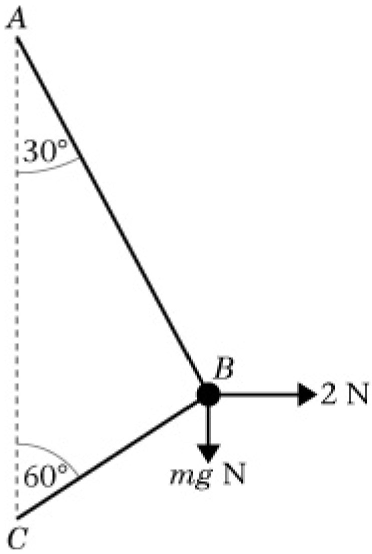
Statics of a particle
Exercise B, Question 6

Question:

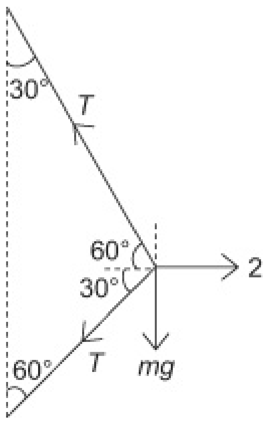
A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C where A is vertically above C. The bead is held in equilibrium by a horizontal force of magnitude 2 N. The sections AB and BC of the string make angles of $30°$ and $60°$ with the vertical respectively.



Find **a** the tension in the string,

b the mass of the bead, giving your answer to the nearest gramme.

Solution:



Let the tension in the string be TN and let the mass of the bead be m kg.

a R(→)

$$2 - T\cos 60° - T\cos 30° = 0$$
$$\therefore T(\cos 60° + \cos 30°) = 2$$

The tension is the same in both sections of the string.
Resolve horizontally first to find T.

$\cos 60° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$

$$\therefore T = \frac{2}{\cos 60° + \cos 30°}$$
$$= \frac{4}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= 2(\sqrt{3}-1)$$
$$= 1.46 \ (3 \text{ s.f.})$$

b R(↑)

$$T \sin 60° - T \sin 30° - mg = 0$$

$$\boxed{\sin 60° = \frac{\sqrt{3}}{2} \sin 30° = \frac{1}{2}}$$

$$\therefore mg = T(\sin 60° - \sin 30°)$$
$$= 2(\sqrt{3}-1)\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$
$$= (\sqrt{3}-1)^2$$
$$= 4 - 2\sqrt{3}$$
$$m = \frac{(4-2\sqrt{3})}{g} = 0.055 \text{ kg} = 55 \text{ g}$$

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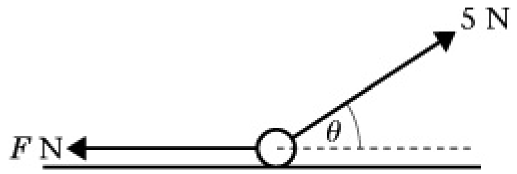
Statics of a particle
Exercise B, Question 7

Question:

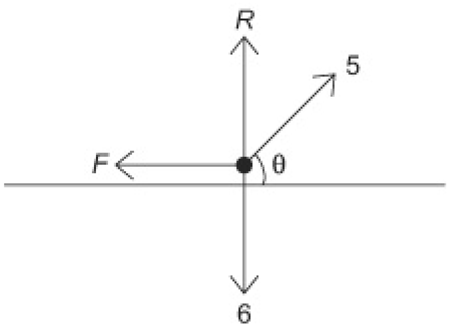
A particle of weight 6 N rests on a smooth horizontal surface. It is acted upon by two external forces as shown in the figure. One of these forces is of magnitude 5 N and acts at an angle θ with the horizontal, where $\tan\theta = \frac{4}{3}$. The other has magnitude F N and acts in a horizontal direction. Find



a the value of F,

b the magnitude of the normal reaction between the particle and the surface.

Solution:



Let the normal reaction be RN.

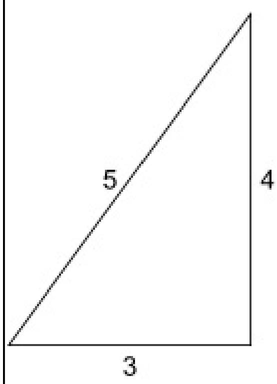
a

R(→)

$5\cos\theta - F = 0$

$\therefore F = 5 \times \frac{3}{5} = 3$ N

The angle θ is defined as satisfying $\tan\theta = \frac{4}{3}$.



From the right-angled triangle shown, $\cos\theta = \frac{3}{5}$ and $\sin\theta = \frac{4}{5}$.

b R(↑)

$R + 5 \sin \theta - 6 = 0$

$\therefore R = 6 - 5 \times \frac{4}{5} = 2 \text{ N}$

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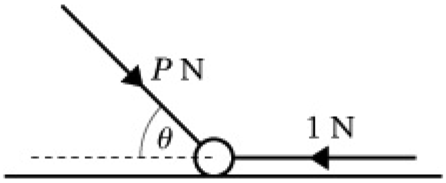
Statics of a particle
Exercise B, Question 8

Question:

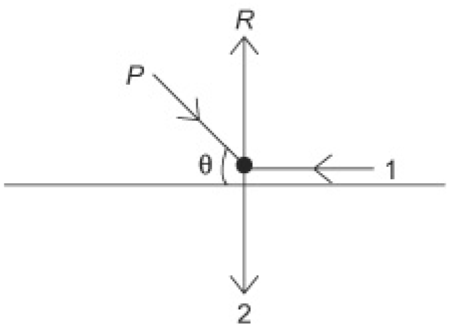
A particle of weight 2 N rests on a smooth horizontal surface and remains in equilibrium under the action of the two external forces shown in the figure. One is a horizontal force of magnitude 1 N and the other is a force P N at an angle θ to the horizontal, where $\tan \theta = \frac{12}{5}$. Find



a the magnitude of P,

b the normal reaction between the particle and the surface.

Solution:



Let the normal reaction be RN.

a

R(→)

$P \cos \theta - 1 = 0$

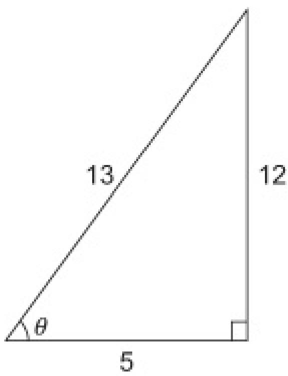
$\therefore P = 1 \div \cos \theta$

$= 1 \div \frac{5}{13}$

$= \frac{13}{5}$

$P = 2.6$

When $\tan \theta = \frac{12}{5}$ we can used the triangle shown to find $\sin \theta = \frac{12}{13}$ and $\cos \theta = \frac{5}{13}$.



b

R(↑)

$R - P\sin\theta - 2 = 0$

$$\begin{aligned}\therefore R &= P\sin\theta + 2\\ &= 2.6 \times \frac{12}{13} + 2\\ &= 2.4 + 2\\ &= 4.4\end{aligned}$$

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Statics of a particle
Exercise B, Question 9

Question:

A particle A of mass m kg rests on a smooth horizontal table. The particle is attached by a light inextensible string to another particle B of mass $2m$ kg, which hangs over the edge of the table.

The string passes over a smooth pulley, which is fixed at the edge of the table so that the string is horizontal between A and the pulley and then is vertical between the pulley and B.

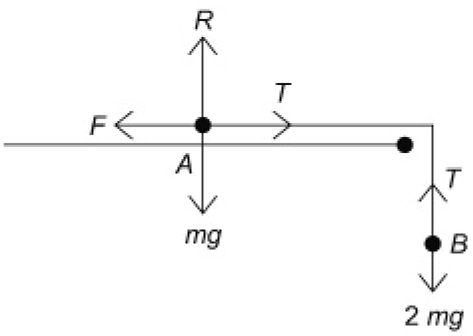
A horizontal force F N applied to A maintains equilibrium. The normal reaction between A and the table is R N.

a Find the magnitudes of F and R in terms of m.

The pulley is now raised to a position above the edge of the table so that the string is inclined at 30 degrees to the horizontal between A and the pulley. Again the string then hangs vertically between the pulley and B. A horizontal force F' N applied to A maintains equilibrium in this new situation. The normal reaction between A and the table is now R' N.

b Find, in terms of m, the magnitudes of F' and R'.

Solution:



a

Consider the mass $2m$ kg.

Consider the mass 2 m kg first, as it has only two forces acting on it. This enables you to find the tension.

R(↑)

$$T - 2m\ g = 0$$
$$\therefore T = 2m\ g.$$

Consider the mass m kg.

R(→)

$$\begin{aligned} T - F &= 0 \\ \therefore F = T &= 2m\ g \\ &= 19.6m\ \text{(accept } 20m\text{)} \end{aligned}$$

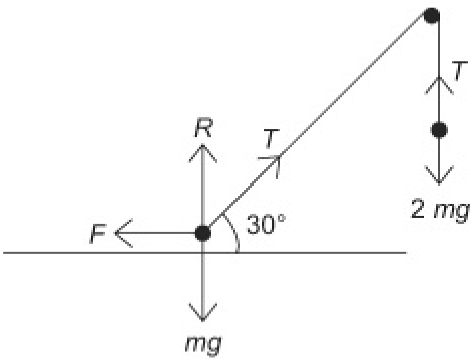
R(↑)

$R - m\ \ g = 0$

$\therefore R = m\ \ g$

$= 9.8m$

b



Consider the 2m kg mass.
Again $T = 2m\ \ g$.
Consider the mass m kg.
R(→)
$T\ \cos\ 30°\ - F = 0$

$\therefore F\ = 2m\ \ g \times \frac{\sqrt{3}}{2} = \sqrt{3}m\ \ g$

$= 17m\ \ (\ 2\text{ s.f.}\ )$

R(↑)
$R + T\ \sin\ 30 - m\ \ g = 0$

$\therefore R\ = m\ \ g - T\ \sin\ 30$

$= m\ \ g - 2m\ \ g \times \frac{1}{2}$

$= 0$

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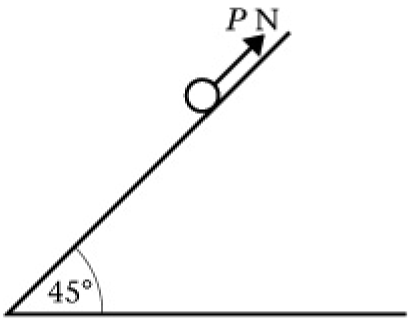
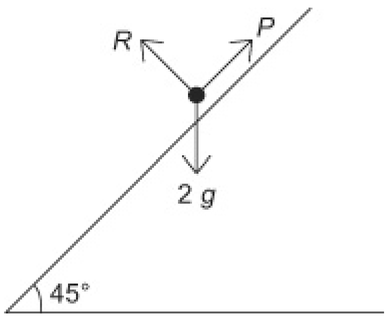
Statics of a particle
Exercise B, Question 10

Question:

A particle of mass 2 kg rests on a smooth inclined plane, which makes an angle of 45 ° with the horizontal. The particle is maintained in equilibrium by a force P N acting up the line of greatest slope of the inclined plane, as shown in the figure. Find the value of P.



Solution:



Let the normal reaction be RN.

R (↗)

Resolve along the plane.

$$P - 2g \sin 45° = 0$$
$$\therefore P = 2g \sin 45°$$
$$= g\sqrt{2}$$
$$= 14 \text{ N} \quad (2 \text{ s.f.})$$

$$\text{sun } 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

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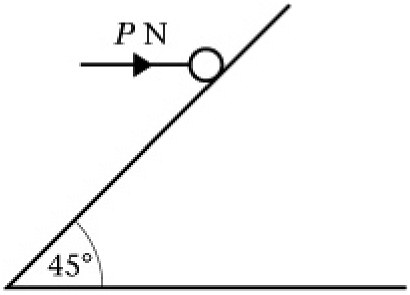
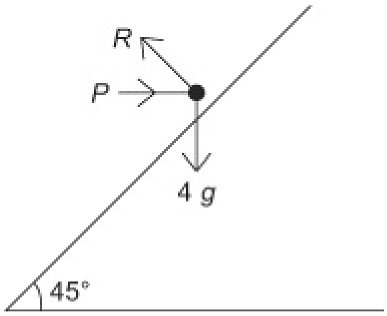
Statics of a particle
Exercise B, Question 11

Question:

A particle of mass 4 kg is held in equilibrium on a smooth plane which is inclined at 45 ° to the horizontal by a horizontal force of magnitude *P* N, as shown in the diagram. Find the value of *P*.



Solution:



Let the normal reaction be *R*N.

R (↗)

$P \cos 45° - 4g \sin 45° = 0$

$\therefore P = \dfrac{4g \sin 45°}{\cos 45°}$

$= 4g$

$= 39 \ (2 \text{ s.f.})$

Resolve along the plane.

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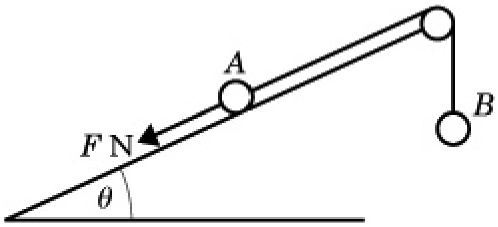
Statics of a particle
Exercise B, Question 12

Question:

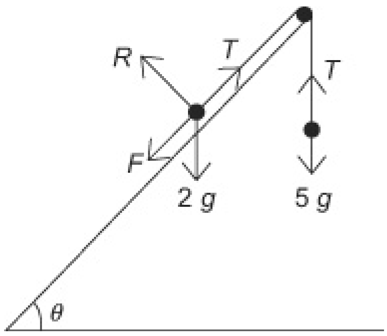
A particle A of mass 2 kg rests in equilibrium on a smooth inclined plane. The plane makes an angle θ with the horizontal, where $\tan\theta = \frac{3}{4}$.



The particle is attached to one end of a light inextensible string which passes over a smooth pulley, as shown in the figure. The other end of the string is attached to a particle B of mass 5 kg. Particle A is also acted upon by a force of magnitude FN down the plane, along a line of greatest slope.

Find **a** the magnitude of the normal reaction between A and the plane, **b** the value F.

Solution:



Let the normal reaction between the particle P and the plane be RN.

Let the tension in the string be TN.
Consider first the 5 kg mass.
R(↑)

$T - 5g = 0$

$\therefore T = 5g$

Consider the 2 kg mass.
R (↖)

$R - 2g\cos\theta = 0$

$\therefore R = 2g \times \frac{4}{5} = \frac{8g}{5} = 16$ N (2 s.f.)

R (↗)

$T - F - 2g\sin\theta = 0$

$\therefore F = T - 2g\sin\theta$

But as $T = 5g$

Consider the 5 kg mass first to find T. Then resolve perpendicular to the plane and parallel to the plane for the forces acting on the 2 kg mass. Use a 3, 4, 5 triangle to find $\sin\theta$ and $\cos\theta$.

$$F = 5g - 2g \times \frac{3}{5}$$

$$= \frac{19g}{5}$$

$$= 37 \text{ N} \ (2 \text{ s.f.})$$

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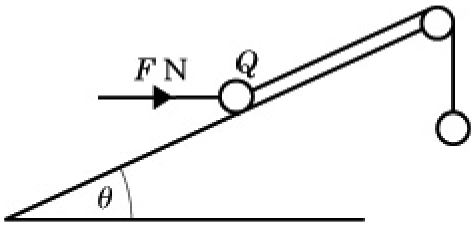
Statics of a particle
Exercise B, Question 13

Question:

A particle Q of mass 5 kg rests in equilibrium on a smooth inclined plane. The plane makes an angle θ with the horizontal, where $\tan \theta = \frac{3}{4}$.



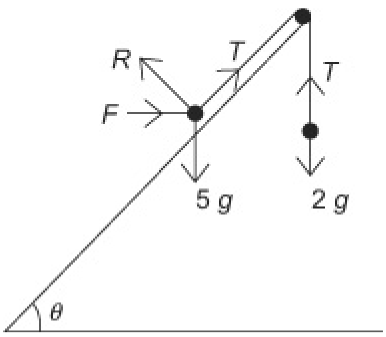
Q is attached to one end of a light inextensible string which passes over a smooth pulley as shown. The other end of the string is attached to a particle of mass 2 kg.

The particle Q is also acted upon by a force of magnitude FN acting horizontally.

Find the magnitude of

a the force FN, **b** the normal reaction between particle Q and the plane.

Solution:



Consider the 2 kg particle.

R(↑)

$$T - 2g = 0$$
$$\therefore T = 2g$$

Consider the 5 kg particle.

R (↗)

$$T + F \cos \theta - 5g \sin \theta = 0$$
$$\therefore F \cos \theta = 5g \sin \theta - T$$

As $T = 2g$, $\cos \theta = \frac{4}{5}$ and $\sin \theta = \frac{3}{5}$

$$F \times \frac{4}{5} = 5g \times \frac{3}{5} - 2g$$

$$\therefore F = g \div \frac{4}{5}$$

$$= \frac{5g}{4}$$

$$= 12 \text{ (2 s.f.)}$$

R (↖)

$R - F \sin \theta - 5g \cos \theta = 0$

$$\therefore R = F \sin \theta + 5g \cos \theta$$

$$= \frac{5g}{4} \times \frac{3}{5} + 5g \times \frac{4}{5}$$

$$= \frac{19g}{4} = 47 \text{ (2 s.f.)}$$

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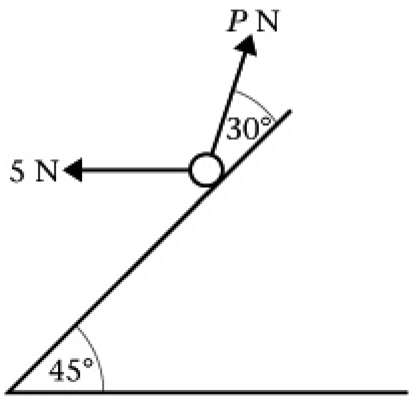
Statics of a particle
Exercise B, Question 14

Question:

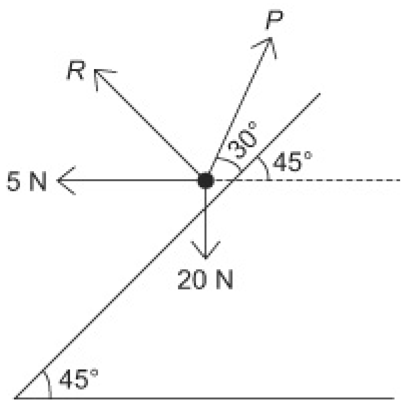
A particle of weight 20 N rests in equilibrium on a smooth inclined plane. It is maintained in equilibrium by the application of two external forces as shown in the diagram. One of the forces is a horizontal force of 5 N, the other is a force PN acting at 75 ° to the horizontal.



Find **a** the value of P,

b the magnitude of the normal reaction between the particle and the plane.

Solution:



Let the normal reaction be RN.

R (↗)

$P\cos 30° - 5\cos 45° - 20\sin 45° = 0$

Resolve along the plane to find P as it is the only unknown in your equation.

$\cos 45° = \frac{1}{\sqrt{2}} \text{sun } 45° = \frac{1}{\sqrt{2}} \cos 30° = \frac{\sqrt{3}}{2}$

$$\therefore P = \frac{5\cos 45° + 20\sin 45°}{\cos 30°}$$

$$= \left(5 \cdot \frac{\sqrt{2}}{2} + 20 \cdot \frac{\sqrt{2}}{2}\right) \div \frac{\sqrt{3}}{2}$$

$$= \frac{25\sqrt{2}}{\sqrt{3}}$$

$$= \frac{25\sqrt{6}}{3}$$

$$= 20.4 \ (3 \text{ s.f.})$$

R (↖)

$R + P\sin 30° + 5\sin 45° - 20\cos 45° = 0$

$\boxed{\text{sun } 30° = \frac{1}{2}}$

$\therefore R = 20\cos 45° - 5\sin 45° - P\sin 30°$

As

$$P = \frac{25\sqrt{6}}{3}$$

$$R = \frac{15}{\sqrt{2}} - \frac{25\sqrt{6}}{6} = \frac{45\sqrt{2} - 25\sqrt{6}}{6} = 0.400 \ (3 \text{ s.f.})$$

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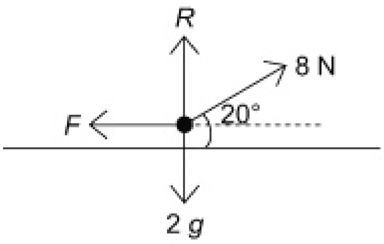
Statics of a particle
Exercise C, Question 1

Question:

A book of mass 2 kg rests on a rough horizontal table. When a force of magnitude 8 N acts on the book, at an angle of 20 ° to the horizontal in an upward direction, the book is on the point of slipping.

Calculate, to three significant figures, the value of the coefficient of friction between the book and the table.

Solution:



Let the normal reaction be RN, the friction force be FN and the coefficient of friction be μ.

R(→)

$8 \cos 20° - F = 0$

$\therefore F = 8 \cos 20°$

Resolve horizontally to find F, vertically to find R and use $F = \mu R$ to find μ.

R(↑)

$R + 8\sin 20° - 2g = 0$

$\therefore R = 2g - 8\sin 20°$

As the book is on the point of slipping the friction is limiting and

$F = \mu R$

$$\therefore \mu = \frac{F}{R} = \frac{8\cos 20°}{2g - 8\sin 20°} = \frac{7.518}{16.86} = 0.446 \text{ (3 s.f.)}$$

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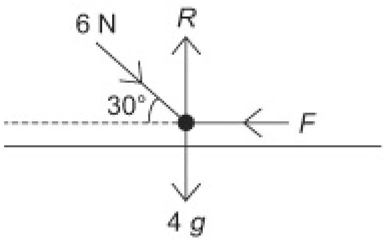
Statics of a particle
Exercise C, Question 2

Question:

A block of mass 4 kg rests on a rough horizontal table. When a force of 6 N acts on the block, at an angle of 30 ° to the horizontal in a downward direction, the block is on the point of slipping. Find the value of the coefficient of friction between the block and the table.

Solution:



Let the normal reaction be RN and the Friction force be FN. Let the coefficient of friction on be μ.

R(→)

$6 \cos 30° - F = 0$

$\therefore F = 6 \cos 30° = 3\sqrt{3} = 5.20$ (3 s.f.)

$R - 6\sin 30° - 4g = 0$

$\therefore R = 6\sin 30° + 4g$

$= 3 + 4 \times 9.8$

$= 42.2$

Resolve horizontally to find F, vertically to find R and use $\mu = \frac{F}{R}$.

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

R(↑)

As the block is on the point of slipping

$F = \mu R$

$\therefore \mu = \frac{F}{R}$

$= 0.123$ (3 s.f.) or 0.12 (2 s.f.)

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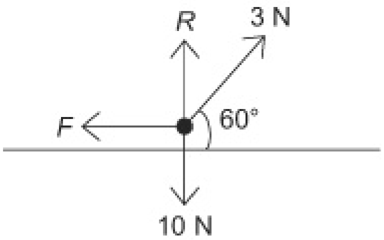
Statics of a particle
Exercise C, Question 3

Question:

A block of weight 10 N is at rest on a rough horizontal surface. A force of magnitude 3 N is applied to the block at an angle of 60 ° above the horizontal in an upward direction. The coefficient of friction between the block and the surface is 0.3.

a Calculate the force of friction, **b** determine whether the friction is limiting.

Solution:



Let the normal reaction force be R and the friction force be F.

a R(→)

$3 \cos 60° - F = 0$

$\therefore F = 3 \cos 60°$

$F = 1.5$ and so friction is 1.5 N

b R(↑)

$R + 3\sin 60° - 10 = 0$

$\therefore R = 10 - 3\sin 60°$

$= 10 - \frac{3\sqrt{3}}{2}$

$= 7.40$ (3 s.f.)

$\therefore \mu R = 0.3 \times 7.40$

$= 2.22$ (3 s.f.)

Find the friction force necessary to maintain equilibrium. Find the normal reaction force. Check whether $F < \mu R$ – non limiting equilibrium or $F = \mu R$ – limiting equilibrium.

$\sin 60° = \frac{\sqrt{3}}{2}$

As $F < \mu R$ the friction force is 1.5 N and is not limiting.

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Statics of a particle
Exercise C, Question 4

Question:

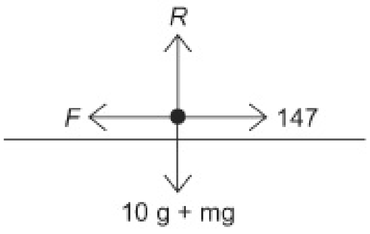
A packing crate of mass 10 kg rests on rough ground. It is filled with books which are evenly distributed through the crate. The coefficient of friction between the crate and the ground is 0.3.

a Find the mass of the books if the crate is in limiting equilibrium under the effect of a horizontal force of magnitude 147 N.

b State what modelling assumptions you have made.

Solution:

a



Let the normal reaction be RN and the friction force be FN.

Let the mass of the books be m kg.

Find F and R by resolving and use $F = \mu R$ for limiting friction.

R(→)

$$147 - F = 0$$
$$\therefore F = 147$$

R(↑)

$$R - 10g - mg = 0$$
$$\therefore R = 10g + mg$$

As the equilibrium is limiting, $F = \mu R$

$$\therefore 147 = 0.3(10g + mg)$$
$$\therefore 147 = 3g + 0.3mg$$
$$\therefore m = \frac{147 - 3g}{0.3g}$$
$$= 40$$

b The assumption is that the crate and books may be modelled as a particle.

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Statics of a particle
Exercise C, Question 5

Question:

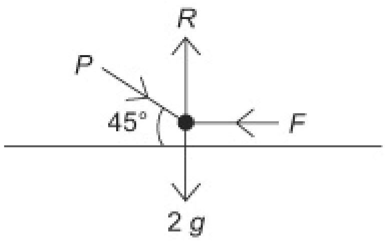
A block of mass 2 kg rests on a rough horizontal plane. A force P acts on the block at an angle of 45 ° to the horizontal. The equilibrium is limiting, with μ = 0.3.

Find the magnitude of P if

a P acts in a downward direction, **b** P acts in an upward direction.

Solution:

a



Let R be the normal reaction and F be the force of friction.

Resolve horizontally and vertically to find F and R, then use the condition for limiting friction.

R(→)

$$P\cos 45° - F = 0$$
$$\therefore F = P\cos 45°$$

R(↑)

$$R - P\sin 45° - 2g = 0$$
$$\therefore R = P\sin 45° + 2g$$

As equilibrium is limiting, $F = \mu R$

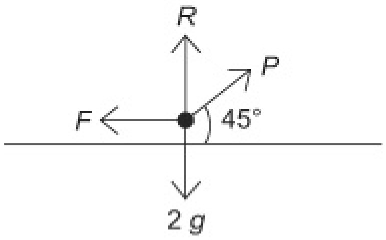
$$\therefore P\cos 45° = 0.3(P\sin 45° + 2g)$$
$$\therefore P\cos 45° - 0.3P\sin 45° = 0.6g$$
$$\therefore P(\cos 45° - 0.3\sin 45°) = 0.6g$$

$$\cos 45° = \sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$P = \frac{0.6g}{\cos 45° - 0.3\sin 45°} = \frac{6g\sqrt{2}}{7} = 11.9\text{ N (3 s.f.) or 12 N (2 s.f.)}$$

b

Let R be the normal reaction and F be the force of friction.



R(→)

$$P\cos 45° - F = 0$$
$$\therefore F = P\cos 45°$$

R(↑)

$$R + P\sin 45° - 2g = 0$$
$$\therefore R = 2g - P\sin 45°$$

As equilibrium is limiting, $F = \mu R$.

$$\therefore P\cos 45° = 0.3(2g - P\sin 45°)$$
$$\therefore P\cos 45° + 0.3P\sin 45° = 0.6g$$
$$\therefore P(\cos 45° + 0.3\sin 45°) = 0.6g$$

$$\therefore P = \frac{6g\sqrt{2}}{13}$$
$$= 6.40 \text{ N } (3 \text{ s.f.}) \text{ or } 6.4 \text{ N } (2 \text{ s.f.})$$

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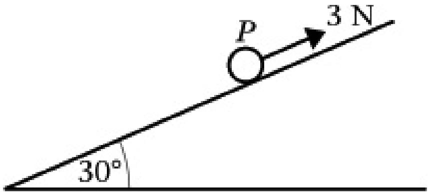
Statics of a particle

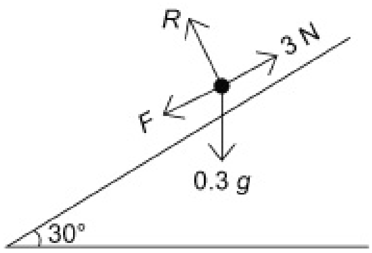
Exercise C, Question 6

Question:

A particle P of mass 0.3 kg is on a rough plane which is inclined at an angle 30 ° to the horizontal. The particle is held at rest on the plane by a force of magnitude 3 N acting up the plane, in a direction parallel to a line of greatest slope of the plane. The particle is on the point of slipping up the plane. Find the coefficient of friction between P and the plane.



Solution:



Let R be the normal reaction and F be the force of friction.

R (↗)

$3 - F - 0.3g\sin 30° = 0$

$\therefore F = 3 - 0.3g\sin 30°$

$= 1.53$ N

Note that the force of friction acts down the plane. Resolve parallel and perpendicular to the plane to obtain F and R and use $F = \mu R$.

R (↖)

$R - 0.3g\cos 30° = 0$

$\therefore R = 0.3g\cos 30°$

$= 2.546$ N

As the particle is on the point of slipping

$F = \mu R$

$\therefore 1.53 = \mu \times 2.546$

$\therefore \mu = \frac{1.53}{2.546}$

$= 0.601$ (3 s.f.) (accept 0.6)

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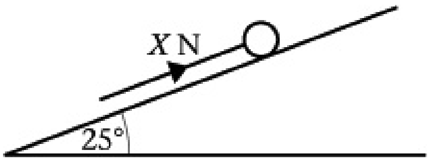
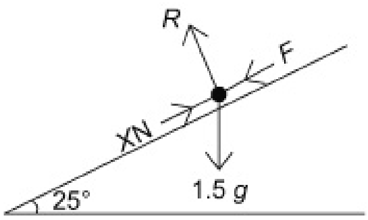
Statics of a particle
Exercise C, Question 7

Question:

A particle of mass 1.5 kg rests in equilibrium on a rough plane under the action of a force of magnitude XN acting up a line of greatest slope of the plane. The plane is inclined at 25 ° to the horizontal. The particle is in limiting equilibrium and on the point of moving up the plane. The coefficient of friction between the particle and the plane is 0.25.
Calculate **a** the normal reaction of the plane on P, **b** the value of X.



Solution:



Let R be the normal reaction and F be the force of friction.

a R (↖)

$R - 1.5 \ g\cos \ 25° = 0$

$\therefore R = 1.5 \ g\cos \ 25° = 13.3$ N (3 s.f.) or 13 N (2 s.f.)

The force of friction acts down the plane, and as the friction is limiting $F = \mu R$.

b R (↗)

$X - F - 1.5g\sin \ 25° = 0$

$\therefore X = F + 1.5 \ g\sin \ 25°$

But the friction is limiting

$$\begin{aligned}\therefore F &= \mu R\\ &= 0.25 \times 13.3227\\ &= 3.3306...\\ \therefore X &= 3.33 + 1.5g\sin \ 25°\\ &= 9.54 \text{ N } (3 \text{ s.f.}) \text{ or } 9.5 \text{ N } (2 \text{ s.f.})\end{aligned}$$

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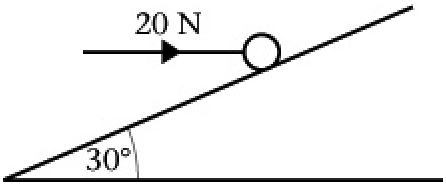
Statics of a particle
Exercise C, Question 8

Question:

A horizontal force of magnitude 20 N acts on a block of mass 1.5 kg, which is in equilibrium resting on a rough plane inclined at 30 ° to the horizontal. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane.

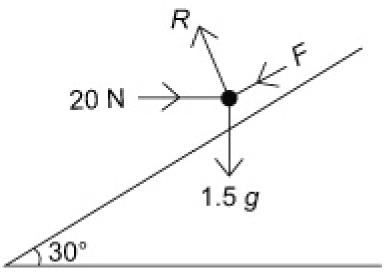


a Find the magnitude and direction of the frictional force acting on the block.

b Find the normal reaction between the block and the plane.

c What can you deduce about the coefficient of friction between the block and the plane?

Solution:



Let the normal reaction be R and the friction force be F acting down the plane.

a R (↗)

$20 \cos 30° - F - 1.5 g \sin 30° = 0$

$\therefore F = 20 \cos 30° - 1.5 g \sin 30°$

$= 9.97 \text{ (3 s.f.)}$

Draw a diagram showing all the forces acting with friction down the plane.
Resolve along the plane. If $F > 0$ then you have chosen the correct direction. If $F < 0$ then friction acts up the plane.

The friction force has magnitude 9.97 N or 10 N (2 s.f.) and acts down the plane.

b R (↖)

$R - 20 \sin 30° - 1.5 g \cos 30° = 0$

$\therefore R = 20 \sin 30° + 1.5 g \cos 30°$

$= 22.7 \text{ (3 s.f.)}$

The normal reaction has magnitude 22.7 N or 23 N (2 s.f.).

c For equilibrium $F \le \mu R$

$\therefore \mu \geq \frac{F}{R}$

i.e. : $\mu \geq 0.439$ or $\mu \geq 0.44$ (2 s.f.)

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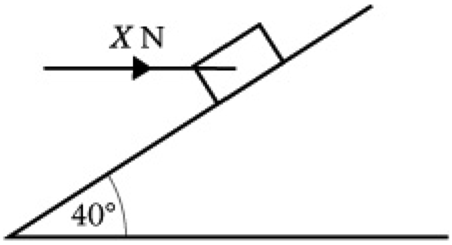
Statics of a particle
Exercise C, Question 9

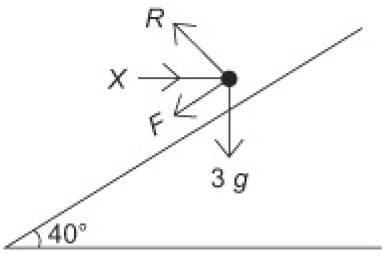
Question:

A box of mass 3 kg lies on a rough plane inclined at 40 ° to the horizontal. The box is held in equilibrium by means of a horizontal force of magnitude XN. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane. The coefficient of friction between the box and the plane is 0.3 and the box is in limiting equilibrium and is about to move up the plane.



a Find the normal reaction between the box and the plane. **b** Find X.

Solution:



Let the normal reaction be R and the friction force be F acting down the plane.

R (↖)

$R - X\sin 40° - 3g\cos 40° = 0$

$\therefore R = X\sin 40° + 3g\cos 40°$ *

> You may decide to find X first and then use your answer to find R.
> The simplest way to find R first is to resolve vertically as there is no X term in the equation obtained.

R (↗)

$X\cos 40° - F - 3g\sin 40° = 0$

$\therefore F = X\cos 40° - 3g\sin 40°$

As the friction is limiting, $F = \mu R$

$\therefore X\cos 40° - 3g\sin 40° = 0.3(X\sin 40° + 3g\cos 40°)$

$\therefore X\cos 40° - 0.3X\sin 40° = 0.9g\cos 40° + 3g\sin 40°$

$\therefore X(\cos 40° - 0.3\sin 40°) = 0.9g\cos 40° + 3g\sin 40°$

$$\therefore X = \frac{0.9g\cos 40° + 3g\sin 40°}{\cos 40° - 0.3\sin 40°}$$

$$= \frac{25.65}{0.5732}$$

$$\therefore X = 44.8 \quad (\text{accept } 44.7) \text{ or } x = 45 \ (2 \text{ s.f.})$$

Substitute into equation $*$ to give

$R = 51.3$ or $R = 51$ (2 s.f.)

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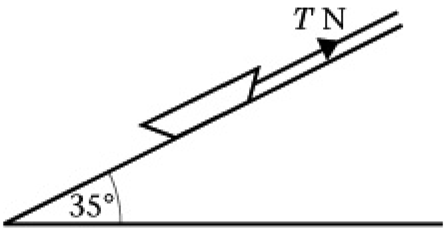
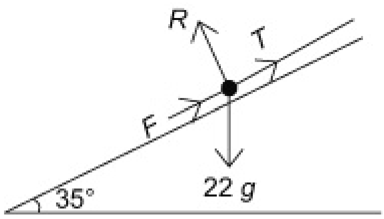
Statics of a particle
Exercise C, Question 10

Question:

A small child, sitting on a sledge, rests in equilibrium on an inclined slope. The sledge is held by a rope which lies along the slope and is under tension. The sledge is on the point of slipping down the plane. Modelling the child and sledge as a particle and the rope as a light inextensible string, calculate the tension in the rope, given that the mass of the child and sledge is 22 kg, the coefficient of friction is 0.125 and that the slope is a plane inclined at 35 ° with the horizontal. The direction of the rope is along a line of greatest slope of the plane.



Solution:



Let the normal reaction be R and the friction be F acting up the plane.

The friction acts up the plane, as the sledge is on the point of slipping down the plane.

R (↗)

$T + F - 22g\sin 35° = 0$ *

R (↖)

$$
\begin{aligned}
R & - 22g\cos 35° = 0 \\
\therefore R & = 22g\cos 35° \\
R & = 176.6
\end{aligned}
$$

Normal reaction is 180 N (2 s.f.).

As the friction is limiting, $F = \mu R$

$$
\begin{aligned}
\therefore F & = 0.125 \times 176.6 \\
& = 22.1 \ (3 \text{ s.f.})
\end{aligned}
$$

Friction is 22 N (2 s.f.)

Substitute into equation ∗ to give

$$\begin{aligned} T &= 22g\sin 35 - 22.0.76... \\ &= 101.6 \\ &= 102 \ (3 \text{ s.f.}) \end{aligned}$$

Tension is 100 N (2 s.f.)

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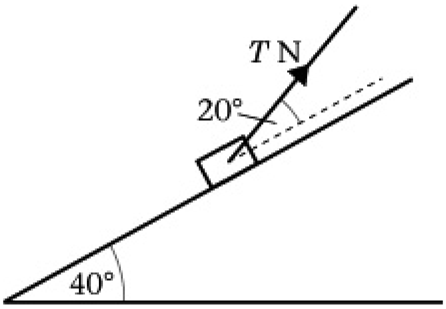
Statics of a particle
Exercise C, Question 11

Question:

A box of mass 0.5 kg is placed on a plane which is inclined at an angle of 40 ° to the horizontal. The coefficient of friction between the box and the plane is $\frac{1}{5}$. The box is kept in equilibrium by a light inextensible string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20 ° with the plane, as shown in the diagram. The box is in limiting equilibrium and may be modelled as a particle. The tension in the string is *T* N. Find *T*

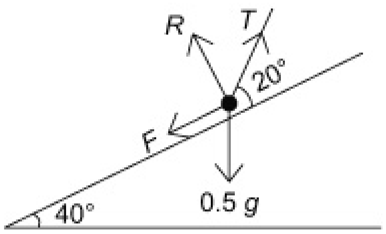


a if the box is about to move up the plane,

b if the box is about to move down the plane.

Solution:

a



Let the normal reaction be *R* and the friction force be *F*.

In part **a** *F* acts down the plane and in part **b** *F* ' acts up the plane.

R (↖)

$$R + T\sin\ 20° - 0.5g\cos\ 40° = 0$$
$$\therefore R = 0.5g\cos\ 40° - T\sin\ 20°$$

R (↗)

$$T\ \cos\ 20° - F - 0.5g\sin\ 40° = 0$$
$$\therefore F = T\ \cos\ 20° - 0.5g\sin\ 40°$$

As the friction is limiting $F = \mu R$

$$\therefore T\cos 20° - 0.5g\sin 40° = \frac{1}{5}(0.5g\cos 40° - T\sin 20°)$$

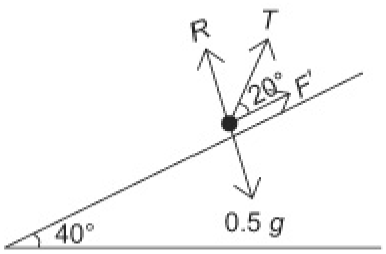
$$\therefore T\cos 20° + \frac{1}{5}T\sin 20° = 0.1g\cos 40° + 0.5g\sin 40°$$

$$\therefore T(\cos 20° + \frac{1}{5}\sin 20°) = 3.900$$

$$\therefore T = \frac{3.900}{\cos 20° + 0.2\sin 20°} = 3.87\text{ N}$$

Tension is 3.9 N (2 s.f.)

b



Let the normal reaction be R and the friction force be F'.

The change from **a** is the direction of the friction force.

$\therefore R = 0.5g\cos 40° - T\sin 20°$ as before
and $F' = 0.5g\sin 40° - T\cos 20°$ (i.e. $F' = -F$)

As $F' = \mu R$

$$0.5g\sin 40° - T\cos 20° = \frac{1}{5}(0.5g\cos 40° - T\sin 20°)$$

$$\therefore 0.5g\sin 40° - 0.1g\cos 40° = T\cos 20° - \frac{1}{5}T\sin 20°$$

$$\therefore 2.399 = T(\cos 20° - 0.2\sin 20°)$$

$$\begin{aligned}\therefore T &= \frac{2.399}{(\cos 20° - 0.2\sin 20°)}\\ &= 2.75\\ &= 2.8\text{N } (2\text{ s.f.})\end{aligned}$$

Tension is 2.8 N (2 s.f.).

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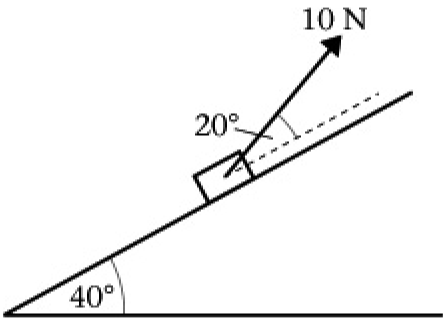
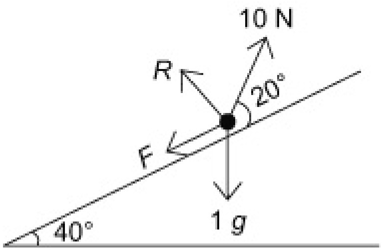
Statics of a particle
Exercise C, Question 12

Question:

A box of mass 1 kg is placed on a plane, which is inclined at an angle of 40 ° to the horizontal. The box is kept in equilibrium by a light inextensible string, which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20 ° with the plane, as shown in the diagram. The box is in limiting equilibrium and may be modelled as a particle. The tension in the string is 10 N and the coefficient of friction between the box and the plane is μ. Find μ if the box is about to move up the plane.



Solution:



R (↖)

$R + 10\sin 20° - g\cos 40° = 0$

$$\begin{aligned}\therefore R &= g\cos 40° - 10\sin 20° \\ &= 4.087\end{aligned}$$

R (↗)

$$10 \cos 20° - F - g\sin 40° = 0$$

$$\begin{aligned}\therefore F &= 10 \cos 20° - g\sin 40° \\ &= 3.0976...\end{aligned}$$

As the friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{3.0976}{4.087}$$

$$= 0.758 \ (\text{3 s.f.})$$

So $\mu = 0.76 \ (\text{2 s.f.})$

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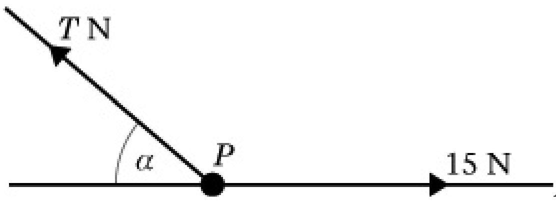
Statics of a particle
Exercise D, Question 1

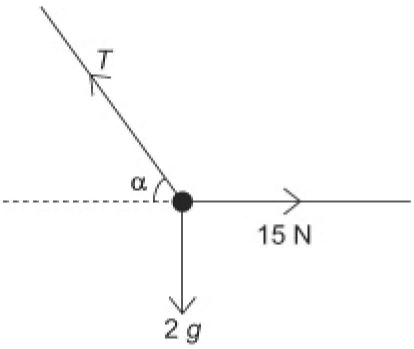
Question:



A particle P of mass 2 kg is held in equilibrium under gravity by two light inextensible strings. One string is horizontal and the other is inclined at an angle α to the horizontal, as shown in the diagram. The tension in the horizontal string is 15 N. The tension in the other string is T newtons.

a Find the size of the angle α. **b** Find the value of T.

Solution:



R(→)

$$15 - T\cos\alpha = 0$$
$$\therefore T\cos\alpha = 15 \qquad (1)$$

R(↑)

$$T\sin\alpha - 2g = 0$$
$$\therefore T\sin\alpha = 2g \qquad (2)$$

Divide equation (2) by equation (1)

$$\frac{T\sin\alpha}{T\cos\alpha} = \frac{2g}{15}$$
$$\therefore \tan\alpha = \frac{2g}{15}$$
$$= 1.307$$
$$\therefore \alpha = 52.6°$$

Substitute into equation (1)

$T \cos 52.6° = 15$

$\therefore T = \frac{15}{\cos 52.6°} = 24.7$

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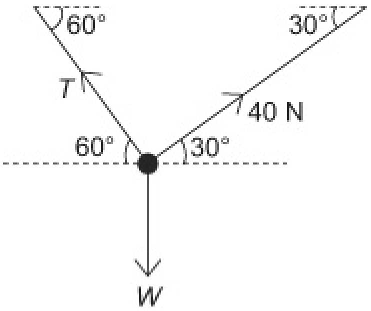
Statics of a particle
Exercise D, Question 2

Question:

A particle is suspended by two light inextensible strings and hangs in equilibrium. One string is inclined at 30 ° to the horizontal and the tension in that string is of magnitude 40 N. the second string is inclined at 60 ° to the horizontal. Calculate in N

a the weight of the particle, **b** the magnitude of the tension in the second string.

Solution:



Let the weight of the particle be WN and the tension in the second string be TN.

R(→)

$40 \cos 30° - T \cos 60° = 0$

$$\therefore T = \frac{40 \cos 30°}{\cos 60°}$$
$$= 40\sqrt{3} \ (= 69.3 \text{ N})$$

R(↑)

$T \sin 60° + 40 \sin 30° - W = 0$

$\therefore W = T \sin 60° + 40 \sin 30°$

Substitute $T = 40\sqrt{3}$, $\sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 30° = \frac{1}{2}$

$$\text{Then } W = 60 + 20$$
$$= 80$$

$\therefore$ the weight of the particle is 80 N and the tension in the second string is 69.3 N

Alternative method

R (↗)

$$40 - W\cos 60° = 0$$
$$\therefore W\cos 60° = 40$$

$$\therefore W = \frac{40}{\cos 60°}$$
$$= 80.$$

b R (↖)

$T - W\sin 60° = 0$

$$\therefore T = W\sin 60°$$
$$= 80\frac{\sqrt{3}}{2}$$
$$= 40\sqrt{3}$$
$$= 69.3 \text{ (3 s.f.)}$$

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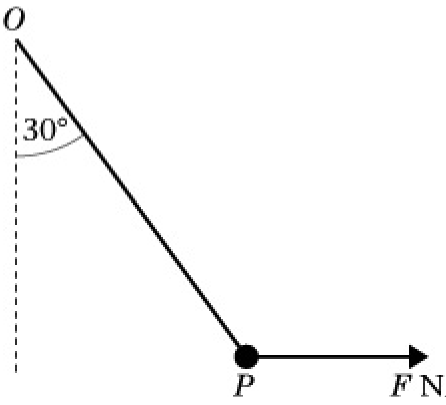
Statics of a particle
Exercise D, Question 3

Question:

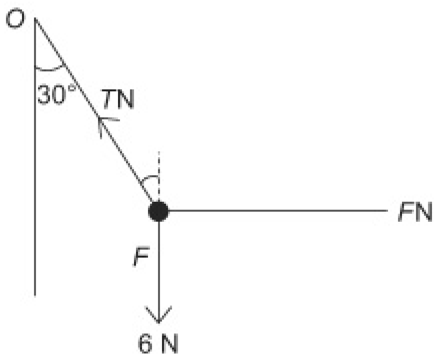


A particle P of weight 6 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. A horizontal force of magnitude F newtons is applied to P. The particle P is in equilibrium under gravity with the string making an angle of 30 ° with the vertical, as shown in the diagram.

Find, to three significant figures,

a the tension in the string, **b** the value of F.

Solution:



Let the tension in the string $= T$ N.

a R(↑)

$T \cos 30° - 6 = 0$

$$\therefore T = \frac{6}{\cos 30°}$$

= The 6.93 tension is 6.93 N (3 s.f.)

b R(→)

$F - T \sin 30° = 0$

$$\therefore F = T \sin 30°$$
$$= T \times \frac{1}{2}$$
$$= 3.46. \ (3 \text{ s.f.})$$

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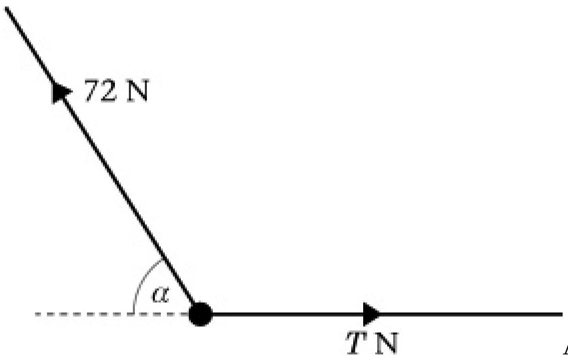
Statics of a particle
Exercise D, Question 4

Question:



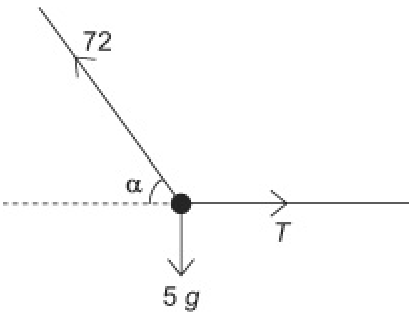
A body of mass 5 kg is held in equilibrium under gravity by two inextensible light ropes. One rope is horizontal, the other is at an angle α to the horizontal, as shown in the diagram.

The tension in the rope inclined at α to the horizontal is 72 N. Find

a the angle α, giving your answer to the nearest degree,

b the value of T to the nearest whole number.

Solution:



a R(↑)

$72 \sin \alpha - 5g = 0$

$$\therefore \sin \alpha = \frac{5g}{72}$$
$$= 0.681$$
$$\therefore \alpha = 42.9 \text{ (3 s.f.)}$$
$$= 43° \text{ (nearest degree)}$$

b R(→)

$T - 72 \cos \alpha = 0$

$$\therefore T = 72 \cos \alpha$$
$$= 52.8$$

$\therefore$ Tension is 53 N to the nearest Newton

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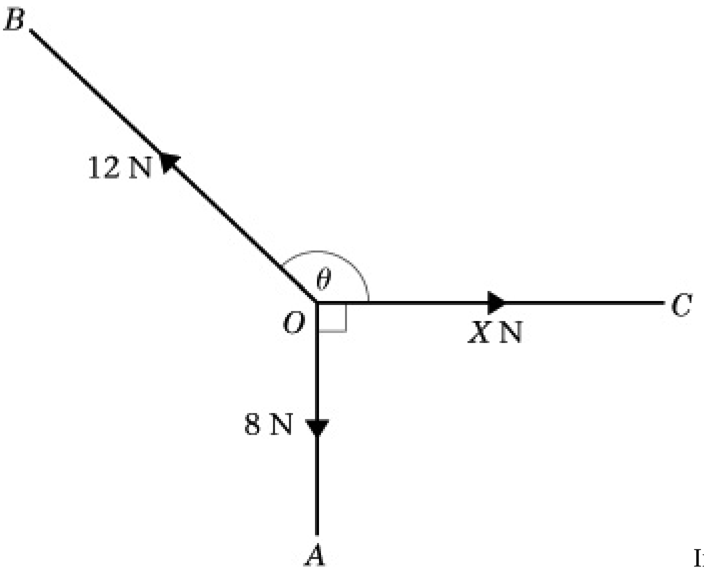
Statics of a particle
Exercise D, Question 5

Question:

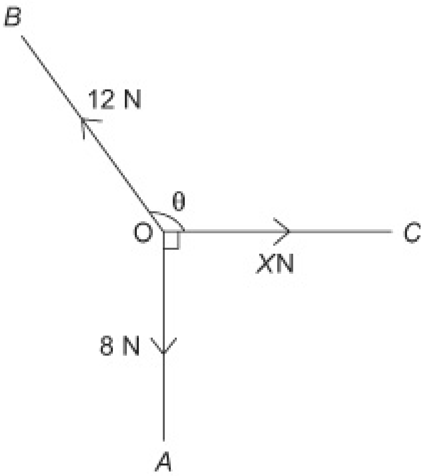


In the diagram, $\angle AOC = 90°$ and $\angle BOC = \theta°$. A particle at *O* is in equilibrium under the action of three coplanar forces. The three forces have magnitudes 8 N, 12 N and *X*N and act along *OA*, *OB* and *OC* respectively. Calculate

a the value, to one decimal place, of θ,

b the value, to two decimal places, of *X*.

Solution:



a

R(↑)

$12 \cos (\theta - 90°) - 8 = 0$

$$\therefore \cos(\theta - 90°) = \frac{8}{12}$$
$$\therefore \theta - 90° = 48.2°$$
$$\therefore \theta = 138.2° \ (1 \text{ d.p.})$$

b R(→)

$X - 12 \sin(\theta - 90°) = 0$

$$\therefore X = 12 \sin 48.2°$$
$$= 8.95 \ (2 \text{ d.p.})$$

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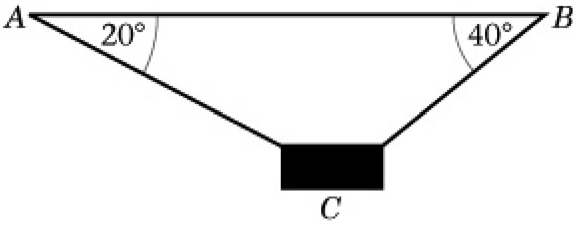
Statics of a particle
Exercise D, Question 6

Question:

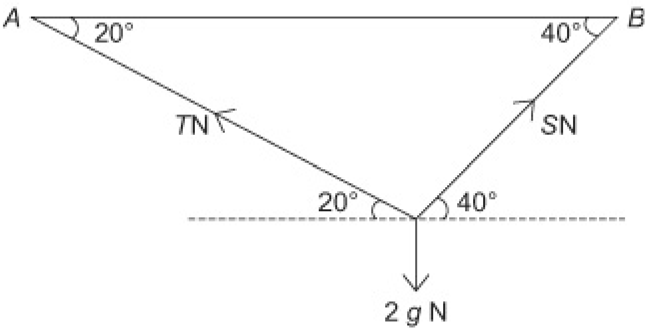
The two ends of a string are attached to two points A and B of a horizontal beam. A package of mass 2 kg is attached to the string at the point C. When the package hangs in equilibrium $\angle BAC = 20°$ and $\angle ABC = 40°$, as shown below.



By modelling the package as a particle and the string as light and inextensible, find, to three significant figures,

a the tension in AC, **b** the tension in BC.

Solution:



Let the tension in AC be TN and the tension in BC be SN.

R(→)

$S \cos 40° - T \cos 20° = 0$ (1)

R(↑)

$S \sin 40° + T \sin 20° - 2g = 0$ (2)

Solve the simultaneous equations (1) and (2)

$(1) \times \sin 40° - (2) \times \cos 40°$

$-T \sin 40° \cos 20° - T \sin 20° \cos 40° + 2g \cos 40° = 0$

$\therefore T(\sin 40° \cos 20° + \cos 40° \sin 20°) = 2g \cos 40°$

$$\therefore T = \frac{2g \cos 40°}{\sin 40° \cos 20° + \cos 40° \sin 20°}$$

$$= 17.3 \ [\text{NB } \sin 40° \cos 20° + \cos 40° \sin 20° = \sin 60°]$$

Substitute the value of *T* into equation (1)

$$\text{Then } S = \frac{T \cos 20°}{\cos 40°}$$

$$= 21.3$$

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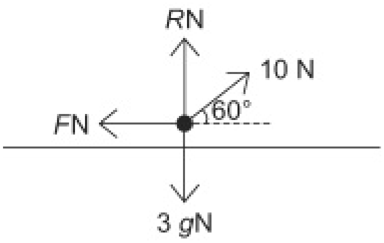
Statics of a particle

Exercise D, Question 7

Question:

A block of mass 3 kg rests on a rough, horizontal table. When a force of magnitude 10 N acts on the block at an angle of 60 ° to the horizontal in an upwards direction, the block is on the point of slipping. Calculate, to two significant figures, the value of the coefficient of friction between the block and the table.

Solution:



Let the normal reaction be R, N the friction force be FN and the coefficient of friction be μ.

R(→)

$$\begin{aligned} 10 \cos 60° - F &= 0 \\ \therefore F &= 10 \cos 60° \\ F &= 5 \end{aligned}$$

R(↑)

$R + 10 \sin 60° - 3g = 0$

$$\begin{aligned} \therefore R &= 3g - 10 \sin 60° \\ &= 3g - 5\sqrt{3} \\ &= 20.7 \end{aligned}$$

As the block is an the point of slipping, the friction is limiting and $F = \mu R$.

$$\begin{aligned} \therefore \mu &= \frac{F}{R} \\ &= \frac{5}{20.7} \\ &= 0.241\ 0.24 \ (2 \text{ s.f.}) \end{aligned}$$

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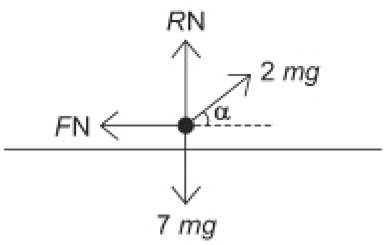
Statics of a particle
Exercise D, Question 8

Question:

A particle P of mass $7m$ is placed on a rough horizontal table, the coefficient of friction between P and the table being μ. A force of magnitude $2mg$, acting upwards at an acute angle α to the horizontal, is applied to P and equilibrium is on the point of being broken by the particle sliding on the table. Given that $\tan\alpha = \frac{5}{12}$, find the value of μ.

Solution:



Let the normal reaction be RN and the friction force be FN.

R(→)

$2mg\cos\alpha - F = 0$

$\therefore F = 2mg\cos\alpha$

R(↑)

$R + 2mg\sin\alpha - 7mg = 0$

$\therefore R = 7mg - 2mg\sin\alpha$

As the friction is limiting, $F = \mu R$.

$$\therefore \mu = \frac{F}{R} = \frac{2mg\cos\alpha}{7mg - 2mg\sin\alpha}$$

As $\tan\alpha = \frac{5}{12}$, $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$

$$\therefore \mu = \frac{2mg \times \frac{12}{13}}{7mg - 2mg \times \frac{5}{13}}$$

$$= \frac{24}{81}$$

$$= \frac{8}{27}$$

$$= 0.296 \ (3 \text{ s.f.})$$

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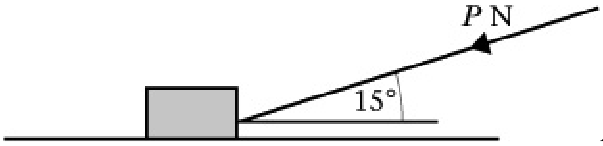
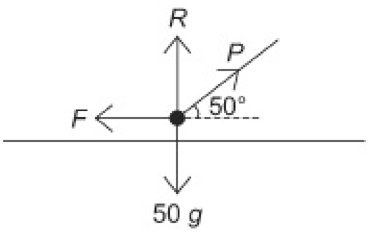
Statics of a particle
Exercise D, Question 9

Question:



A box of mass 50 kg rests on rough horizontal ground. The coefficient of friction between the box and the ground is 0.6. A force of magnitude P newtons is applied to the box at an angle of 15 ° to the horizontal, as shown in the diagram, and the box is now in limiting equilibrium. By modelling the box as a particle find, to three significant figures, the value of P.

Solution:



Let the normal reaction be RN and the frictional force be FN.

R(→)

$$F - P\cos 15^\circ = 0$$
$$\therefore F = P\cos 15^\circ$$

R(↑)

$R - P\sin 15^\circ - 50g = 0$

$\therefore R = P\sin 15^\circ + 50g$

As the friction is limiting, $F = \mu R$

$\therefore P\cos 15^\circ = 0.6(P\sin 15^\circ + 50g)$

$\therefore P\cos 15^\circ - 0.6P\sin 15^\circ = 30g$

$\therefore P(\cos 15^\circ - 0.6\sin 15^\circ) = 30g$

$$\therefore P = \frac{30g}{\cos 15^\circ - 0.6\sin 15^\circ}$$
$$\therefore P = 363$$

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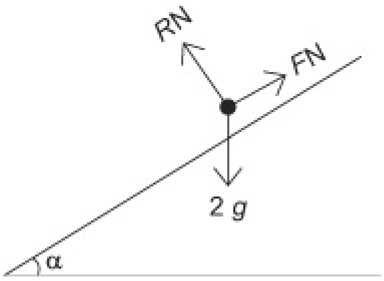
Statics of a particle
Exercise D, Question 10

Question:

A book of mass 2 kg rests on a rough plane inclined at an angle $\alpha°$ to the horizontal. Given that the coefficient of friction between the book and the plane is 0.2, and that the book is on the point of slipping down the plane, find, to the nearest degree, the value of α.

Solution:



R (↗)

$F - 2g \sin \alpha = 0$

$\therefore F = 2g \sin \alpha$

R (↖)

$R - 2g \cos \alpha = 0$

$\therefore R = 2g \cos \alpha$

As the friction is limiting, $F = \mu R$

$\therefore 2g \sin \alpha = 0.2 \times 2g \cos \alpha$

Divide both sides by $2g \cos \alpha$

$$\therefore \tan \alpha = 0.2$$
$$\therefore \alpha = 11.3°$$
$$\alpha = 11° \text{ (to the nearest degree)}$$

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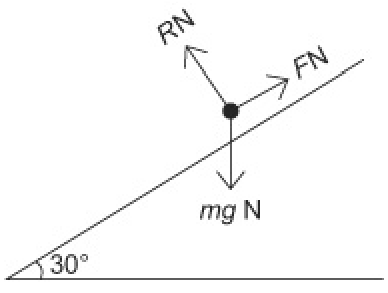
Statics of a particle
Exercise D, Question 11

Question:

a A book is placed on a desk lid which is slowly tilted. Given that the book begins to slide when the inclination of the lid to the horizontal is 30 ° , find the coefficient of friction between the book and the desk lid.

b State an assumption you have made about the book when forming the mathematical model you used to solve part **a**.

Solution:



Let the normal reaction be RN, the friction be FN and the mass be m kg.

a

R (↗)

$F - mg \sin 30° = 0$

R (↖)

$R - mg \cos 30° = 0$

As the friction is limiting $F = \mu R$

$\therefore mg \sin 30° = \mu mg \cos 30°$

Divide both sides by $mg \cos 30°$.

$$\frac{mg \sin 30°}{mg \cos 30°} = \mu$$

$$\begin{aligned} \therefore \mu &= \tan 30° \\ &= \frac{\sqrt{3}}{3} \\ &= 0.577 \end{aligned}$$

b The book was modelled as a particle.

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Statics of a particle
Exercise D, Question 12

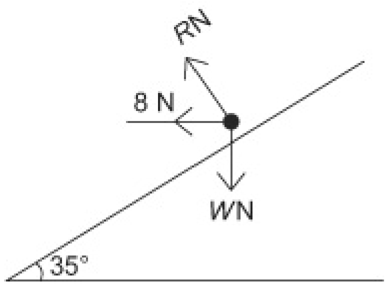
Question:

A particle is placed on a smooth plane inclined at 35 ° to the horizontal. The particle is kept in equilibrium by a horizontal force, of magnitude 8 N, acting in the vertical plane containing the line of greatest slope of the inclined plane through the particle. Calculate, in N to one decimal place,

a the weight of the particle,

b the magnitude of the force exerted by the plane on the particle.

Solution:



Let the normal reaction be RN and let the weight of the particle be WN.

a R (↗)

$8 \cos 35° - W \sin 35° = 0$

$$\therefore W \sin 35° = 8 \cos 35°$$

$$\therefore W = 8 \frac{\cos 35°}{\sin 35°}$$

$$W = 11.4$$

b R (↖)

$R - 8 \sin 35° - W \cos 35° = 0$

$$\therefore R = 8 \sin 35° + W \cos 35°$$

$$R = 13.9$$

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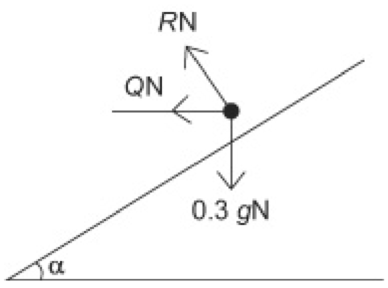
Statics of a particle
Exercise D, Question 13

Question:

A particle of mass 0.3 kg lies on a smooth plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle is held in equilibrium by a horizontal force of magnitude Q newtons. The line of action of this force is in the same vertical plane as a line of greatest slope of the inclined plane. Calculate the value of Q, to one decimal place.

Solution:



Let the normal reaction force be RN.

R (↗)

$$
\begin{aligned}
Q\cos\alpha - 0.3g\sin\alpha &= 0 \\
\therefore Q\cos\alpha &= 0.3g\sin\alpha \\
\therefore Q &= \frac{0.3g\sin\alpha}{\cos\alpha} \\
&= 0.3g\tan\alpha
\end{aligned}
$$

But $\tan\alpha = \frac{3}{4}$

$$
\begin{aligned}
\therefore Q &= 0.3g \times \frac{3}{4} \\
&= 2.2 \ (1 \text{ d.p.})
\end{aligned}
$$

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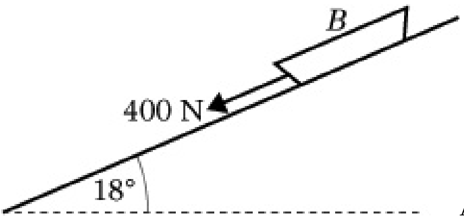
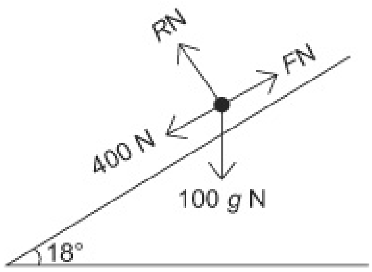
Statics of a particle
Exercise D, Question 14

Question:



A small boat B of mass 100 kg is standing on a ramp which is inclined at 18° to the horizontal. A force of magnitude 400 N is applied to B and acts down the ramp as shown in the diagram. The boat is in limiting equilibrium on the point of sliding down the ramp. Find the coefficient of friction between B and the ramp, giving your answer to two decimal places.

Solution:



Let the normal reaction be RN and the friction be FN.

R (↗)

$$F - 400 - 100g \sin 18^\circ = 0$$
$$\therefore F = 400 + 100g \sin 18^\circ$$
$$= 702.8$$

R (↖)

$$R - 100g \cos 18^\circ = 0$$
$$\therefore R = 100g \cos 18^\circ$$
$$= 932.0$$

As the Friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$
$$= 0.754$$
$$= 0.75 \ (2 \text{ d.p.}).$$

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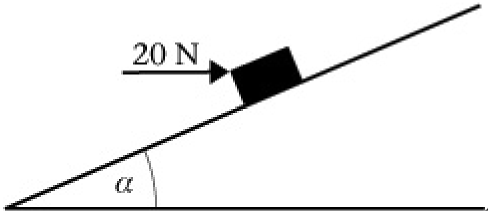
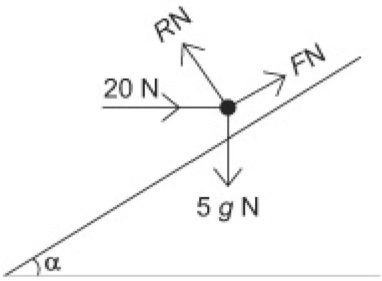
Statics of a particle
Exercise D, Question 15

Question:



A parcel of mass 5 kg lies on a rough plane inclined at an angle of α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The parcel is held in equilibrium by the action of a horizontal force of magnitude 20 N, as shown in the diagram. The force acts in a vertical plane through a line of greatest slope of the plane. The parcel is on the point of sliding down the plane. Find the coefficient of friction between the parcel and the plane.

Solution:



Let the normal reaction be RN and the friction be FN.

R (↗)

$$F - 5g \sin \alpha + 20 \cos \alpha = 0$$
$$\therefore F = 5g \sin \alpha - 20 \cos \alpha$$

As $\tan \alpha = \frac{3}{4}$, $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$

$\therefore F = 3g - 16$

R (↖)

$$R - 20 \sin \alpha - 5g \cos \alpha = 0$$
$$\therefore R = 20 \sin \alpha + 5g \cos \alpha$$
$$= 12 + 4g$$

As the friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{3g - 16}{12 + 4g}$$

$$= \frac{13.4}{51.2}$$

$$= 0.262 \text{ (3 s.f.)}$$

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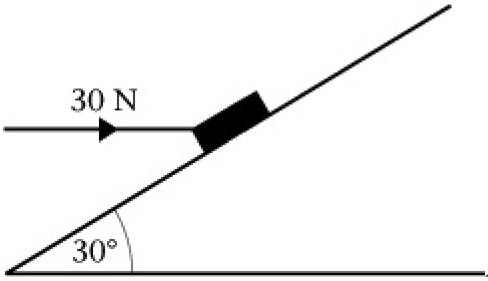
Statics of a particle
Exercise D, Question 16

Question:



A small parcel of mass 3 kg is held in equilibrium on a rough plane by the action of a horizontal force of magnitude 30 N acting in a vertical plane through a line of greatest slope. The plane is inclined at an angle of 30 ° to the horizontal, as shown in the diagram.

The parcel is modelled as a particle. The parcel is on the point of moving up the slope.

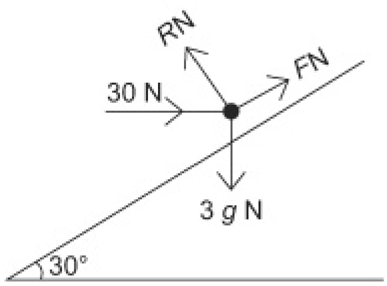
a Draw a diagram showing all the forces acting on the parcel.

b Find the normal reaction on the parcel.

c Find the coefficient of friction between the parcel and the plane.

Solution:

a



Let the normal reaction be *R*N and the friction be *F*N.

b R (↗)

$30 \cos 30° - F - 3g \sin 30° = 0$

$$\begin{aligned} \therefore F &= 30 \cos 30° - 3g \sin 30° \\ &= 11.28 \end{aligned}$$

R (↖)

$$\begin{aligned} R &- 30 \sin 30° - 3g \cos 30° = 0 \\ \therefore R &= 30 \sin 30° + 3g \cos 30° \\ &= 40.46 \end{aligned}$$

c As the friction is limiting, $F = \mu R$.

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{11.28}{40.46}$$

$$= 0.2788$$

$$= 0.279 \ (3 \text{ s.f.}) .$$

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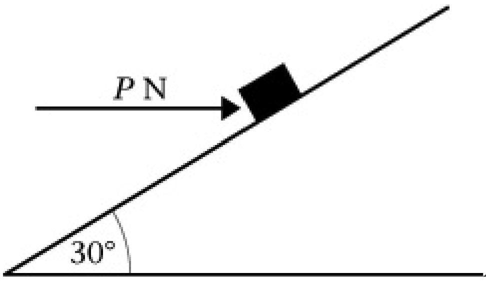
Statics of a particle
Exercise D, Question 17

Question:



A box of mass 6 kg lies on a rough plane inclined at an angle of 30 ° to the horizontal. The box is held in equilibrium by means of a horizontal force of magnitude *P* newtons, as shown in the diagram.

The line of action of the force is in the same vertical plane as a line of greatest slope of the plane. The coefficient of friction between the box and the plane is 0.4. The box is modelled as a particle. Given that the box is in limiting equilibrium and on the point of moving up the plane, find,

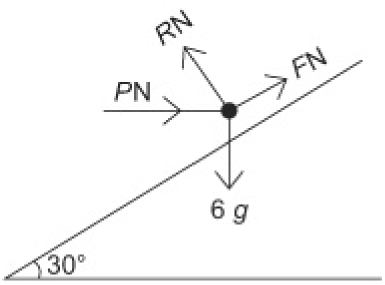
a the normal reaction exerted on the box by the plane,

b the value of *P*.

The horizontal force is removed.

c Show that the box will now start to move down the plane.

Solution:



Let the normal reaction be *R*N and the friction be *F*N.

a R(↑)

$R \cos 30° - F \sin 30° - 6g = 0$ (1)

As the friction is limiting, $F = \mu R$

$\therefore F = 0.4R$

Substitute into equation (1)

$\therefore R \cos 30° - 0.4R \sin 30° = 6g$

$\therefore R(\cos 30° - 0.4 \sin 30°) = 6g$

$\therefore R = \dfrac{6g}{\cos 30° - 0.4 \sin 30°}$

$R = 88.3$

b R(→)

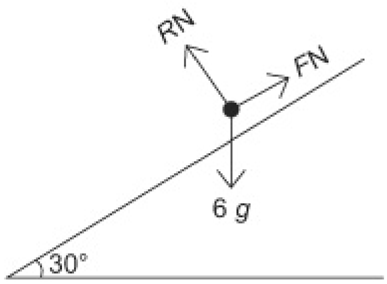
$P - R \sin 30° - F \cos 30° = 0$

$\therefore P = R \sin 30° + 0.4R \cos 30°$ (as $F = 0.4R$)

$= 88.3(\sin 30° + 0.4 \cos 30°)$

$= 74.7$

c



Draw a new sketch with the new normal reaction RN and friction force FN.

R (↖)

$R - 6g \cos 30° = 0$

$\therefore R = 6g \cos 30°$

$= 50.9$

$\therefore \mu R = 20.4$

R (↗)

Resultant force down the plane $= 6g \sin 30° - F$

$= 29.4 - F$

As maximum value F can take is 20.4, there is a resultant force of 9 N down the plane and the box will move.

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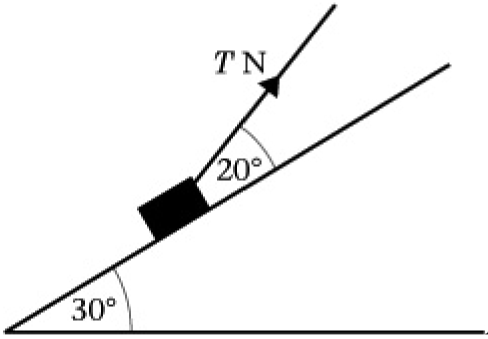
Statics of a particle
Exercise D, Question 18

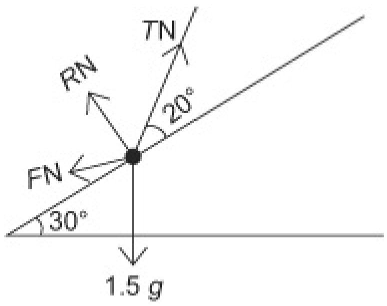
Question:



A box of mass 1.5 kg is placed on a plane which is inclined at an angle of 30° to the horizontal. The coefficient of friction between the box and plane is $\frac{1}{3}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in the diagram.

The box is in limiting equilibrium and is about to move up the plane. The tension in the string is T newtons. The box is modelled as a particle. Find the value of T.

Solution:



Let the normal reaction be RN and the friction force be FN.

R (↗)

$T\cos 20° - F - 1.5g\sin 30° = 0$

$\therefore F = T\cos 20° - 1.5g\sin 30°$

R (↖)

$$R + T\sin 20° - 1.5g\cos 30° = 0$$
$$\therefore R = 1.5g\cos 30° - T\sin 20°$$

As the box is in limiting equilibrium, $F = \mu R$

$$\therefore T\cos 20° - 1.5g\sin 30° = \frac{1}{3}(1.5g\cos 30° - T\sin 20°)$$

$$\therefore T\cos 20° + \frac{1}{3}T\sin 20° = 0.5g\cos 30° + 1.5g\sin 30°$$

$$\therefore T\left(\cos 20° + \frac{1}{3}\sin 20°\right) = 0.5g\cos 30° + 1.5g\sin 30°$$

$$\therefore T = \frac{11.59}{1.054}$$

$$\therefore T = 11.0$$

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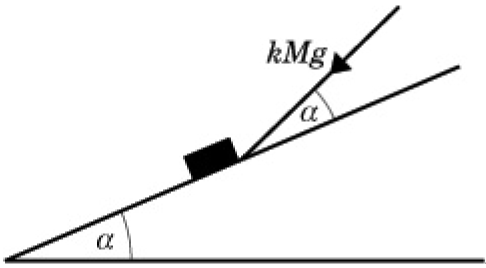
Statics of a particle
Exercise D, Question 19

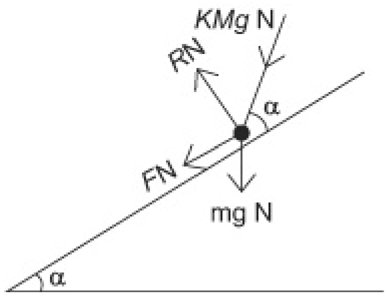
Question:



A rough slope is inclined at an angle α to the horizontal, where $\alpha < 45°$. A small parcel of mass M is at rest on the slope, and the coefficient of friction between the parcel and the slope is μ. A force of magnitude kMg, where k is a constant, is applied to the parcel in a direction making an angle α with a line of greatest slope, as shown in the diagram.

The line of action of the force is in the same vertical plane as the line of greatest slope. Given that the parcel is on the point of moving down the slope, show that: $k = \frac{\mu \cos\alpha - \sin\alpha}{\cos\alpha - \mu \sin\alpha}$

Solution:



Let R be the normal reaction and F be the force of friction.

R (↗)

$$F - kMg\cos\alpha - Mg\sin\alpha = 0$$
$$\therefore F = Mg(k\cos\alpha + \sin\alpha)$$

R (↖)

$$R - kMg\sin\alpha - Mg\cos\alpha = 0$$
$$\therefore R = Mg(k\sin\alpha + \cos\alpha)$$

As the friction is limiting; $F = \mu R$.

$$\therefore Mg(k\cos\alpha + \sin\alpha) = \mu Mg(k\sin\alpha + \cos\alpha)$$

$$\therefore k\cos\alpha - \mu k\sin\alpha = \mu\cos\alpha - \sin\alpha$$

$$\therefore k(\cos\alpha - \mu\sin\alpha) = \mu\cos\alpha - \sin\alpha$$

$$\therefore k = \frac{\mu\cos\alpha - \sin\alpha}{\cos\alpha - \mu\sin\alpha}$$

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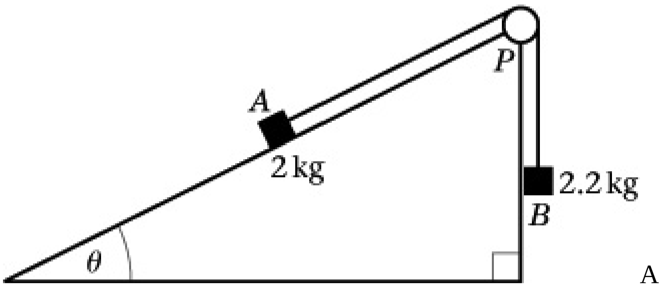
Statics of a particle
Exercise D, Question 20

Question:



A parcel A of mass 2 kg rests on a rough slope inclined at an angle θ to the horizontal, where $\tan\ \theta = \frac{3}{4}$. A string is attached to A and passes over a small smooth pulley fixed at P. The other end of the string is attached to a weight B of mass 2.2 kg, which hangs freely, as shown in the diagram.

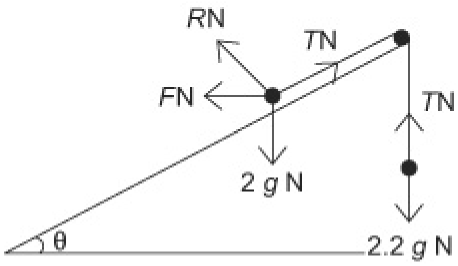
The parcel A is in limiting equilibrium and about to slide up the slope. By modelling A and B as particles and the string as light and inextensible, find

a the normal contact force acting on A,

b the coefficient of friction between A and the slope.

Solution:

a



Let the normal reaction be RN and the friction be FN acting down the plane. Let the tension in the string be TN.

Consider the 2 kg mass.

R (↗)

$$R - 2g\ \cos\ \theta = 0$$
$$\therefore R = 2g\ \cos\ \theta$$

As $\tan\ \theta = \frac{3}{4}$, $\sin\ \theta = \frac{3}{5}$ and $\cos\ \theta = \frac{4}{5}$

$$\therefore R = 2g \times \frac{4}{5}$$

$$= \frac{8g}{5}$$

$$R = 15.68$$
$$= 15.7 \ (3 \text{ s.f.})$$

b Consider the 2.2 kg mass.

R(↑)

$$T - 2.2g = 0$$
$$\therefore T = 2.2g$$

Consider the 2 kg mass.

R (↗)

$$T - F - 2g \sin \theta = 0$$
$$\therefore F = T - 2g \sin \theta$$

But $T = 2.2g$ and $\sin \theta = \frac{3}{5}$

$$\therefore F = 2.2g - 2g \times \frac{3}{5}$$
$$\text{i.e.}: F = g$$
$$\therefore F = 9.8$$

As the Friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{9.8}{15.68} \ \left(\text{or } g \div \frac{8g}{5}\right)$$

$$= 0.625 \text{ or } \frac{5}{8}$$

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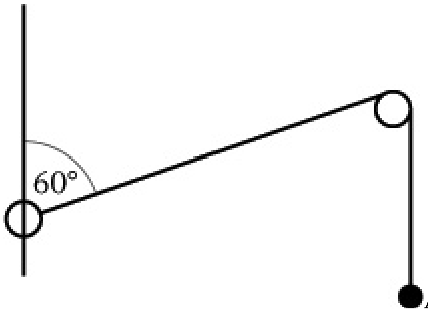
Statics of a particle
Exercise D, Question 21

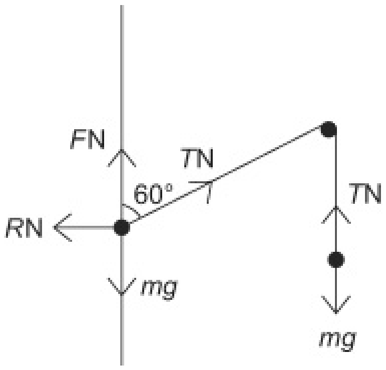
Question:



A light inextensible string passes over a smooth peg, and is attached at one end to a particle of mass *m* kg and at the other end to a ring also of mass *m* kg. The ring is threaded on a rough vertical wire as shown in the diagram. The system is in limiting equilibrium with the part of the string between the ring and the peg making an angle of 60 ° with the vertical wire.

Calculate the coefficient of friction between the ring and the wire, giving your answer to three significant figures.

Solution:



Let the normal reaction, the friction and the tension be *R*N, *F*N and *T*N respectively.

Consider the particle of mass *m*.

R(↑)

$$T - mg = 0$$
$$\therefore T = mg$$

Consider the ring.

R(→)

$$T \sin 60 - R = 0$$
$$\therefore R = T \sin 60°$$
$$= mg \sin 60°$$
$$= mg \frac{\sqrt{3}}{2}$$

R(↑)

$$
\begin{aligned}
F + T\cos 60° - mg &= 0 \\
\therefore F &= mg - T\cos 60° \\
&= mg - \tfrac{1}{2}mg \\
&= \tfrac{1}{2}mg
\end{aligned}
$$

As the friction is limiting $F = \mu R$

$$
\begin{aligned}
\therefore \mu &= \frac{F}{R} \\
&= \frac{1}{2}mg \div mg\frac{\sqrt{3}}{2} \\
&= \frac{1}{\sqrt{3}} \\
&= 0.577 \ (3 \text{ s.f.})
\end{aligned}
$$

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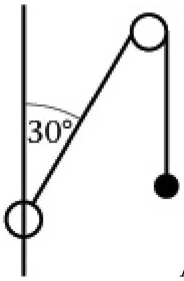
Statics of a particle
Exercise D, Question 22

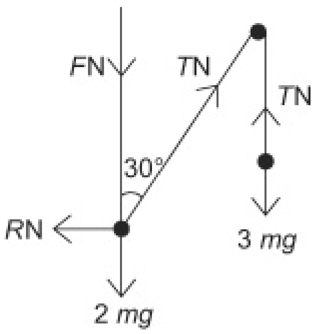
Question:



A light inextensible string passes over a smooth peg, and is attached at one end to a particle of mass $3m$ kg and at the other end to a ring of mass $2m$ kg. The ring is threaded on a rough vertical wire as shown in the diagram. The system is in limiting equilibrium with the part of the string between the ring and the peg making an angle of $30°$ with the vertical wire.

Calculate the coefficient of friction between the ring and the wire, giving your answer to three significant figures.

Solution:



Let the normal reaction, the friction and the tension be RN, FN and TN respectively.

Consider the 3 m kg particle.

R(↑)

$$T - 3mg = 0$$
$$\therefore T = 3mg$$

Consider the 2 m kg ring.

R(→)

$$R - T\sin 30° = 0$$
$$\therefore R = T\sin 30°$$
$$\therefore R = 3mg\sin 30°\text{, as } T = 3mg$$
$$\therefore R = \frac{3mg}{2}$$

R(↑)

$$T\cos 30° - F - 2mg = 0$$

$$\therefore F = T\cos 30° - 2mg$$

$$= 3mg\frac{\sqrt{3}}{2} - 2mg = 0.598mg$$

As friction is limiting: $F = \mu R$

$$\therefore \mu = \frac{F}{R} = \frac{0.598mg}{1.5mg}$$

$$= 0.399 \ (3 \text{ s.f.})$$

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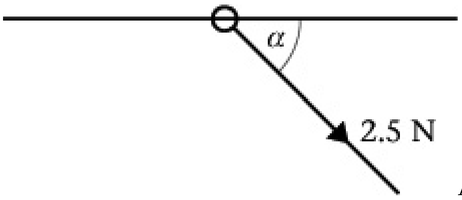
Statics of a particle
Exercise D, Question 23

Question:



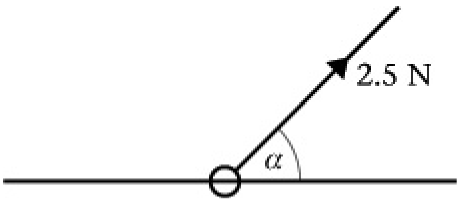
A ring of mass 0.3 kg is threaded on a fixed, rough horizontal curtain pole. A light inextensible string is attached to the ring. The string and the pole lie in the same vertical plane. The ring is pulled downwards by the string which makes an angle α to the horizontal, where $\tan\alpha = \frac{1}{4}$ as shown in the diagram.

The tension in the string is 2.5 N.

Given that, in this position, the ring is in limiting equilibrium,

a find the coefficient of friction between the ring and the pole.

The direction of the string is now altered so that the ring is pulled upwards. The string lies in the same vertical plane as before and again makes an angle α with the horizontal, as shown in the diagram below.



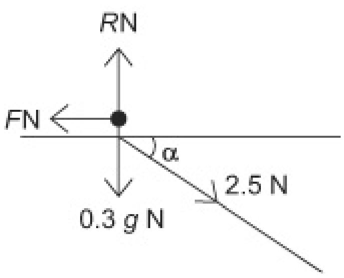
The tension in the string is again 2.5 N.

b Find the normal reaction exerted by the pole on the ring.

c State whether the ring is in equilibrium in the position shown in the second figure, giving a brief justification for your answer. You need make no further detailed calculation of the forces acting.

Solution:

a



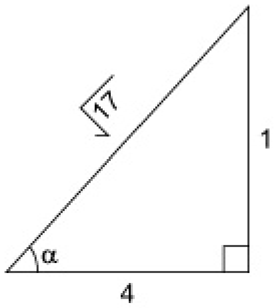
Let the normal reaction and friction force be RN and FN respectively.

R(→)

$2.5 \cos \alpha - F = 0$

$\therefore F = 2.5 \cos \alpha$.

As $\tan \alpha = \frac{1}{4}$, $\sin \alpha = \frac{1}{\sqrt{17}}$ and $\cos \alpha = \frac{4}{\sqrt{17}}$ from Pythagoras' Theorem.



$\therefore F = 2.5 \times \frac{4}{\sqrt{17}} = 2.425$

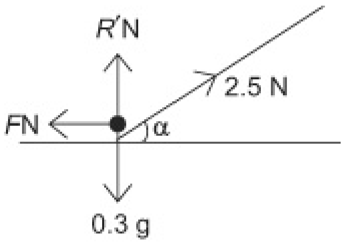
R(↑)

$$R - 0.3g - 2.5 \sin \alpha = 0$$
$$\therefore R = 0.3g + 2.5 \times \frac{1}{\sqrt{17}}$$
$$= 3.546$$

As the friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$
$$= \frac{2.425}{3.546}$$
$$= 0.684 \ (3 \text{ s.f.})$$

b



Let the new normal reaction be R' N, the friction remains FN.

R(↑)

$$R' + 2.5 \sin \alpha = 0.3g$$
$$\therefore R' = 0.3g - 2.5 \times \frac{1}{\sqrt{17}}$$
$$= 2.33$$

c To maintain equilibrium F would need to be 2.425 N (as in part **a**)

But the maximum value F can take is μR

i.e. : 0.684×2.33 (1.596 N)

As $2.425 > 1.596$ the ring is not in equilibrium.

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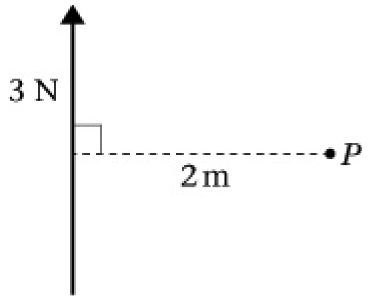
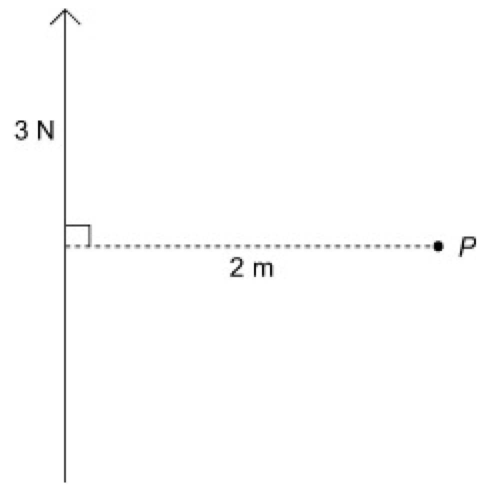
Moments
Exercise A, Question 1

Question:

Calculate the moment about *P* of each of these forces acting on a lamina.



Solution:



Moment $= 3 \times 2 = 6$ Nm clockwise

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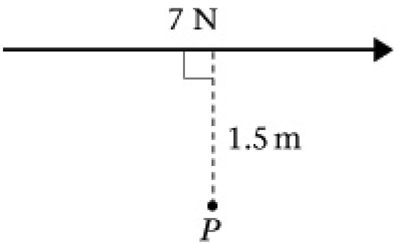
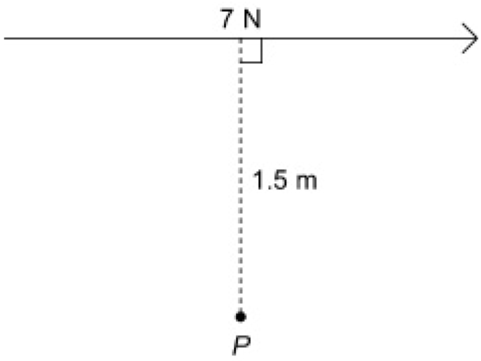
Moments
Exercise A, Question 2

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Moment $= 7 \times 1.5 = 10.5$ Nm clockwise

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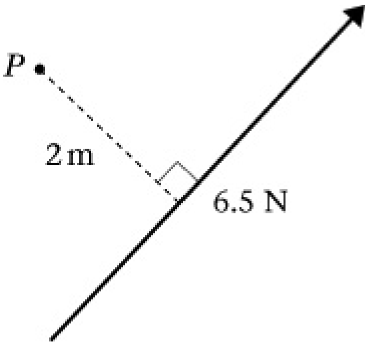
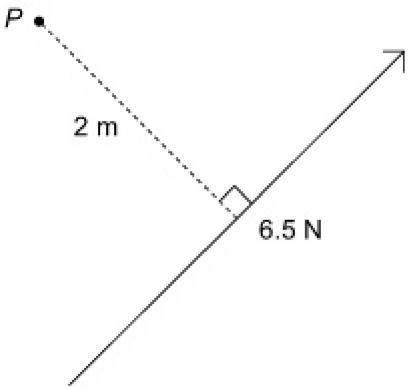
Moments
Exercise A, Question 3

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Moment $= 2 \times 6.5 = 13$ Nm anticlockwise

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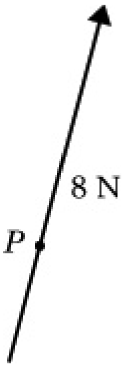
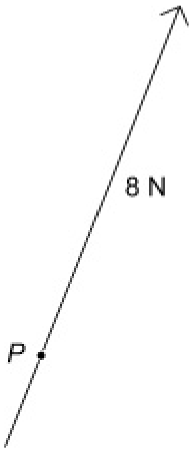
Moments
Exercise A, Question 4

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Line of action passes through P so the distance is zero.
Moment = 0 Nm
(No turning effect)

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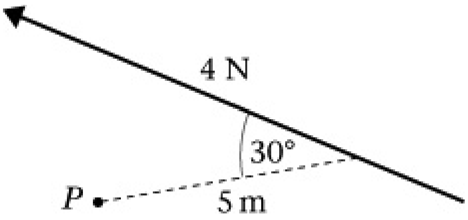
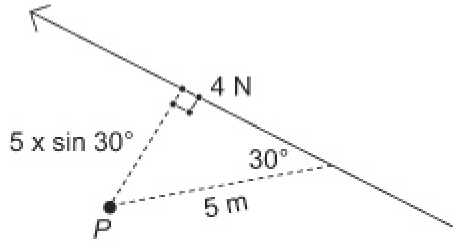
Moments
Exercise A, Question 5

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Draw in the right angled triangle.

Perpendicular distance = $5 \times \sin 30°$
Moment $= 4 \times 5 \sin 30°$
$= 10$ Nm anticlockwise

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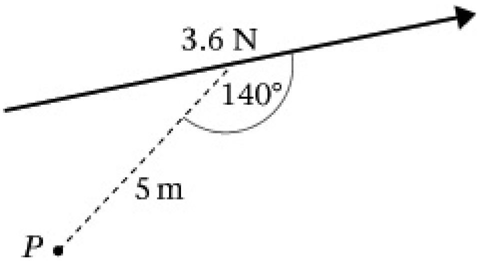
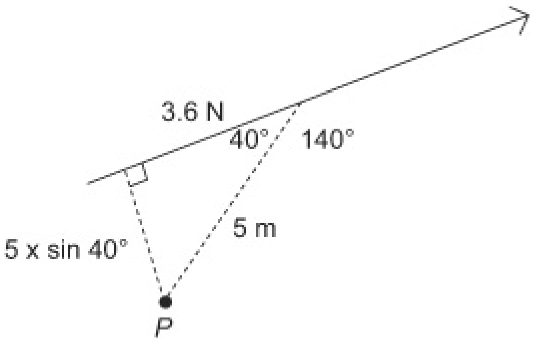
Moments
Exercise A, Question 6

Question:

Calculate the moment about *P* of each of these forces acting on a lamina.



Solution:



$\boxed{\text{Draw in the right angled triangle.}}$

The angle inside the triangle is $180° - 140° = 40°$,
so the distance $= 5 \times \sin 40°$
Moment $= 3.6 \times 5 \sin 40°$
≈ 11.6 Nm clockwise

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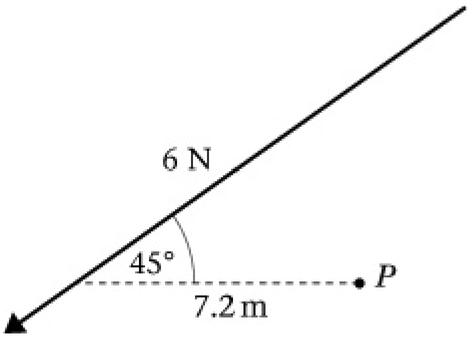
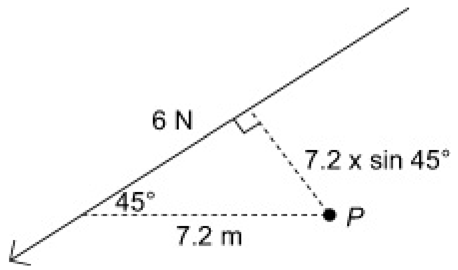
Moments
Exercise A, Question 7

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Distance $= 7.2 \times \sin 45°$
Moment $= 6 \times 7.2 \sin 45°$
≈ 30.5 Nm anticlockwise

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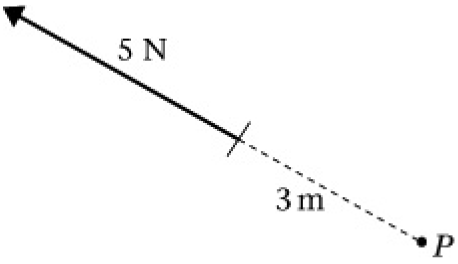
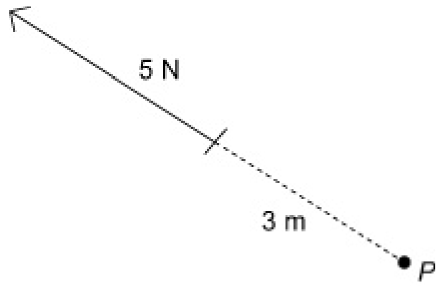
Moments
Exercise A, Question 8

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



The line of action of the force acts through P, so
moment = 0 Nm

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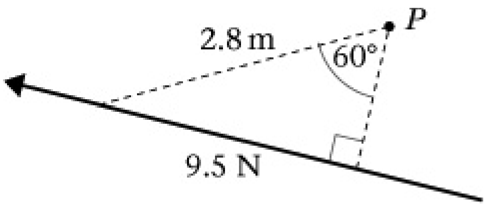
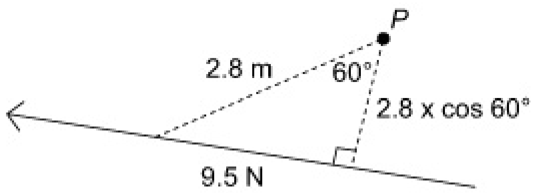
Moments
Exercise A, Question 9

Question:

Calculate the moment about *P* of each of these forces acting on a lamina.



Solution:



Distance $= 2.8 \times \cos 60°$

Moment $= 9.5 \times 2.8 \cos 60°$

$= 13.3$ Nm clockwise

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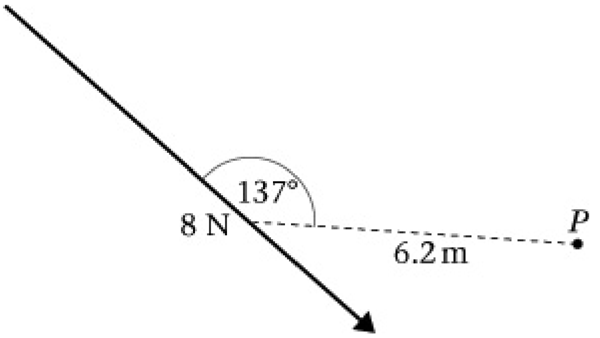
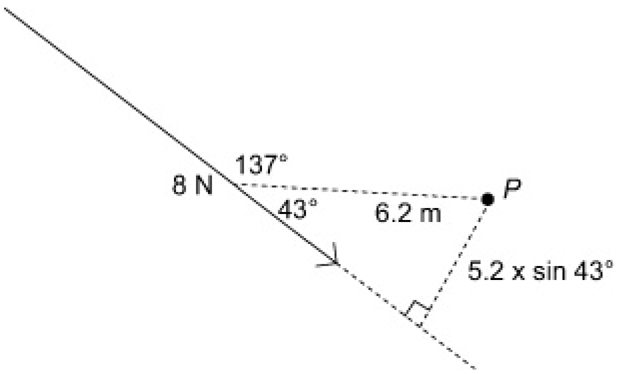
Moments
Exercise A, Question 10

Question:

Calculate the moment about *P* of each of these forces acting on a lamina.



Solution:



Draw in the right angled triangle.

The angle inside the triangle $= 180° - 137°$
$= 43°$
Distance $= 6.2 \times \sin 43°$
Moment $= 8 \times 6.2 \sin 43°$
≈ 33.8 Nm anticlockwise

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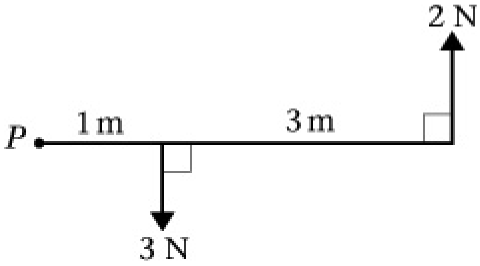
Moments
Exercise B, Question 1

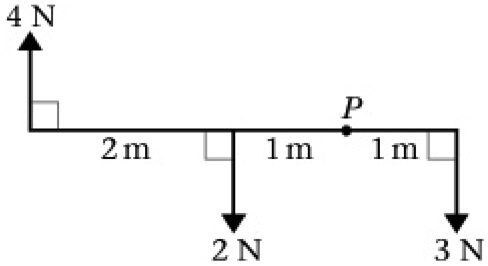
Question:

These diagrams show sets of forces acting on a light rod. For each rod, calculate the sum of the moments about *P*.

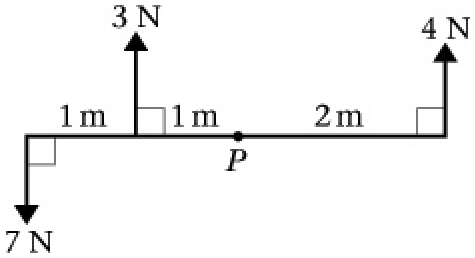
a



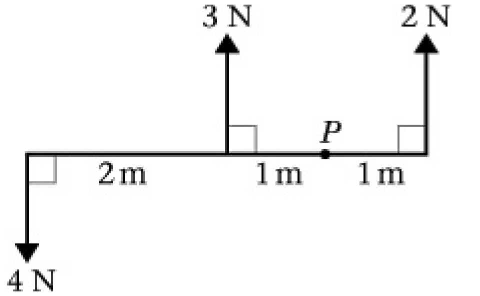
b



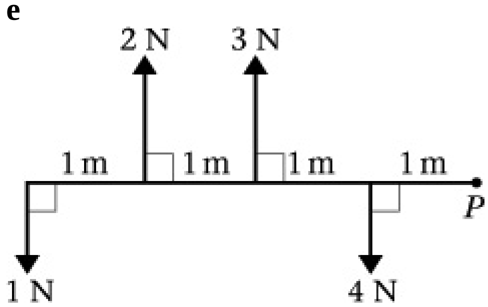
c



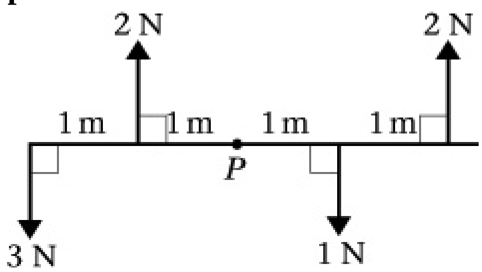
d



e

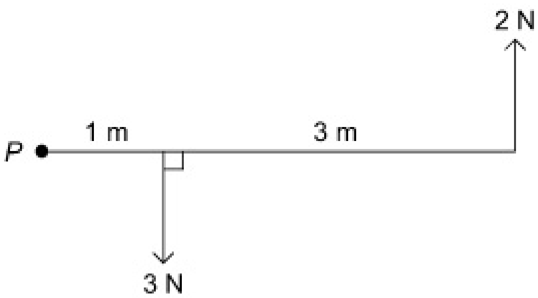


f



Solution:

a



↻ (Total of moments for forces acting clockwise about P): $3 \times 1 = 3$ Nm

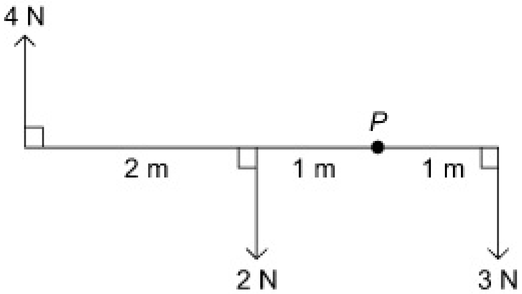
↺ (Total of moments for forces acting anticlockwise about P):

$(1 + 3) \times 2 = 8$ Nm

Sum of moments $= 8 - 3$

$= 5$ Nm anticlockwise

b



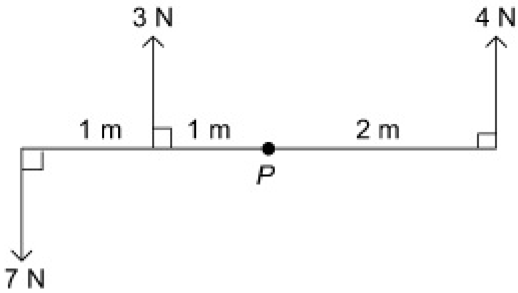
↻ $4 \times (2+1) + 3 \times 1 = 15$ Nm

↺ $2 \times 1 = 2$ Nm

Sum of moments $= 15 - 2$

$= 13$ Nm clockwise

c



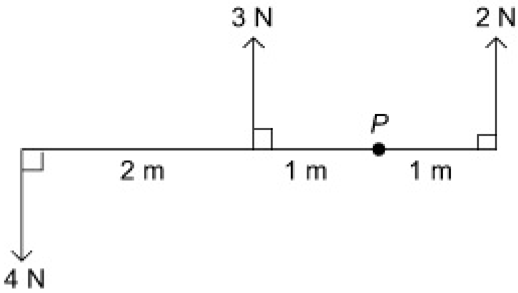
↻ $3 \times 1 = 3$ Nm

↺ $7 \times (1+1) + 4 \times 2 = 22$ Nm

Sum of moments $= 22 - 3$

$= 19$ Nm anticlockwise

d



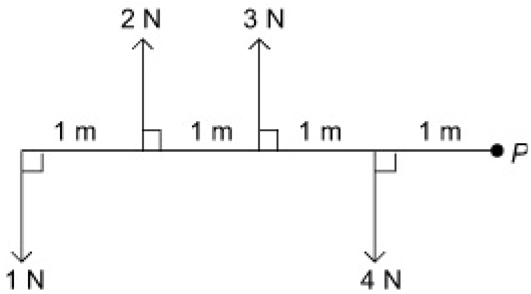
↻ $3 \times 1 = 3$ Nm

↺ $2 \times 1 + 4 \times (2+1) = 14$ Nm

Sum of moments $= 14 - 3$

$= 11$ Nm anticlockwise

e



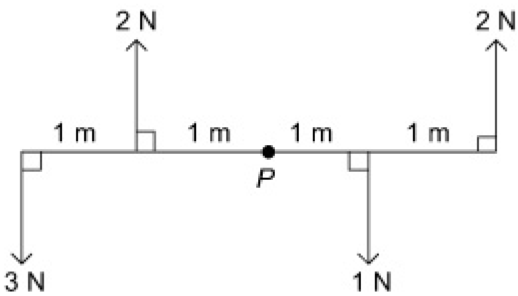
↻ $3 \times (1+1) + 2 \times (1+1+1) = 12$ Nm

↺ $4 \times 1 + 1 \times (1+1+1+1) = 8$ Nm

Sum of moments $= 12 - 8$

$= 4$ Nm clockwise

f



↻ $1 \times 1 + 2 \times 1 = 3$ Nm

↺ $2 \times (1+1) + 3 \times (1+1) = 10$ Nm

Sum of moments $= 10 - 3$

$= 7$ Nm anticlockwise

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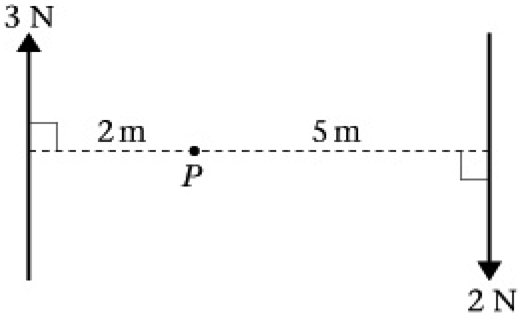
Moments
Exercise B, Question 2

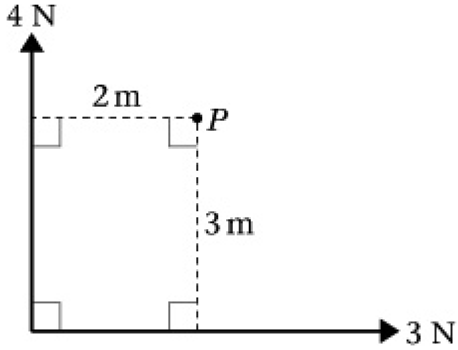
Question:

These diagrams show forces acting on a lamina. In each case, find the sum of the moments of the set of forces about *P*.

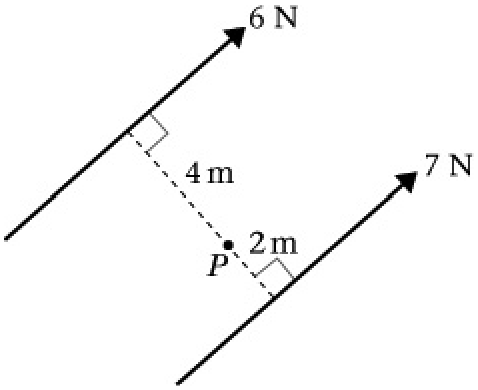
a



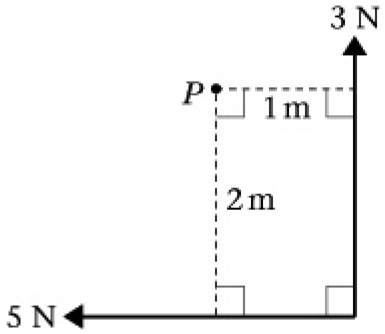
b



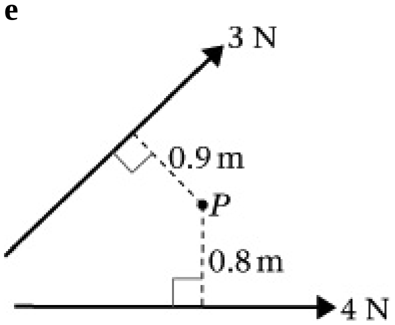
c



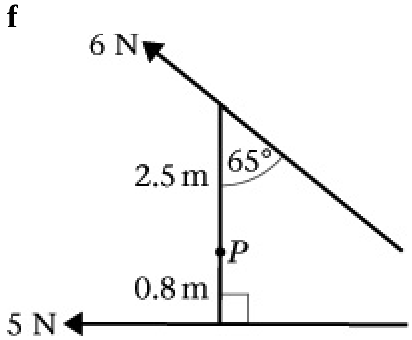
d



e

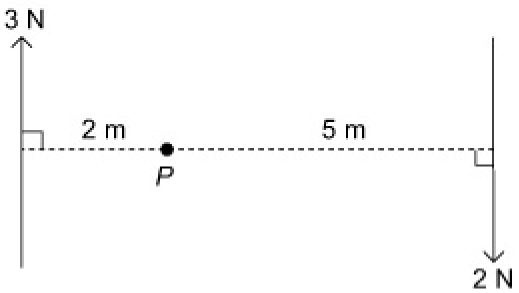


f



Solution:

a

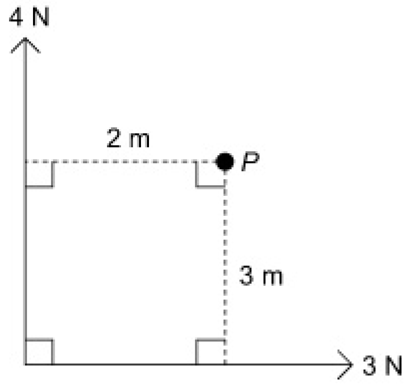


$\circlearrowright$ $3 \times 2 + 2 \times 5 = 16$ Nm

$\circlearrowleft$ None

Sum of moments $= 16$ Nm clockwise

b



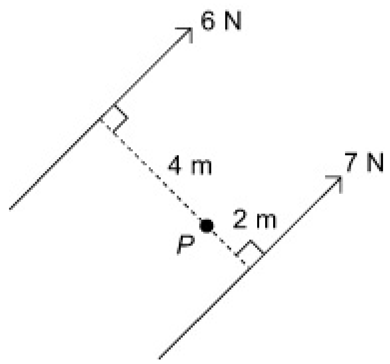
$\circlearrowright$ $4 \times 2 = 8$ Nm

$\circlearrowleft$ $3 \times 3 = 9$ Nm

Sum of moments $= 9 - 8$

$= 1$ Nm anticlockwise

c



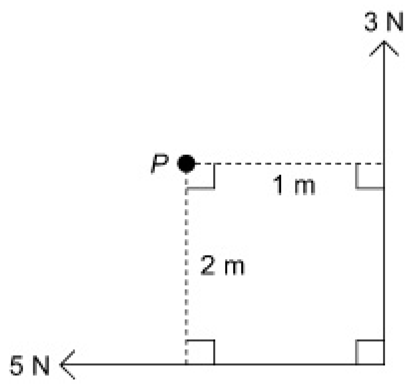
$\circlearrowright$ $6 \times 4 = 24$ Nm

$\circlearrowleft$ $7 \times 2 = 14$ Nm

Sum of moments $= 24 - 14$

$= 10$ Nm clockwise

d



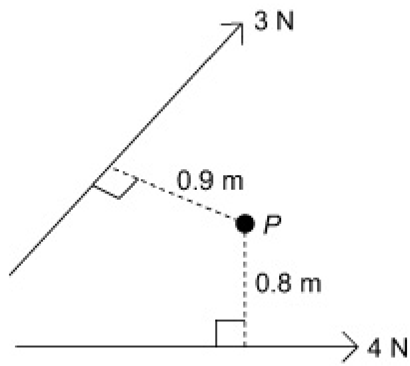
$\circlearrowright$ $5 \times 2 = 10$ Nm

$\circlearrowleft$ $3 \times 1 = 3$ Nm

Sum of moments $= 10 - 3$

$= 7$ Nm clockwise

e



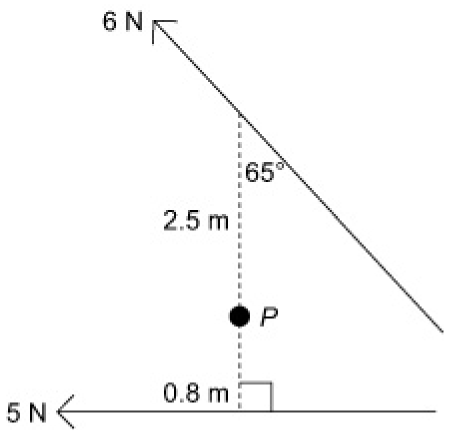
$\circlearrowright$ $3 \times 0.9 = 2.7$ Nm

$\circlearrowleft$ $4 \times 0.8 = 3.2$ Nm

Sum of moments $= 3.2 - 2.7$

$= 0.5$ Nm anticlockwise

f



↻ 5×0.8	$= 4$ Nm
↺ $6 \times (2.5 \times \sin 65°)$	$= 13.594\ldots$ Nm
or $(6 \times \sin 65°) \times 2.5$	
or $(6 \times \cos 25°) \times 2.5$	
Sum of moments	$= 13.594.\ldots - 4$
	≈ 9.59 Nm anticlockwise

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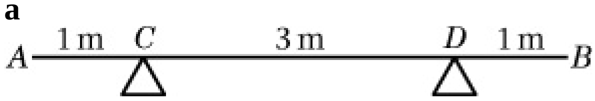
Moments
Exercise C, Question 1

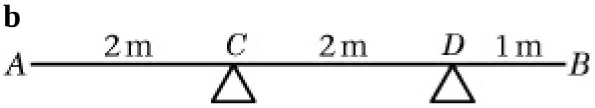
Question:

AB is a uniform rod of length 5 m and weight 20 N. In these diagrams *AB* is resting in a horizontal position on supports at *C* and *D*. In each case, find the magnitudes of the reactions at *C* and *D*.

a

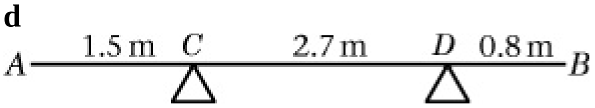


b



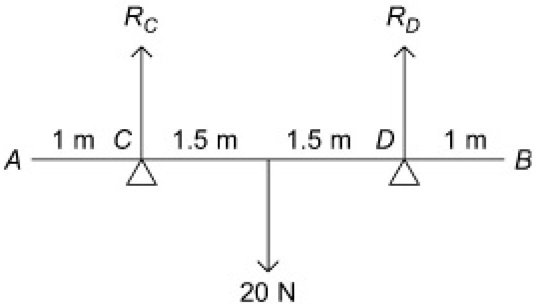
c

d



Solution:

a



R(↑)

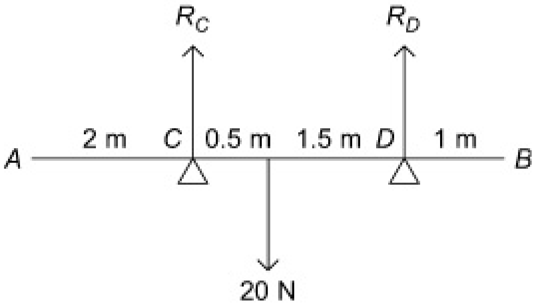
$R_C + R_D = 20$

Taking moments about *C*:

$3 \times R_D = 20 \times 1.5 = 30$

$\Rightarrow R_D = 10$ N and $R_C = 10$ N

b



R(↑)

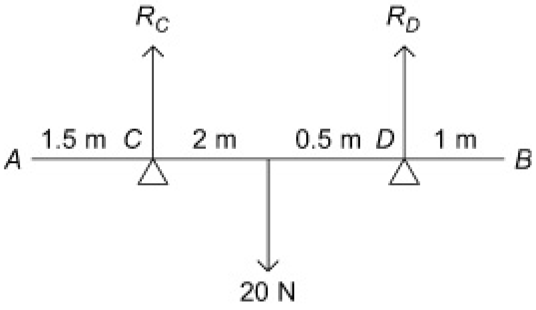
$R_C + R_D = 20$

Taking moments about *C*:

$R_D \times 2 = 20 \times 0.5 = 10$

$\Rightarrow R_D = 5$ N and $R_C = 15$ N

c



R(↑)

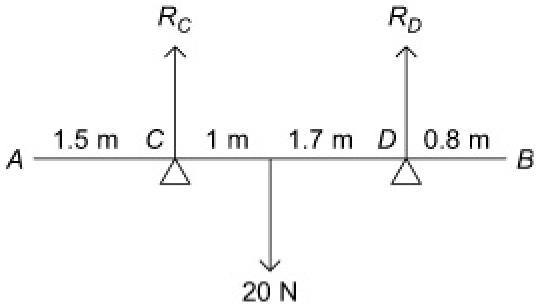
$R_C + R_D = 20$

Taking moments about *C*:

$R_D \times 2.5 = 20 \times 1 = 20$

$\Rightarrow R_D = \frac{20}{2.5} = 8$ N and $R_C = 12$ N

d



R(↑)

$R_C + R_D = 20$

Taking moments about C:

$2.7 \times R_D = 20 \times 1 = 20$

$\Rightarrow R_D = \frac{20}{2.7} \approx 7.4$ N and $R_C \approx 12.6$ N

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Edexcel AS and A Level Modular Mathematics

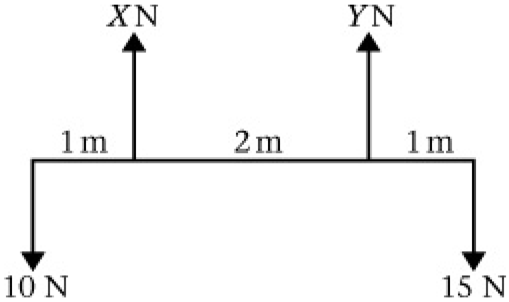
Moments

Exercise C, Question 2

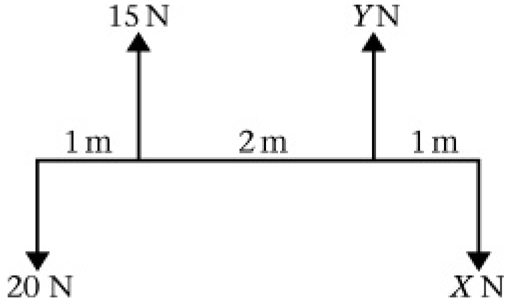
Question:

Each of these diagrams shows a light rod in equilibrium in a horizontal position under the action of a set of forces.

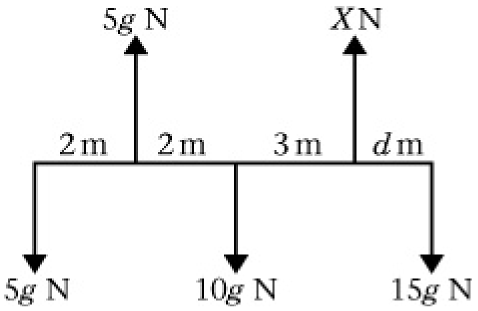
a



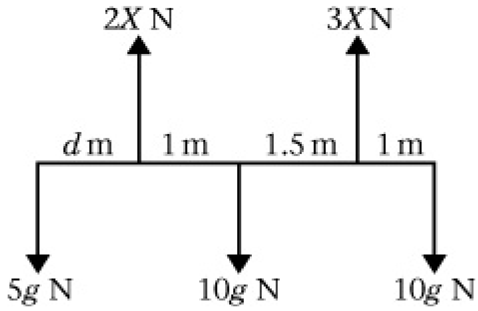
b



c

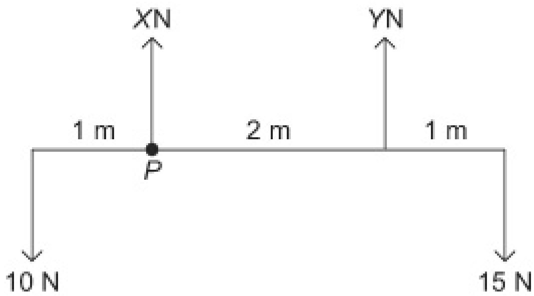


d



Solution:

a



R(↑)

$X + Y = 10 + 15 = 25$

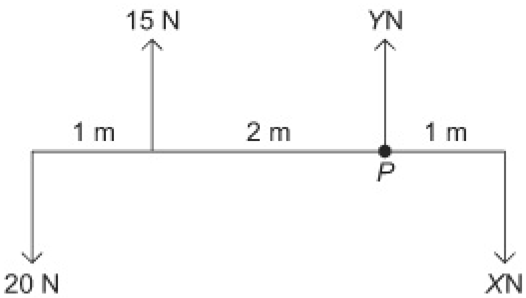
Taking moments about P:

$15 \times (2 + 1) = 10 \times 1 + Y \times 2$

$\Rightarrow 45 = 10 + 2Y, \quad 2Y = 35, Y = 17.5$

$\Rightarrow X = 7.5$ and $Y = 17.5$

b



R(↑)

$15 + Y = 20 + X, \; Y - X = 5$

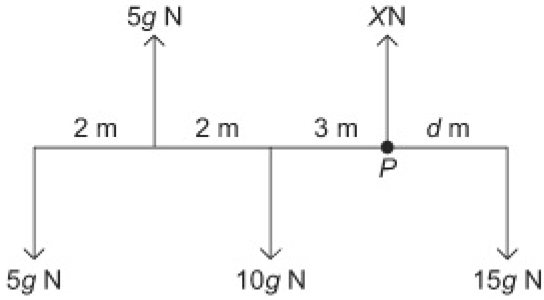
Taking moments about P:

$20 \times (2 + 1) = 15 \times 2 + X \times 1$

$\Rightarrow 60 = 30 + X, \quad X = 30$

$\Rightarrow X = 30$ and $Y = 35$

c



R(↑)

$5g + X = 5g + 10g + 15g = 30g$

$\Rightarrow X = 25g = 245$

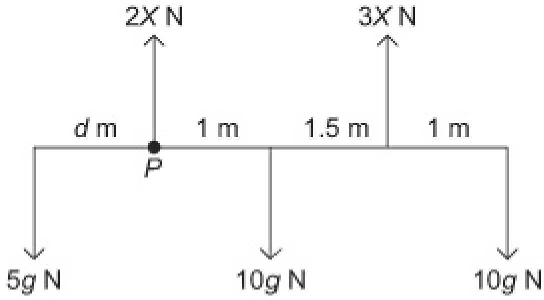
Taking moments about P:

$15g \times d + 5g \times (2 + 3) = 10g \times 3 + 5g \times (2 + 2 + 3)$

$\Rightarrow 15g\ d + 25g = 30g + 35g$

$\Rightarrow 15d = 40$, $d = 2\frac{2}{3}$

d



R(↑)

$2X + 3X = 5g + 10g + 10g = 25g$

$\Rightarrow X = 5g = 49$

Taking moments about P:

$5g \times d + 15g \times (1 + 1.5)$

$= 10g \times 1 + 10g \times (1 + 1.5 + 1)$

$\Rightarrow 5d + 15 \times 2.5 = 10 + 35$, $5d = 7.5$,

$\Rightarrow d = 1.5$

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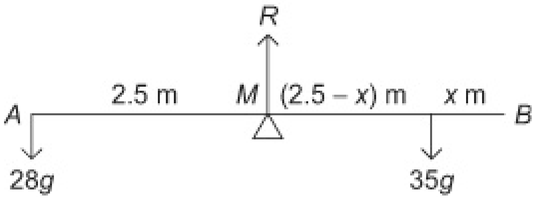
Moments
Exercise C, Question 3

Question:

Jack and Jill are playing on a see-saw made from a uniform plank *AB* of length 5 m pivoted at *M*, the mid-point of *AB*. Jack has mass 35 kg and Jill has mass 28 kg. Jill sits at *A*. Where must Jack sit for the plank to be in equilibrium when horizontal?

Solution:



Suppose that Jack sits x m from B.
Taking moments about the pivot (M):
$28g \times 2.5 = 35g \times (2.5 - x)$
$\Rightarrow 28 \times 2.5 = 35(2.5 - x)$
$5(2.5 - x) = 4 \times 2.5 = 10$
$2.5 - x = 2, \Rightarrow x = 0.5$
Jack sits 0.5 m from B

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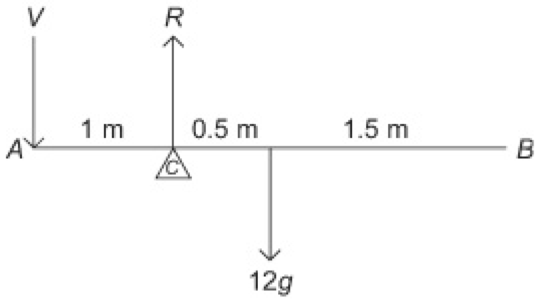
Moments
Exercise C, Question 4

Question:

A uniform rod AB of length 3 m and mass 12 kg is pivoted at C, where $AC = 1$ m. Calculate the vertical force that must be applied at A to maintain equilibrium with the rod horizontal.

Solution:



Suppose that the force required is V N acting vertically downwards at A.
Taking moments about the pivot (C):

$$V \times 1 = 0.5 \times 12g$$

$$\Rightarrow V = 6g = 59 \text{ N (2 s.f.)}$$

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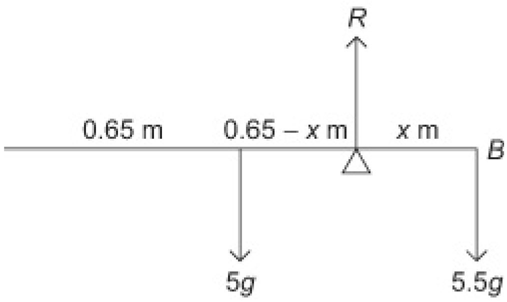
Moments
Exercise C, Question 5

Question:

A broom consists of a broomstick of length 130 cm and mass 5 kg and a broomhead of mass 5.5 kg attached at one end. By modelling the broomstick as a rod and the broomhead as a particle, find where a support should be placed so that the broom will balance horizontally.

Solution:



Let the support be x m from the broomhead.
Taking moments about the support:

$$5.5g \times x = 5g \times (0.65 - x)$$
$$5.5x = 5 \times 0.65 - 5x$$
$$10.5x = 3.25$$
$$x \approx 0.31$$

The support should be 31 cm from the broomhead.

Solutionbank M1
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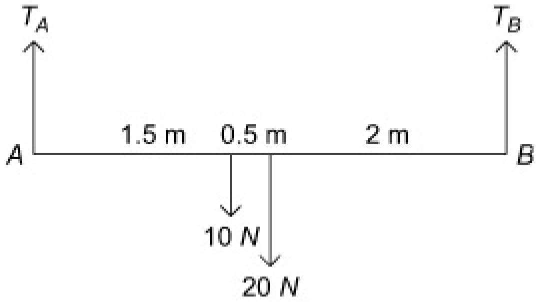
Moments
Exercise C, Question 6

Question:

A uniform rod AB of length 4 m and weight 20 N is suspended horizontally by two vertical strings attached at A and at B. A particle of weight 10 N is attached to the rod at point C, where $AC = 1.5$ m. Find the magnitudes of the tensions in the two strings.

Solution:



Let the tensions in the two strings be T_A and T_B respectively.

R(↑)

$T_A + T_B = 10 + 20 = 30$

Taking moments about point A:

$10 \times 1.5 + 20 \times (1.5 + 0.5) = 4 \times T_B$

$\Rightarrow 4T_B = 15 + 40 = 55$, $T_B = 13.75$ N

and $T_A = 16.25$ N

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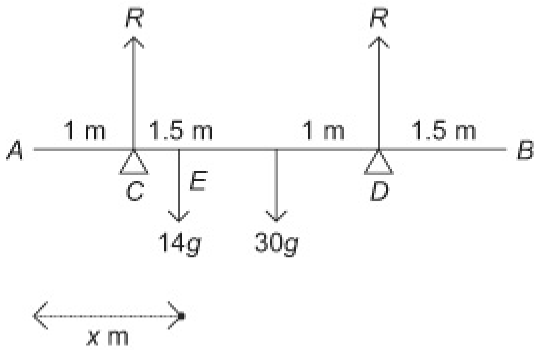
Moments
Exercise C, Question 7

Question:

A uniform plank AB of length 5 m and mass 30 kg is resting horizontally on supports at C and D, where $AC = 1$ m and $AD = 3.5$ m. When a particle of mass 14 kg is attached to the rod at point E the magnitude of the reaction at C is equal to the magnitude of the reaction at D. Find the distance AE.

Solution:



R(↑)

$R + R = 30g + 14g = 44g \Rightarrow R = 22g$

Let distance $AE = x$ m

Taking moments about A:

$R \times 1 + R \times 3.5 = 30g \times 2.5 + 14g \times x$

$\Rightarrow \quad 4.5 \times 22g = 75g + 14gx$

$99 = 75 + 14x$

$\Rightarrow \quad 24 = 14x \text{ , } x \approx 1.71$

$AE \approx 1.71$ m

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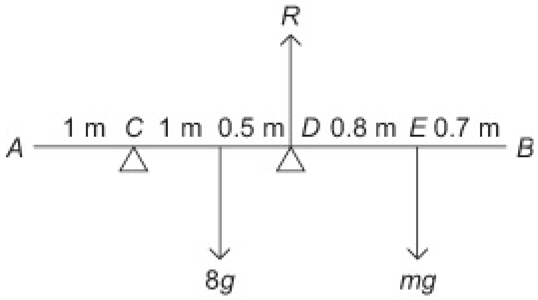
Moments
Exercise C, Question 8

Question:

A uniform rod *AB* has length 4 m and mass 8 kg. It is resting in a horizontal position on supports at points *C* and *D* were *AC* = 1 m and *AD* = 2.5 m. A particle of mass m kg is placed at point *E* where *AE* = 3.3 m. Given that rod is about to tilt about *D*, calculate the value of m.

Solution:



If the rod is about to turn about *D* then the reaction at *C* is zero.
Taking moments about point *D*:

$$8g \times 0.5 = m\ g \times 0.8$$

$$\Rightarrow m = 5$$

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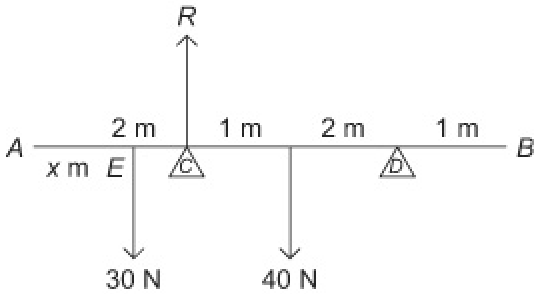
Moments
Exercise C, Question 9

Question:

A uniform bar AB of length 6 m and weight 40 N is resting in a horizontal position on supports at points C and D where $AC = 2$ m and $AD = 5$ m. When a particle of weight 30 N is attached to the bar at point E the bar is on the point of tilting about C. Calculate the distance AE.

Solution:



If the bar is about to tilt about C then the reaction at D is zero.
Let the distance $AE = x$ m
Taking moments about C:
$40 \times 1 = 30 \times (2 - x)$, $40 = 60 - 30x$

$\Rightarrow 30x = 20$, $x = \frac{2}{3}$

The distance $AE = \frac{2}{3}$ m

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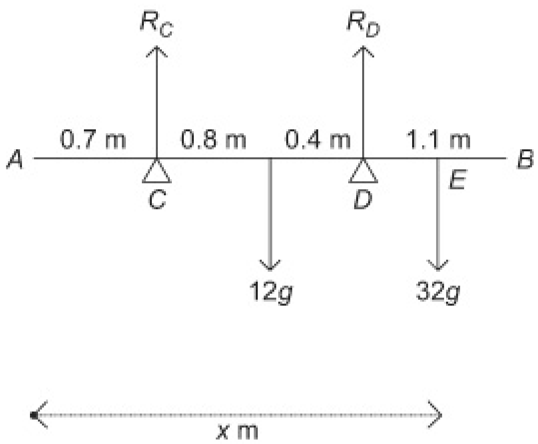
Moments
Exercise C, Question 10

Question:

A plank AB of mass 12 kg and length 3 m is in equilibrium in a horizontal position resting on supports at C and D where $AC = 0.7$ m and $DB = 1.1$ m. A boy of mass 32 kg stands on the plank at point E. The plank is about to tilt about D. By modelling the plank as a uniform rod and the boy as a particle, calculate the distance AE.

Solution:



Let the distance AE be x m.
If the plank is about to tilt about D then $R_C = 0$.
Taking moments about D:

$12g \times 0.4 = 32g \times (x - 1.9)$

$12 \times 0.4 = 32x - 32 \times 1.9$

$32x = 4.8 + 60.8 = 65.6$

$\Rightarrow x = 65.6 \div 32 = 2.05$ m

E is 2.05 m from A

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Moments
Exercise C, Question 11

Question:

A uniform rod *AB* has length 5 m and weight 20 N. The rod is resting on supports at points *C* and *D* where $AC = 2$ m and $BD = 1$ m.

a Find the magnitudes of the reactions at *C* and *D*.

A particle of weight 12 N is placed on the rod at point *A*.

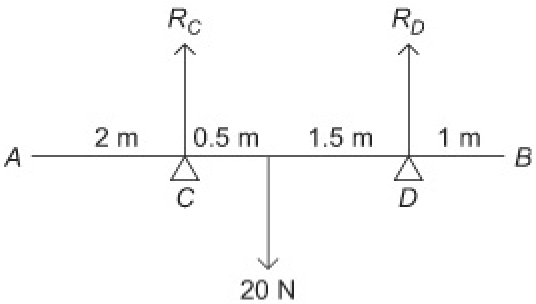
b Show that this causes the rod to tilt about *C*.

A second particle of weight 12 N is placed on the rod at *E* to hold it in equilibrium.

c How far must *E* be from *A*?

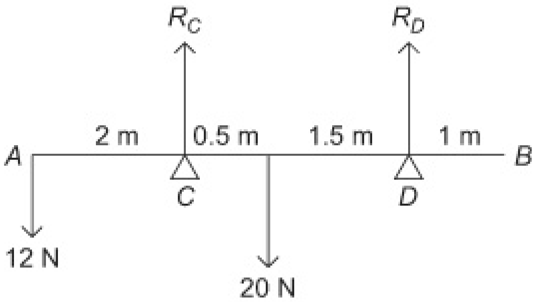
Solution:

a



R(↑)
$R_C + R_D = 20$
Taking moments about C:
$20 \times 0.5 = R_D \times 2$, $R_D = 5$ N
$R_C = 15$ N

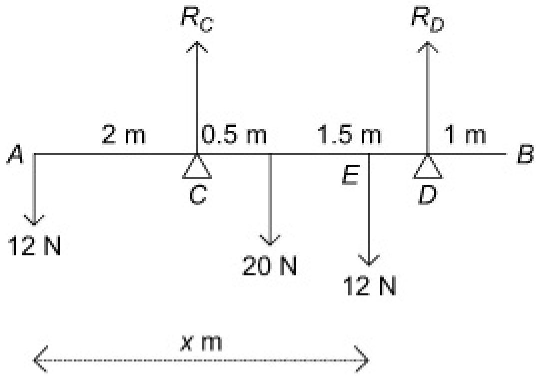
b Adding the weight of 12 N:



Taking moments about *C*:
$20 \times 0.5 = 12 \times 2 + R_D \times 2$
$10 = 24 + 2R_D$
$\Rightarrow R_D$ is negative, which is impossible, therefore there is an anticlockwise moment about *C* – the rod will tilt.

c Adding the second particle:

Let the distance *AE* be *x* m.
If the system is just about to tilt about *C*, taking moments about *C* and $R_D = 0$



$$12 \times (x - 2) + 20 \times 0.5 = 12 \times 2$$
$$12x - 24 + 10 = 24$$
$$12x = 38$$
$$\Rightarrow x = 38 \div 12 \approx 3.17$$

The second particle needs to be 3.17 m from *A* to prevent tilting.

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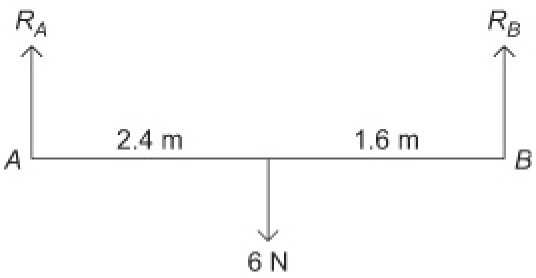
Moments
Exercise D, Question 1

Question:

A non-uniform rod *AB* of length 4 m and weight 6 N rests horizontally on two supports at *A* and *B*. Given that the centre of mass of the rod is 2.4 m from the end *A*, find the reactions at the two supports.

Solution:



R(↑)

$6 = R_A + R_B$

Taking moments about *A*:

$6 \times 2.4 = 4 \times R_B$

$\Rightarrow R_B = 3.6$ N

$R_A = 2.4$ N

The reactions at *A* and *B* are 2.4 N and 3.6 N respectively.

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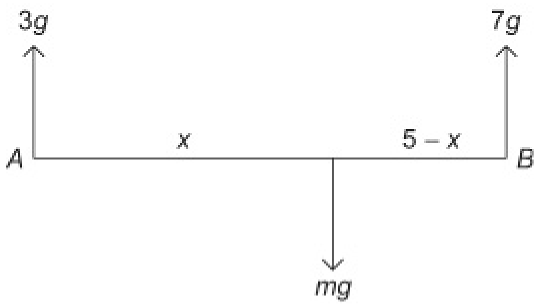
Moments
Exercise D, Question 2

Question:

A non-uniform bar *AB* of length 5 m is supported horizontally on supports at *A* and *B*. The reactions at these supports are 3g N and 7g N respectively. Find the position of the centre of mass.

Solution:



Let m be the mass of the bar.
R(↑) $m\ \ g = 3g + 7g$
$\Rightarrow$ the mass of the bar is 10 kg
Let the centre of mass be x m from A:
Taking moments about A:
$m\ \ g \times x = 7g \times 5$
$\Rightarrow m\ \ x = 35$, $10x = 35$, $x = 3.5$ m
The centre of mass is 3.5 m from A.

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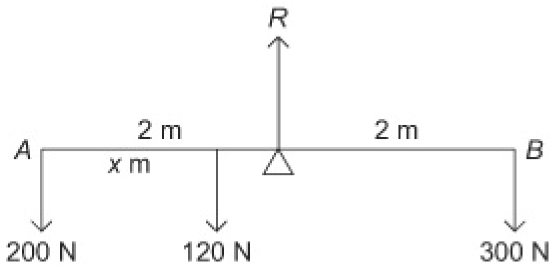
Moments
Exercise D, Question 3

Question:

A non-uniform plank *AB* of length 4 m and weight 120 N is pivoted at its mid-point. The plank is in equilibrium in a horizontal position with a child of weight 200 N sitting at *A* and a child of weight 300 N sitting at *B*. By modelling the plank as a rod and the two children as particles find the distance of the centre of mass of the plank from *A*.

Solution:



Let the centre of mass be x m from A.
Taking moments about the mid-point:

$$120 \times (2 - x) + 200 \times 2 = 300 \times 2$$

$$240 - 120x + 400 = 600$$

$$120x = 40$$

$$\Rightarrow x = \frac{40}{120} = \frac{1}{3}$$

The centre of mass is $\frac{1}{3}$ m from A.

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Moments
Exercise D, Question 4

Question:

A non-uniform rod AB of length 5 m and mass 15 kg rests horizontally suspended from the ceiling by two vertical strings attached to C and D, where $AC = 1\,\text{m}$ and $AD = 3.5\,\text{m}$.

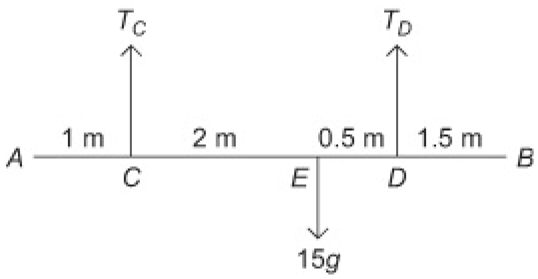
a Given that the centre of mass is at E where $AE = 3\,\text{m}$, find the magnitudes of the tensions in the strings.

When a particle of mass 10 kg is attached to the rod at F the rod is just about to rotate about D.

b Find the distance AF.

Solution:

a



Taking moments about C:

$T_D \times 2.5 = 15g \times 2$

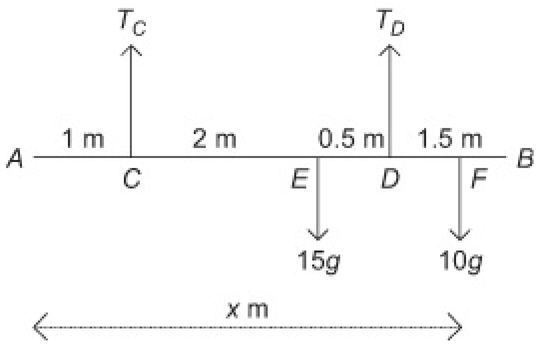
$2.5T_D = 30g$

$\Rightarrow T_D = 12g = 118\text{ N (3 s.f.)}$

R(↑)

$T_C + T_D = 15g\ ,\ T_C = 3g = 29.4\text{ N}$

b



With the particle attached at F, T_C is zero because the rod is about to rotate about D.
Let the distance $AF = x$ m.
Taking moments about D:

$15g \times 0.5 = 10g \times (x - (1 + 2 + 0.5))$

$= 10g \times (x - 3.5)$

$7.5g = 10gx - 35g$

$42.5 = 10x$

$\Rightarrow x = 4.25$

The distance AF is 4.25 m

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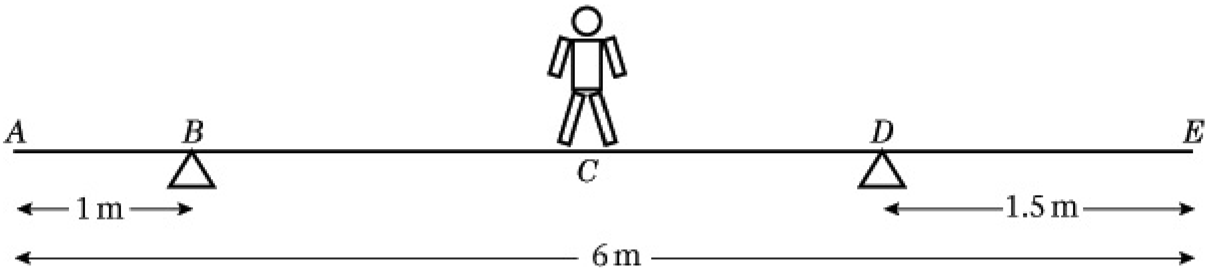
Moments
Exercise E, Question 1

Question:



A plank *AE*, of length 6 m and weight 100 N, rests in a horizontal position on supports at *B* and *D*, where *AB* = 1 m and *DE* = 1.5 m. A child of weight 145 N stands at *C*, the mid-point of *AE*, as shown in the diagram above. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate

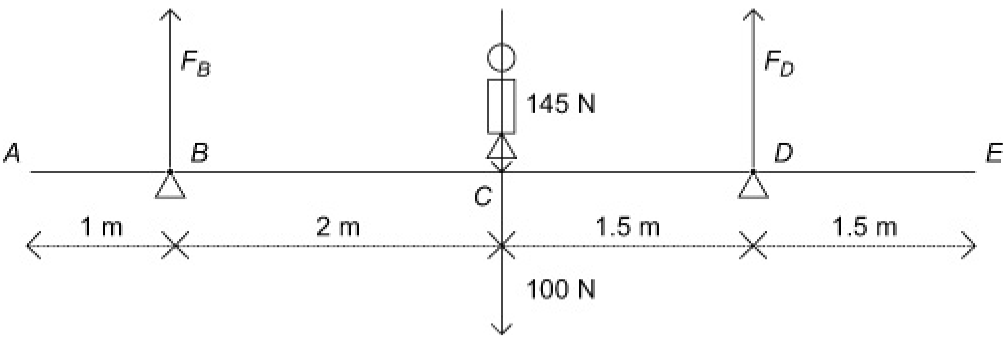
a the magnitude of the force exerted by the support on the plank at *B*,

b the magnitude of the force exerted by the support on the plank at *D*.

The child now stands at a different point *F* on the plank. The plank is in equilibrium and on the point of tilting about *D*.

c Calculate the distance *DF*.

Solution:



a Taking moments about the point *D*:
⇒ since the child and the plank are in equilibrium,
$1.5 \times 100 + 1.5 \times 145 = 3.5 \times F_B$, $150 + 217.5 = 3.5 \times F_B$, ⇒ $F_B = 367.5 \div 3.5 = 105$ N

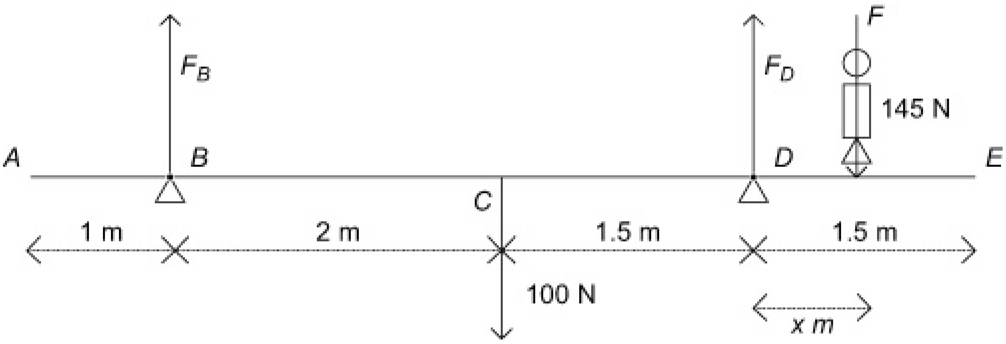
b R(↑), the child and the plank are in equilibrium, so

$100 + 145 = F_B + F_D$,

$245 = 105 + F_D$

⇒ $F_D = 245 - 105 = 140$ N

c



If the plank is about to tilt about *D*, then $F_B = 0$ and the child must be standing to the right of *D*.

Let the distance *DF* be *x* m. Taking moments about *D*:

$$\Rightarrow 100 \times 1.5 = 145 \times x\,, \quad x = \frac{100 \times 1.5}{145} \approx 1.03 \text{ m} \quad \left(= 103 \text{ cm} \right)$$

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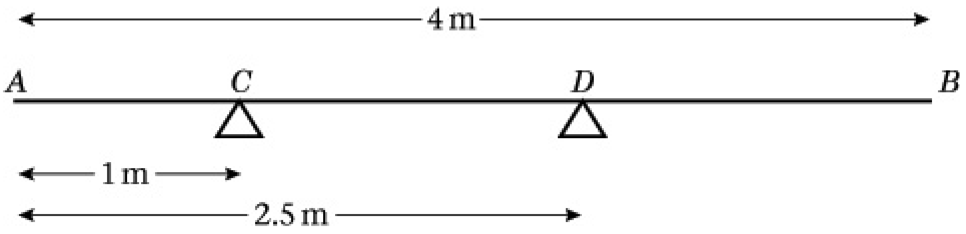
Moments
Exercise E, Question 2

Question:

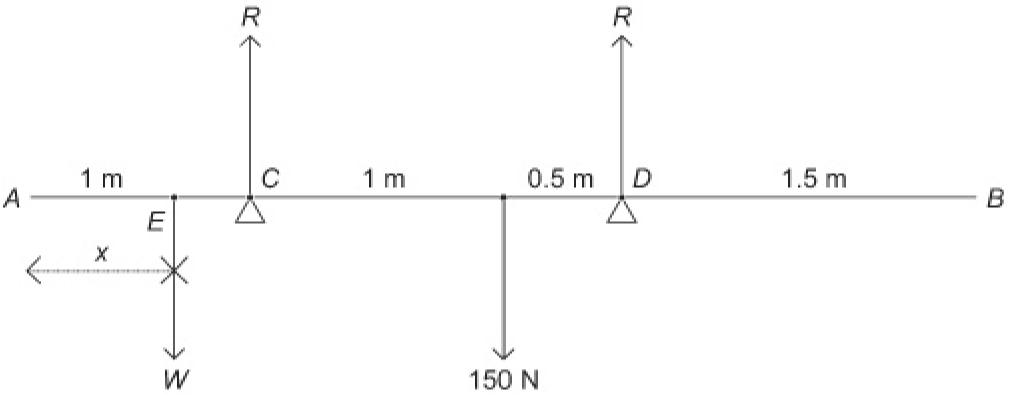


A uniform rod AB has length 4 m and weight 150 N. The rod rests in equilibrium in a horizontal position, smoothly supported at points C and D, where AC = 1 m and AD = 2.5 m as shown in the diagram above. A particle of weight W N is attached to the rod at a point E where $AE = x$ metres. The rod remains in equilibrium and the magnitude of the reaction at C is now equal to the magnitude of the reaction at D.

a Show that $W = \frac{150}{7 - 4x}$

b Hence deduce the range of possible values of x.

Solution:



a Since the rod is uniform, the centre of mass is at the mid-point.
Taking moments about A:

$$Wx + 150 \times 2 = R \times 1 + R \times 2.5 \,,$$

$$Wx + 300 = 3.5R$$

R(↑), equilibrium $\Rightarrow W + 150 = R + R \,,\ 2R = W + 150$

Hence $R = \frac{W + 150}{2}$, and $Wx + 300 = \frac{7}{2} \times \frac{W + 150}{2}$

$$\Rightarrow 4\left(Wx + 300 \right) = 7W + 7 \times 150 \,,\ 4Wx + 1200 = 7W + 1050$$

$$1200 - 1050 = 7W - 4Wx$$

$$W\left(7 - 4x \right) = 150 \,,\ W = \frac{150}{7 - 4x}$$

b $x \geq 0$ and $\frac{150}{7-4x} > 0$

$$\begin{aligned} \Rightarrow 7-4x &> 0 \\ 4x &< 7 \\ x &< \frac{7}{4} \\ x &< 1.75 \\ \text{So } 0 \leq x &< 1.75 \end{aligned}$$

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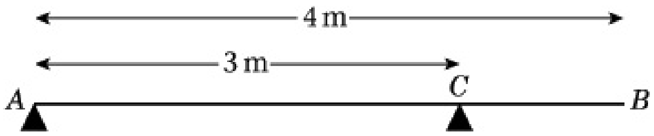
Moments
Exercise E, Question 3

Question:



A uniform plank AB has mass 40 kg and length 4 m. It is supported in a horizontal position by two smooth pivots. One pivot is at the end A and the other is at the point C where $AC = 3$ m, as shown in the diagram above. A man of mass 80 kg stands on the plank which remains in equilibrium. The magnitude of the reaction at A is twice the magnitude of the reaction at C. The magnitude of the reaction at C is RN. The plank is modelled as a rod and the man is modelled as a particle.

a Find the value of R.

b Find the distance of the man from A.

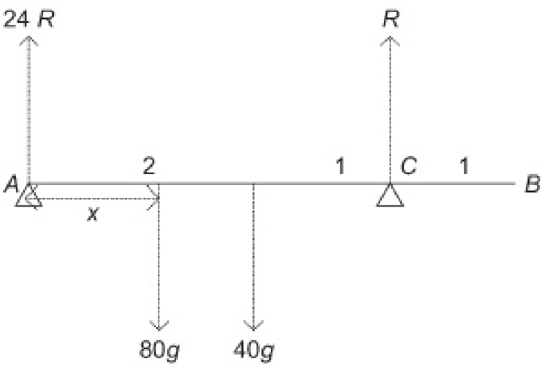
c State how you have used the modelling assumption that

i the plank is uniform,

ii the plank is a rod,

iii the man is a particle.

Solution:



a

R (↑) $3R = 80g + 40g$

$R = 40g = 392$ N

b Taking moments about A: $80g \times x + 40g \times 2 = 40g \times 3$

$80g \times x = 40g$, $\Rightarrow x = \frac{1}{2} = 0.5\ m$

c (i) Since the plank is uniform, the weight acts at centre of plank.
(ii) Since the plank is a rod, the plank remains straight.
(iii) Since the man is a particle, his weight acts at a single point.

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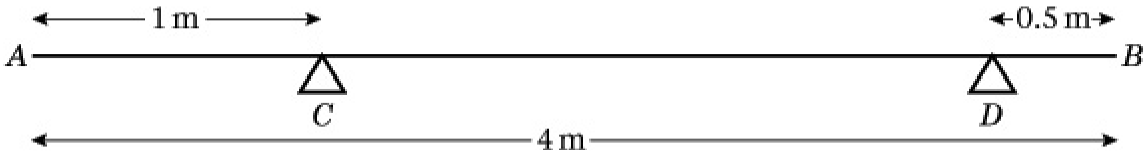
Moments
Exercise E, Question 4

Question:



A non-uniform rod AB has length 4 m and weight 150 N. The rod rests horizontally in equilibrium on two smooth supports C and D, where $AC = 1$ m and $DB = 0.5$ m, as shown in the diagram above. The centre of mass of AB is x metres from A. A particle of weight WN is placed on the rod at A. The rod remains in equilibrium and the magnitude of the reaction of C on the rod is 100 N.

a Show that $550 + 7W = 300x$.

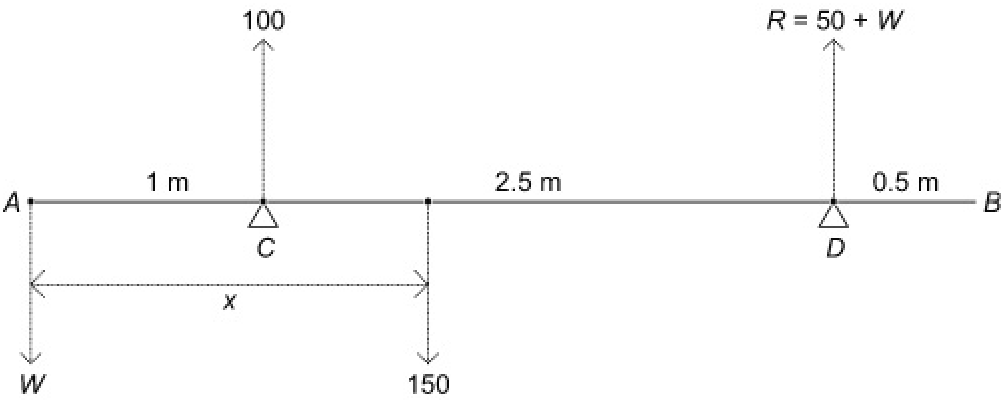
The particle is now removed from A and placed on the rod at B. The rod remains in equilibrium and the reaction of C on the rod now has magnitude 52 N.

b Obtain another equation connecting W and x.

c Calculate the value of x and the value of W.

Solution:

a

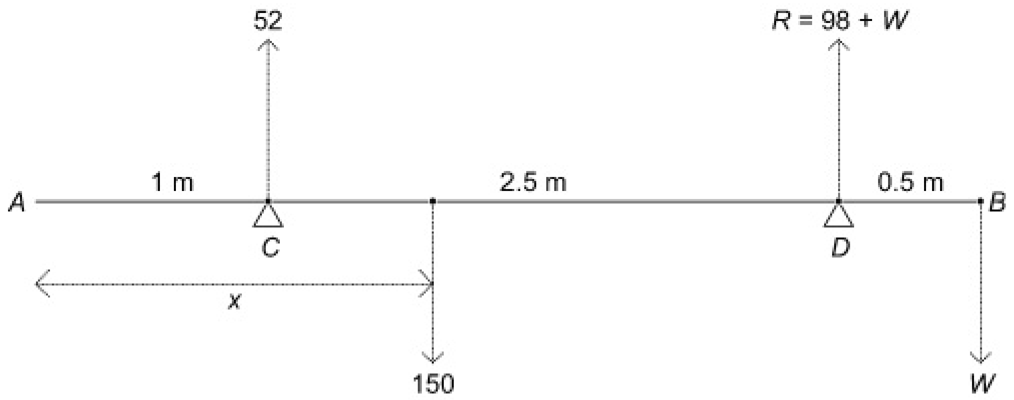


R(↑) $100 + R = W + 150$, $R = W + 50$

Taking moments about A,

$$
\begin{aligned}
100 \times 1 + (W + 50) \times 3.5 &= 150 \times x \\
150x &= 100 + 175 + 3.5W \\
275 + 3.5W &= 150x \\
550 + 7W &= 300x
\end{aligned}
$$

b



R(↑) $52 + R = 150 + W$, $R = 150 + W - 52 = 98 + W$
Taking moments about B: $52 \times 3 + (98 + W) \times 0.5 = 150 \times (4 - x)$
$156 + 49 + 0.5W = 600 - 150x$
doubling, $410 + W = 1200 - 300x$, $W = 790 - 300x$

c Solving the simultaneous equations $\rightarrow W = 790 - (550 + 7W)$,
$8W = 790 - 550 = 240 \Rightarrow W = 30$
$\Rightarrow 410 + 30 = 1200 - 300x$, $300x = 760$, $x = 2.53$ (3 s.f.)

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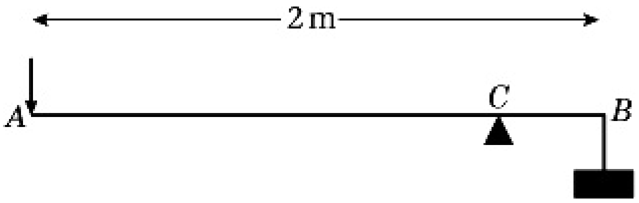
Moments
Exercise E, Question 5

Question:



A lever consists of a uniform steel rod *AB* of weight 100 N and length 2 m, which rests on a small smooth pivot at a point *C*. A load of weight 1700 N is suspended from the end *B* of the rod by a rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at the end *A*, as shown in the diagram above. The rope is modelled as a light string.

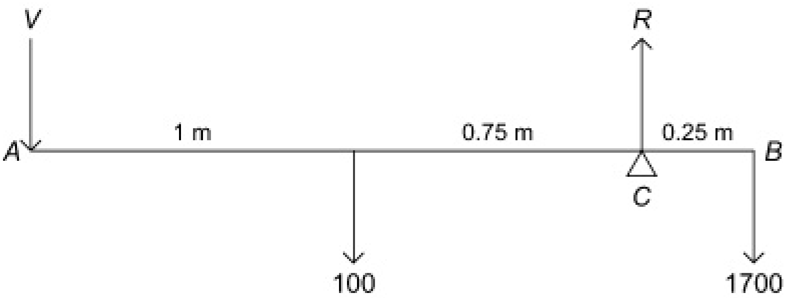
a Given that $BC = 0.25$ m find the magnitude of the force applied at *A*.

The position of the pivot is changed so that the rod remains in equilibrium when the force at *A* has magnitude 150 N.

b Find, to the nearest centimetre, the new distance of the pivot from *B*.

Solution:

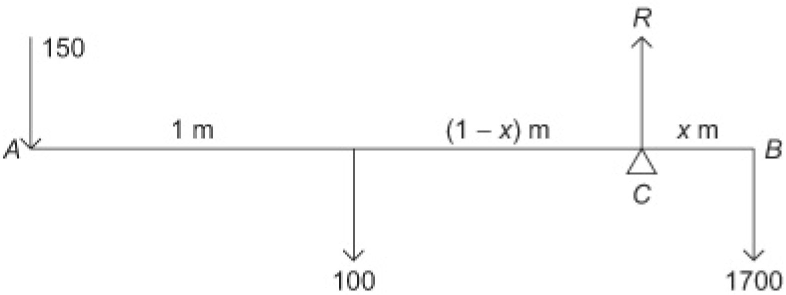
a Let the force applied at *A* be *V*.



Taking moments about *C*: $V \times 1.75 + 100 \times 0.75 = 1700 \times 0.25$

$\Rightarrow 1.75V + 75 = 425$, $1.75V = 350$, $V = 200$ N

b If the distance $BC = x$



Taking moments about *C*: $150 (1 + 1 - x) + 100 (1 - x) = 1700x$

$\Rightarrow 300 - 150x + 100 - 100x = 1700x \Rightarrow 400 - 250x = 1700x$

$400 = 1950x$, $x = \frac{400}{1950} \approx 0.21$ m (2 s.f.)

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Moments
Exercise E, Question 6

Question:

A plank AB has length 4 m. It lies on a horizontal platform, with the end A lying on the platform and the end B projecting over the edge, as shown above. The edge of the platform is at the point C.

Jack and Jill are experimenting with the plank. Jack has mass 48 kg and Jill has mass 36 kg. They discover that if Jack stands at B and Jill stands at A and $BC = 1.8$ m, the plank is in equilibrium and on the point of tilting about C.

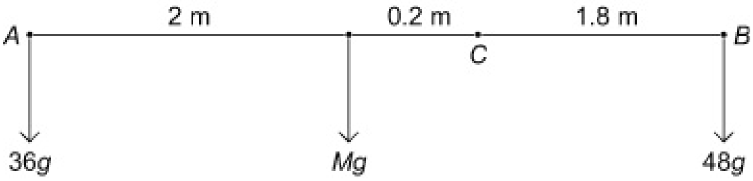
a By modelling the plank as a uniform rod, and Jack and Jill as particles, find the mass of the plank.

They now alter the position of the plank in relation to the platform so that, when Jill stands at B and Jack stands at A, the plank is again in equilibrium and on the point of tilting about C.

b Find the distance BC in this position.

Solution:

a Let the mass of the plank be M. Since the plank is uniform, its centre of mass is at its mid-point.

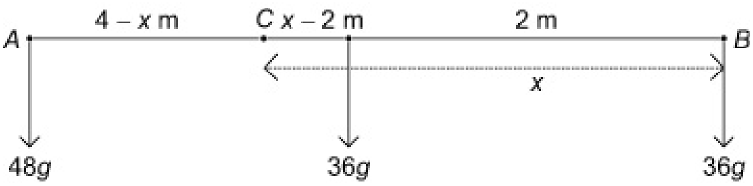


Taking moments about C: $48g \times 1.8 = Mg \times 0.2 + 36g \times 2.2$

$$86.4\,g = 0.2Mg + 79.2g\ ,\ 86.4 = 0.2\,M + 79.2$$

$$0.2\,M = 86.4 - 79.2 = 7.2 \Rightarrow M = 36\text{ kg}$$

b Let the distance BC be x



Taking moments about C: $36gx + 36g\,(x - 2) = 48g\,(4 - x)$

$\Rightarrow$ (dividing by the common factor $12g$)

$3x + 3\,(x - 2) = 4\,(4 - x)$, $6x - 6 = 16 - 4x$

$\Rightarrow 10x = 22$, $x = 2.2$ m

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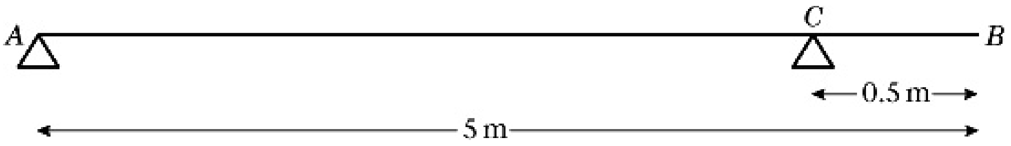
Moments
Exercise E, Question 7

Question:



A plank of wood AB has mass 12 kg and length 5 m. It rests in a horizontal position on two smooth supports. One support is at the end A. The other is at the point C, 0.5 m from B, as shown in the diagram above. A girl of mass 30 kg stands at B with the plank in equilibrium.

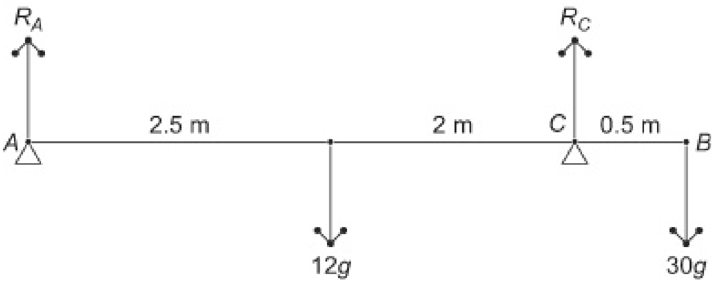
a By modelling the plank as a uniform rod and the girl as a particle, find the reaction on the plank at A.

The girl gets off the plank. A boulder of mass m kg is placed on the plank at A and a man of mass 93 kg stands on the plank at B. The plank remains in equilibrium and is on the point of tilting about C.

b By modelling the plank again as a uniform rod, and the man and the boulder as particles, find the value of m.

Solution:

a

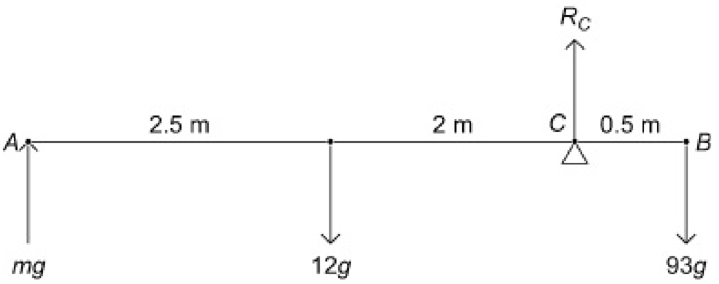


Taking moments about C: $R_A \times 4.5 + 30g \times 0.5 = 12g \times 2$

$$R_A \times 4.5 = 24g - 15g = 9g$$

$$\Rightarrow R_A = 2g = 19.6\ N$$

b



The plank is about to tilt about $C \Rightarrow$ reaction at $A = 0$

Taking moments about C: $mg \times 4.5 + 12g \times 2 = 93g \times 0.5$

$\Rightarrow 4.5m = 93 \times 0.5 - 24 = 22.5$, m = ie 5

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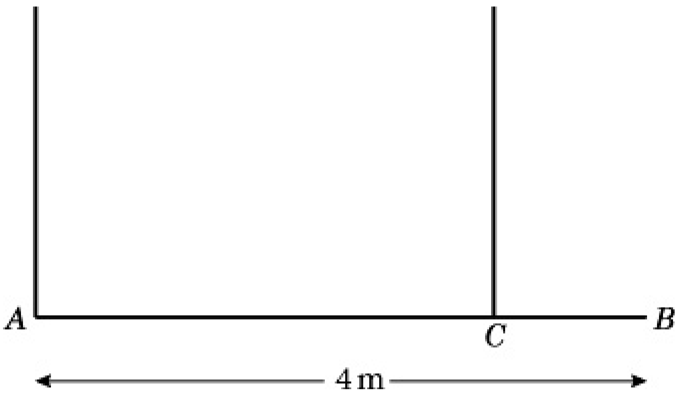
Moments
Exercise E, Question 8

Question:

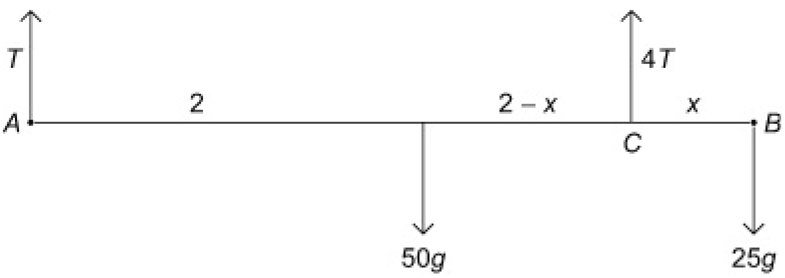


A plank *AB* has mass 50 kg and length 4 m. A load of mass 25 kg is attached to the plank at *B*. The loaded plank is held in equilibrium, with *AB* horizontal, by two vertical ropes attached at *A* and *C*, as shown in the diagram. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at *C* is four times the tension in the rope at *A*, calculate

a the tension in the rope at *C*,

b the distance *CB*.

Solution:



a Let the tension in the rope at *A* be *T* N

R(↑) $T + 4T = 50g + 25g$, $5T = 75g$
$\Rightarrow$ $T = 15g$, so tension at *C* is $60g$ N = 588 N

b Let the distance *BC* be *x*

Taking moments about *C*: $15g \times (4 - x) + 25g \times x = 50g \times (2 - x)$

$$60 - 15x + 25x = 100 - 50x$$

$$60x = 40 , x = \frac{2}{3} \text{ m}$$

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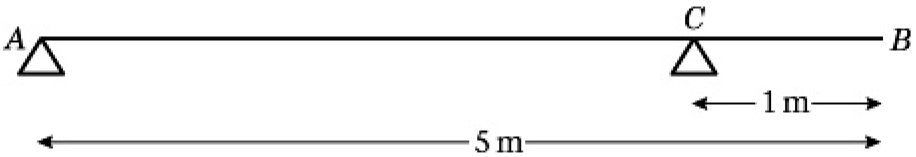
Moments
Exercise E, Question 9

Question:



A uniform beam *AB* has weight 200 N and length 5 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at end *A* and the other is at a point *C* on the beam, where *BC* = 1 m, as shown in the diagram. The beam is modelled as a uniform rod.

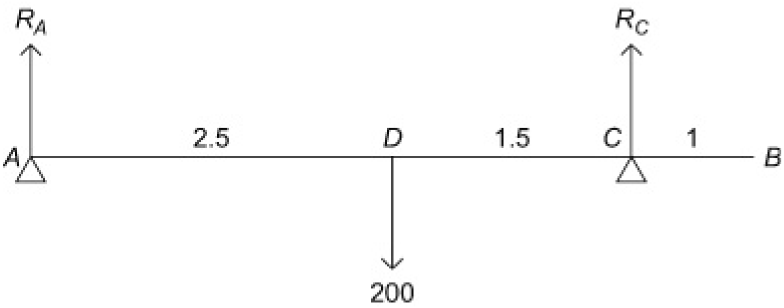
a Find the reaction on the beam at *C*.

A woman of weight 500 N stands on the beam at the point *D*. The beam remains in equilibrium. The reactions on the beam at *A* and *C* are now equal.

b Find the distance *AD*.

Solution:

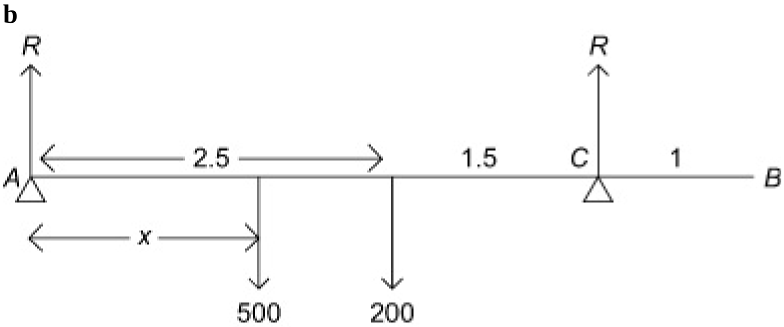
a



Taking moments about *A*:

$$200 \times 2.5 = R_C \times 4$$
$$R_C = 125 \text{ N}$$

b



Let the distance *AD* be *x*

R(↑) $2R = 500 + 200 = 700$

$R = 350$ N

Taking moments about *A*: $R \times 4 = 200 \times 2.5 + 500 \times x$,

$1400 = 4R = 500 + 500x$, $900 = 500x$, $x = 1.8$ m

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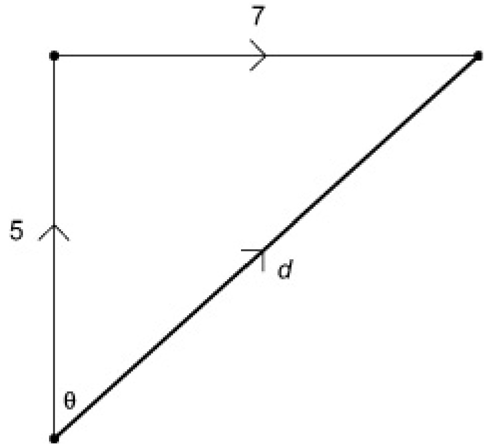
Vectors
Exercise A, Question 1

Question:

A bird flies 5 km due north and then 7 km due east. How far is the bird from its original position, and in what direction?

Solution:



$d = \sqrt{5^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74} \approx 8.60$ km

$\theta = \tan^{-1}\frac{7}{5} = \tan^{-1}1.4 = 54.46 \ldots {}^\circ$

The bird is 8.60 km (3 s.f.) from the starting point on a bearing of 054 ° (nearest degree).

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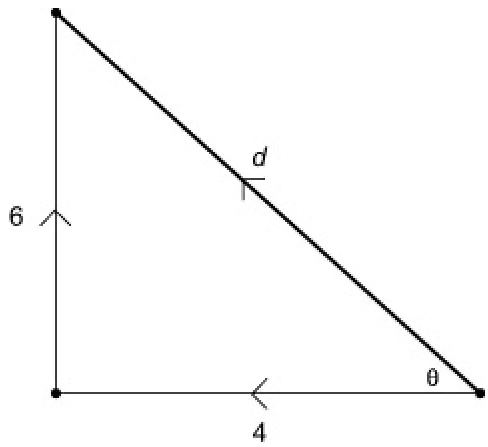
Vectors
Exercise A, Question 2

Question:

A girl cycles 4 km due west then 6 km due north. Calculate the total distance she has cycled and her displacement from her starting point.

Solution:



Distance cycled = 4 km + 6 km = 10 km

$d = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} \approx 7.2 \ldots$

$\theta = \tan^{-1} \frac{6}{4} = \tan^{-1} 1.5 = 56.3°$

bearing = 270 ° + 56 ° = 326 °

The displacement is 7.2 km (3 s.f.) on a bearing of 326 ° (nearest degree).

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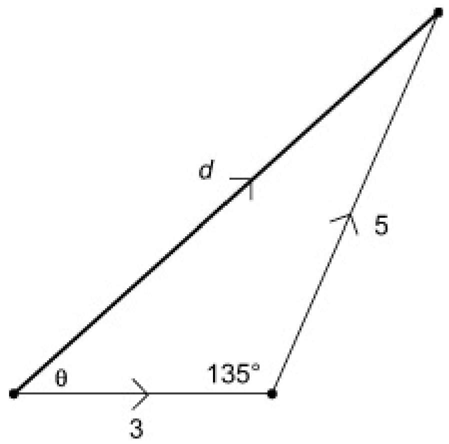
Vectors
Exercise A, Question 3

Question:

A man walks 3 km due east and then 5 km northeast. Find his distance and bearing from his original position.

Solution:



$d^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 135° = 55.21 \ldots$

$d = 7.43$ km (3 s.f.)

$\frac{\sin \theta}{5} = \frac{\sin 135°}{d}$

$\sin \theta = \frac{5 \times \sin 135°}{d} = 0.476, \theta = 28.4°$ (3 s.f.)

$\Rightarrow$ bearing is $90° - 28° = 062°$ (nearest degree)

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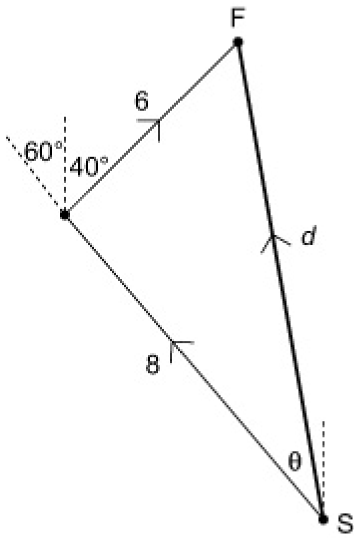
Vectors
Exercise A, Question 4

Question:

In an orienteering exercise, a team hike 8 km from the starting point, *S*, on a bearing of 300 ° then 6 km on a bearing 040 ° to the finishing point, *F*. Find the magnitude and direction of the displacement from *S* to *F*.

Solution:



$d^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 80° = 83.3 \ldots$

$d = 9.13$ km (3 s.f.)

$$\frac{\sin \theta}{6} = \frac{\sin 80°}{d}$$

$$\sin \theta = \frac{6 \times \sin 80°}{d} = 0.647 \ldots, \theta = 40.3° \text{ (3 s.f.)}$$

$\Rightarrow$ bearing is $300° + 40° = 340°$ (nearest degree)

$\Rightarrow$ the vector *SF* is 9.13 km (3 s.f.) on a bearing of 340 ° (nearest degree)

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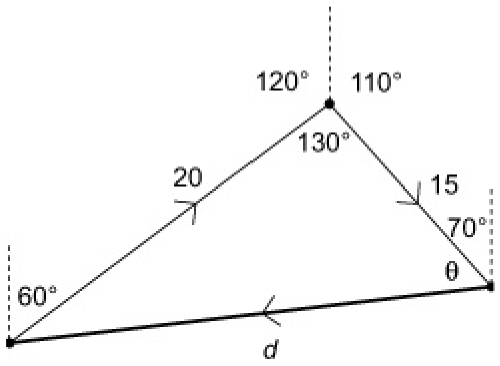
Vectors
Exercise A, Question 5

Question:

A boat travels 20 km on a bearing of 060 ° , followed by 15 km on a bearing of 110 ° . What course should it take to return to its starting point by the shortest route?

Solution:



$$d^2 = 20^2 + 15^2 - 2 \times 20 \times 15 \times \cos 130° = 1010.67 \ldots$$

$$d = 31.8 \text{ km (3 s.f.)}$$

$$\frac{\sin \theta}{20} = \frac{\sin 130°}{d}$$

$$\sin \theta = \frac{20 \times \sin 130°}{d} = 0.4819 \ldots , \theta = 28.8° \text{ (3 s.f.)}$$

$\Rightarrow$ bearing of the start $= 360° - 29° - 70° = 261°$ (nearest degree)

$\Rightarrow$ the return course is 31.8 km (3 s.f.) on a bearing of 261 ° (nearest degree)

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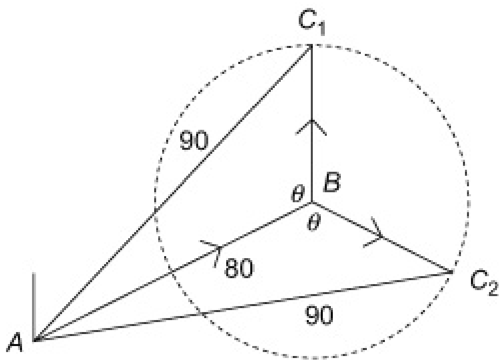
Vectors
Exercise A, Question 6

Question:

An aeroplane flies from airport A to airport B 80 km away on a bearing of 070 ° . From B the aeroplane flies to airport C, 60 km from B. Airport C is 90 km from A. Find the two possible directions for the course set by the aeroplane on the second stage of its journey.

Solution:



$\cos\theta = \frac{6^2 + 8^2 - 9^2}{2 \times 6 \times 8} = 0.19791 \ldots$ $\Rightarrow$ bearing

$\theta = 78.6°$ (3 s.f.)

of C from B is

$180° + 70° - \theta = 171.4°$ (1 d.p.) or $180° + 70° + \theta = 323.6°$ (1 d.p.)

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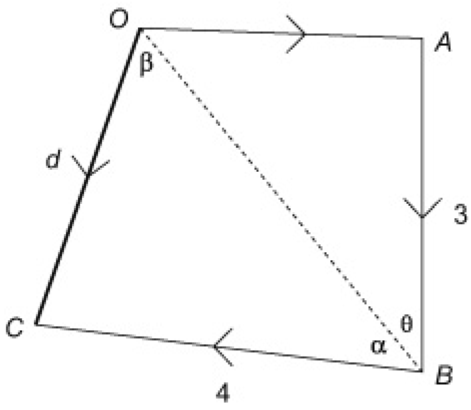
Vectors
Exercise A, Question 7

Question:

In a regatta, a yacht starts at point O, sails 2 km due east to A, 3 km due south from A to B, and then 4 km on a bearing of $280°$ from B to C. Find the displacement vector of C from O.

Solution:



Starting with the displacement OB, distance

$OB = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\theta = \tan^{-1} \frac{2}{3} = 33.7°$ (3 s.f.)

Now, looking at triangle OBC,

$\alpha = 80 - 33.7 = 46.3°$ (3 s.f.)

$d^2 = 4^2 + 13 - 2 \times 4 \times \sqrt{13} \times \cos 46.3° = 9.07 \ldots$

$d = 3.01$ km (3 s.f.)

$$\frac{\sin \beta}{4} = \frac{\sin 46.3°}{d}$$

$$\sin \beta = \frac{4 \times \sin 46.3°}{d} = 0.960 \ldots, \beta = 73.59°$$

$\Rightarrow$ bearing $= 90 + (90 - \theta) + \beta = 220°$ (nearest degree)

$\Rightarrow$ vector ie OC is 3.01 km (3 s.f.) on a bearing of $220°$ (nearest degree)

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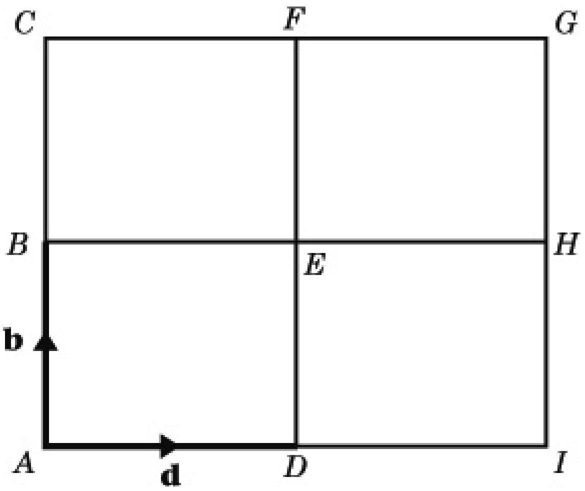
Vectors
Exercise B, Question 1

Question:



ACGI is a square, *B* is the mid-point of *AC*, *F* is the mid-point of *CG*, *H* is the mid-point of *GI*, and *D* is the mid-point of *AI*.

Vectors **b** and **d** are represented in magnitude and direction by *AB* and *AD* respectively. Find, in terms of **b** and **d**, the vectors represented in magnitude and direction by

a *AC*,	**b** *BE*,	**c** *HG*,	**d** *DF*,
e *AE*,	**f** *DH*,	**g** *HB*,	**h** *FE*,
i *AH*,	**j** *BI*,	**k** *EI*,	**l** *FB*.

Solution:

In this exercise there will usually be several correct routes to the answers because the addition law for vectors allows several options for equivalent vectors. You might reach the correct answers by a different routes to those used in these solutions.

a $AC = 2AB = 2b$

b $BE = AD$ (parallel and equal in length) $= d$

c $HG = BC$ (parallel and equal in length) $= AB$ (B is midpoint of AC) $= b$

d $DF = AC$ (parallel and equal in length) $= 2b$

e $AE = AD + DE$ (triangle law of addition)

$= AD + AB$ (DE and AB parallel and equal in length) $= d + b$

f $DH = DI + IH$ (triangle law of addition)

$= AD + AB$ ($AD = DI$ because D is the mid F point of AI, and AB is parallel and equal to IH)

$= d + b$

g $HB = -BH$ (same length, opposite direction)

$= -AI$ (parallel and equal in length) $= -2d$

h $FE = -EF$ (same length, opposite direction)

$= -HG$ (parallel and equal in length) $= -b$ (from part c)

i $AH = AI + IH$ (triangle law of addition) $= 2d + b$

j $BI = BA + AI = -AB + AI = -b + 2d$

k $EI = EB + BA + AI = -BE - AB + AI = -d - b + 2d = -b + d$

l $FB = FD + DA + AB = -DF - AD + AB = -2b - d + b = -b - d$

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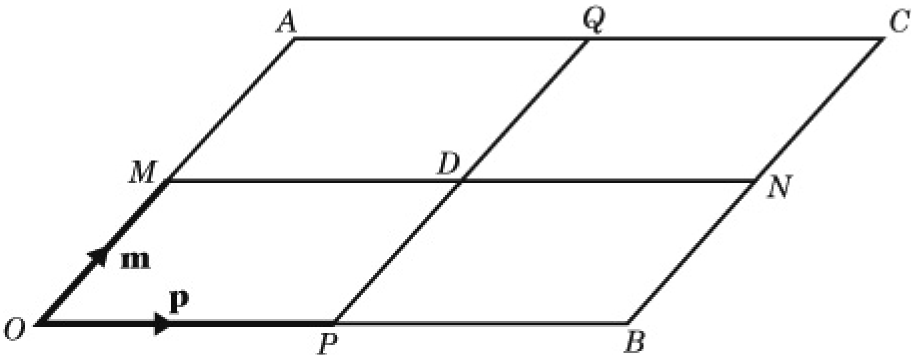
Vectors
Exercise B, Question 2

Question:



OACB is a parallelogram. *M*, *Q*, *N* and *P* are the mid-points of *OA*, *AC*, *BC* and *OB* respectively. Vectors **p** and **m** are equal to *OP* and *OM* respectively. Express in terms of **p** and **m**

a *OA*	**b** *OB*	**c** *BN*	**d** *DQ*
e *OD*	**f** *MQ*	**g** *OQ*	**h** *AD*
i *CD*	**j** *AP*	**k** *BM*	**l** *NO*.

Solution:

a $OA = 2OM$ (M is the mid F point of OA) $= 2m$

b $OB = 2OP$ (P is the mid F point of OB) $= 2p$

c $BN = \frac{1}{2}BC = \frac{1}{2}OA$ (opposite sides parallel and equal) $= m$

d $DQ = PD$ (MN and PQ bisect each other)

$= OM$ (line segments parallel and equal in length) $= m$

e $OD = OP + PD$ (addition of vectors)

$= OP + OM$ (PD and OM are parallel and equal in length) $= p + m$

f $MQ = MO + OP + PQ$ (vector addition)

$= -OM + OP + OA$ (PQ and OA are parallel and equal in length)

$= -m + p + 2m = p + m$

g $OQ = OP + PQ = p + 2m$

h $AD = AO + OD$ (vector addition) $= -OA + OD = -2m + (p + m) = p - m$

i $CD = CN + ND$ (vector addition)

$= MO + PO$ (line segments parallel and equal in length)

$= -OM + -OP = -m - p$

j $AP = AO + OP$ (vector addition) $= -OA + OP = -2m + p$

k $BM = BO + OM$ (vector addition) $= -OB + OM = -2p + m$

l $NO = NB + BO$ (vector addition)

$= MO + BO$ (MO and NB are parallel and equal in length)

$= -OM + -OB = -m - 2p$

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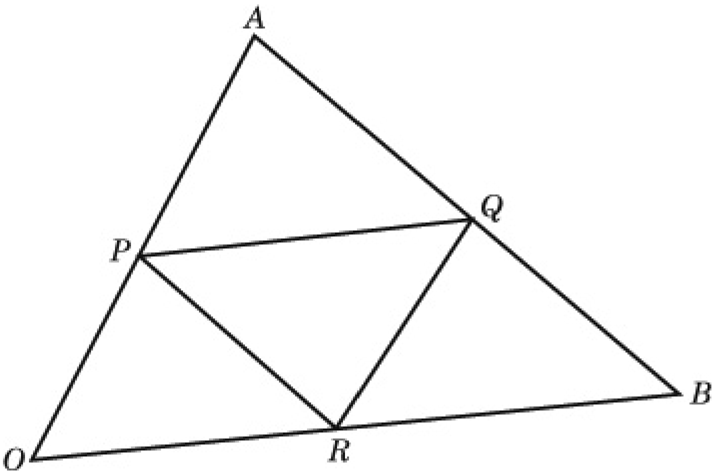
Vectors

Exercise B, Question 3

Question:



OAB is a triangle. *P*, *Q* and *R* are the mid-points of *OA*, *AB* and *OB* respectively. *OP* and *OR* are equal to **p** and **r** respectively. Find, in terms of **p** and **r**

a *OA* **b** *OB* **c** *AB* **d** *AQ*
e *OQ* **f** *PQ* **g** *QR* **h** *BP*.

Use parts **b** and **f** to prove that triangle *PAQ* is similar to triangle *OAB*.

Solution:

a $OA = 2OP$ (P is the mid *F* point of OA) $= 2p$

b $OB = 2OR$ (R is the mid *F* point of OB) $= 2r$

c $AB = AO + OB$ (addition of vectors) $= -OA + OB = -2p + 2r$

d $AQ = \frac{1}{2}AB$ (Q is the mid *F* point of AB) $= \frac{1}{2}\left(-2p + 2r\right) = -p + r$

e $OQ = OA + AQ$ (addition of vectors) $= 2p + \left(-p + r\right) = p + r$

f $PQ = PO + OQ$ (addition of vectors) $= -OP + OQ = -p + \left(p + r\right) = r$

g $QR = QO + OR$ (addition of vectors) $= -OQ + OR = -\left(p + r\right) + r = -p$

h $BP = BO + OP$ (addition of vectors) $= -OB + OP = -2r + p$

From **b** $OB = 2r$, and from **f** $PQ = r$,

$\Rightarrow$ OB and PQ are parallel

$\Rightarrow$ $\angle AOB = \angle APQ$ and $\angle ABO = \angle AQP$ (corresponding angles, parallel lines)

Angle A is common to both triangles

$\Rightarrow$ triangles PAQ and OAB are similar (three equal angles)

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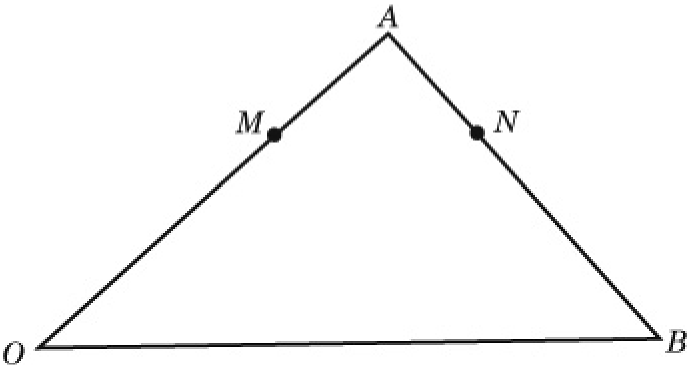
Vectors
Exercise B, Question 4

Question:



OAB is a triangle. $OA = a$ and $OB = b$. The point *M* divides *OA* in the ratio 2:1. *MN* is parallel to *OB*. Express the vector *ON* in terms of **a** and **b**.

Solution:

M divides *OA* in the ratio 2:1 $\Rightarrow OM = \frac{2}{3} a$

Using vector addition,

$ON = OA + AN = OA + \lambda AB$ (***N*** lies on ***AB***, so $AN = \lambda AB$) $= a + \lambda (- a + b)$ and $ON = OM + MN = OM + \mu OB$ (*MN* is parallel to *OB*) $= \frac{2}{3} a + \mu b$

$$\Rightarrow a + \lambda \left(- a + b \right) = \frac{2}{3} a + \mu b$$

$\Rightarrow$ (by comparing coefficients of **a** and **b**), $1 - \lambda = \frac{2}{3}$ and $\lambda = \mu$

so $\lambda = \mu = \frac{1}{3}$ and $ON = \frac{2}{3} a + \frac{1}{3} b$

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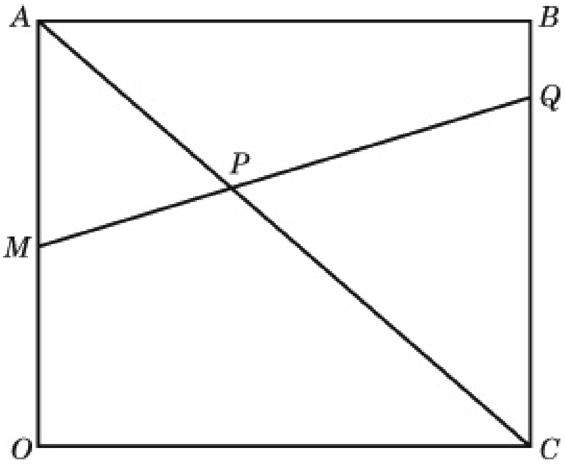
Vectors
Exercise B, Question 5

Question:



OABC is a square. *M* is the mid-point of *OA*, and *Q* divides *BC* in the ratio 1:3. *AP* and *MQ* meet at *P*. If *OA* = *a* and *OC* = *c*, express *OP* in terms of **a** and **c**.

Solution:

M is the mid F point of OA, so $OM = \frac{1}{2}OA = \frac{1}{2}a$

$$\begin{aligned} \text{Using vector addition, } MQ &= MA + AB + BQ \\ &= MA + AB + \frac{1}{4}BC = \frac{1}{2}a + c - \frac{1}{4}a = \frac{1}{4}a + c \\ \text{and } AC &= AO + OC = -a + c \end{aligned}$$

P lies on both AC and MQ, so

$OP = OM + \lambda MQ = \frac{1}{2}a + \lambda\left(\frac{1}{4}a + c\right)$

and $OP = OA + \mu AC = a + \mu(-a + c)$

$\Rightarrow \quad \frac{1}{2}a + \lambda\left(\frac{1}{4}a + c\right) = a + \mu\left(-a + c\right)$

by comparing coefficients of **a** and **c**, we get $\frac{1}{2} + \lambda\frac{1}{4} = 1 - \mu$ and $\lambda = \mu$

$\Rightarrow \frac{5}{4}\lambda = \frac{1}{2}, \lambda = \mu = \frac{2}{5}$ and $OP = \frac{3}{5}a + \frac{2}{5}b$

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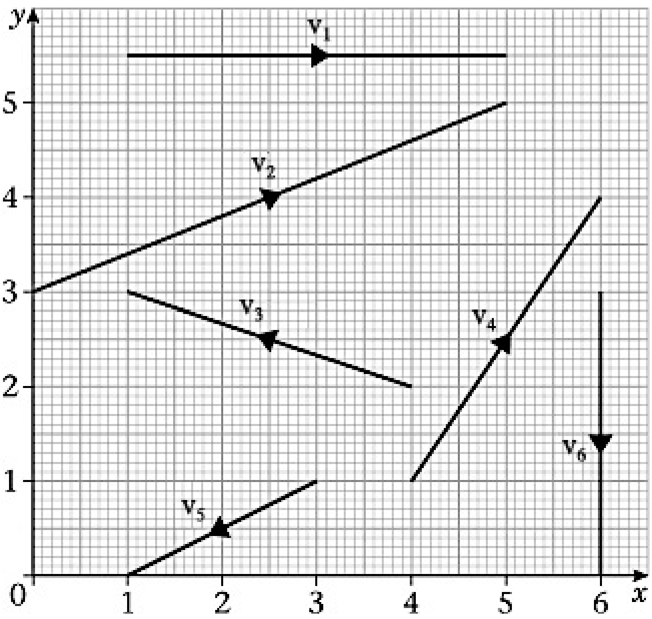
Vectors
Exercise C, Question 1

Question:

Express the vectors v_1 , v_2 , v_3 , v_4 , v_5 and v_6 using the **i**, **j** notation.



Solution:

$v_1 = 4i$, $v_2 = 5i + 2j$, $v_3 = -3i + j$, $v_4 = 2i + 3j$, $v_5 = -2i - j$, $v_6 = -3j$.

Note that some people prefer to describe vectors as ‘column vectors’. In this case, the answers would be

$v_1 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, $v_4 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $v_5 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, $v_6 = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

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Vectors
Exercise D, Question 1

Question:

Given that $a = 2i + 3j$ and $b = 4i - j$, find these terms of **i** and **j**.

a $a + b$	**b** $3a + b$	**c** $2a - b$	**d** $2b + a$
e $3a - 2b$	**f** $b - 3a$	**g** $4b - a$	**h** $2a - 3b$

Solution:

a $a + b = (2i + 3j) + (4i - j) = (2 + 4)i + (3 - 1)j = 6i + 2j$

b $3a + b = 3(2i + 3j) + (4i - j) = (6i + 9j) + (4i - j) = (6 + 4)i + (9 - 1)j = 10i + 8j$

c $2a - b = 2(2i + 3j) - (4i - j) = (4i + 6j) - (4i - j) = (4 - 4)i + (6 - (-1))j = 7j$

d $2b + a = 2(4i - j) + (2i + 3j) = (8i - 2j) + (2i + 3j) = (8 + 2)i + (-2 + 3)j = 10i + j$

e

$$3a - 2b = 3(2i + 3j) - 2(4i - j) = (6i + 9j) - (8i - 2j) = (6 - 8)i + (9 + 2)j = -2i + 11j$$

f $b - 3a = (4i - j) - 3(2i + 3j) = (4i - j) - (6i + 9j) = (4 - 6)i + (-1 - 9)j = -2i - 10j$

g

$$4b - a = 4(4i - j) - (2i + 3j) = (16i - 4j) - (2i + 3j) = (16 - 2)i + (-4 + (-3))j = 14i - 7j$$

h

$$2a - 3b = 2(2i + 3j) - 3(4i - j) = (4i + 6j) - (12i - 3j) = (4 - 12)i + (6 - (-3))j = -8i + 9j$$

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Vectors
Exercise D, Question 2

Question:

Find the magnitude of each of these vectors.

a $3i + 4j$	**b** $6i - 8j$	**c** $5i + 12j$	**d** $2i + 4j$
e $3i - 5j$	**f** $4i + 7j$	**g** $-3i + 5j$	**h** $-4i - j$

Solution:

a $| 3i + 4j | = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

b $| 6i - 8j | = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

c $| 5i + 12j | = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

d $| 2i + 4j | = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} = 4.47$ (3 s.f.)

e $| 3i - 5j | = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} = 5.83$ (3 s.f.)

f $| 4i + 7j | = \sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65} = 8.06$ (3 s.f.)

g $| -3i + 5j | = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} = 5.83$ (3 s.f.)

h $| -4i - j | = \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17} = 4.12$ (3 s.f.)

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Vectors
Exercise D, Question 3

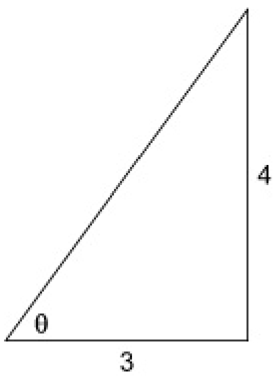
Question:

Find the angle that each of these vectors makes with the positive x-axis.

a $3i + 4j$ **b** $6i - 8j$ **c** $5i + 12j$ **d** $2i + 4j$

Solution:

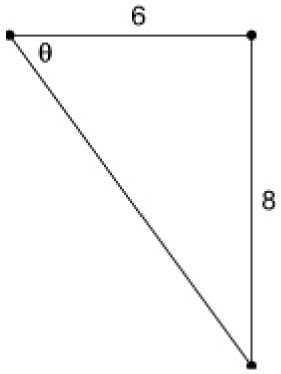
a $3i + 4j$



arc tan $\left(\frac{4}{3} \right)$

$= 53.1°$ above (3 s.f.)

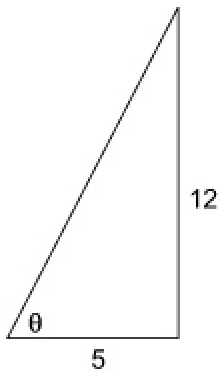
b $6i - 8j$



arctan $\left(\frac{8}{6} \right)$

$= 53.1°$ below (3 s.f.)

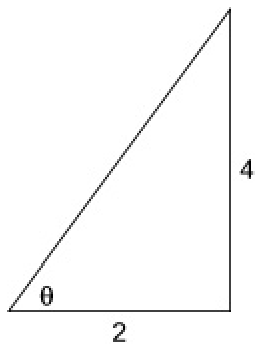
c $5i + 12j$



arctan $\left(\frac{12}{5} \right)$

$= 67.4°$ above (3 s.f.)

d $2i + 4j$



arctan $\left(\frac{4}{2} \right)$

$63.4°$ above (3 s.f.)

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Vectors
Exercise D, Question 4

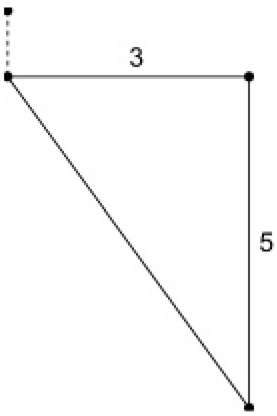
Question:

Find the angle that each of these vectors makes with the positive *y*-axis.

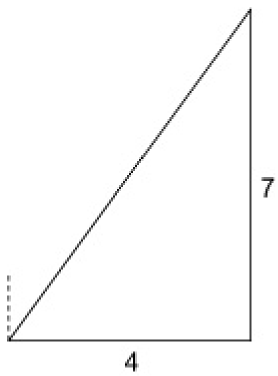
a $3i - 5j$ **b** $4i + 7j$ **c** $-3i + 5j$ **d** $-4i - j$

Solution:

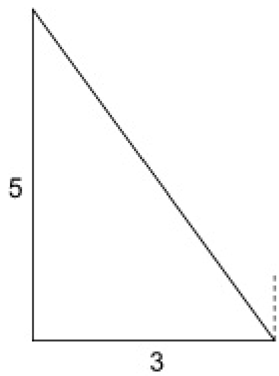
a $3i - 5j$



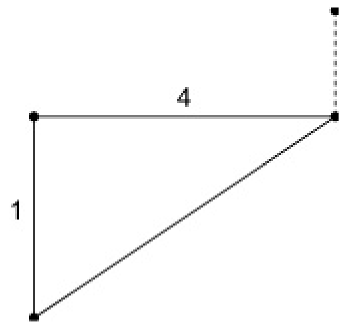
b $4i + 7j$



c $-3i + 5j$



d $-4i - j$



$90° + \arctan\left(\frac{5}{3}\right)$ $\arctan\left(\frac{4}{7}\right)$ $\arctan\left(\frac{3}{5}\right)$ $90° + \arctan\left(\frac{1}{4}\right)$

$= 90° + 59° = 149°$ (3 s.f.) to the right $= 29.7°$ (3 s.f.) to the right $= 31.0°$ (3 s.f.) to the left $= 90° + 14° = 104°$ (3 s.f.) to the left

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise D, Question 5

Question:

Given that $a = 2i + 5j$ and $b = 3i - j$, find

a λ if $a + \lambda b$ is parallel to the vector **i**, **b** μ if $\mu a + b$ is parallel to the vector **j**,

Solution:

a $a + \lambda b = (2i + 5j) + \lambda(3i - j) = (2 + 3\lambda)i + (5 - \lambda)j$

Parallel to **i**, so $5 - \lambda = 0$, $\lambda = 5$.

b $\mu a + b = \mu(2i + 5j) + (3i - j) = (2\mu + 3)i + (5\mu - 1)j$

Parallel to **j**, so $2\mu + 3 = 0$, $\mu = \frac{-3}{2}$

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Vectors
Exercise D, Question 6

Question:

Given that $c = 3i + 4j$ and $d = i - 2j$, find

a λ if $c + \lambda d$ is parallel to $i + j$,
b μ if $\mu c + d$ is parallel to $i + 3j$,
c s if $c - sd$ is parallel to $2i + j$,
d t if $d - tc$ is parallel to $-2i + 3j$.

Solution:

a $c + \lambda d = (3i + 4j) + \lambda(i - 2j) = (3 + \lambda)i + (4 - 2\lambda)j$

Parallel to $i + j$, so $3 + \lambda = 4 - 2\lambda$

$3\lambda = 1$, $\lambda = \frac{1}{3}$

b $\mu c + d = \mu(3i + 4j) + (i - 2j) = (3\mu + 1)i + (4\mu - 2)j$

Parallel to $i + 3j$, so $4\mu - 2 = 3(3\mu + 1)$

$4\mu - 2 = 9\mu + 3$, $5\mu = -5$, $\mu = -1$

c $c - sd = (3i + 4j) - s(i - 2j) = (3 - s)i + (4 + 2s)j$

Parallel to $2i + j$, so $3 - s = 2(4 + 2s)$

$3 - s = 8 + 4s$, $-5 = 5s$, $s = -1$

d $d - tc = (i - 2j) - t(3i + 4j) = (1 - 3t)i + (-2 - 4t)j$

Parallel to $-2i + 3j$, so $-2(-2 - 4t) = 3(1 - 3t)$

$4 + 8t = 3 - 9t$, $1 = -17t$, $t = -\frac{1}{17}$

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Vectors
Exercise D, Question 7

Question:

In this question, the horizontal unit vectors **i** and **j** are directed due east and due north respectively.

Find the magnitude and bearing of these vectors.

a $2i + 3j$ **b** $4i - j$ **c** $-3i + 2j$ **d** $-2i - j$

Solution:

a $|2i + 3j| = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} = 3.61 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{2}{3}\right) = 33.7°, \text{ so bearing} \approx 034°$$

b $|4i - j| = \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17} = 4.12 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{1}{4}\right) = 14.0°, \text{ so bearing} \approx 90° + 14° = 104°$$

c $|-3i + 2j| = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13} = 3.61 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{2}{3}\right) = 33.7°, \text{ so bearing} \approx 270° + 34° = 304°$$

d $|-2i - j| = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5} = 2.24 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{1}{2}\right) = 26.6°, \text{ so bearing} \approx 270° - 27° = 243°$$

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Vectors
Exercise E, Question 1

Question:

Find the speed of a particle moving with these velocities:

a $3i + 4j \text{ m s}^{-1}$

b $24i - 7j \text{ km h}^{-1}$

c $5i + 2j \text{ m s}^{-1}$

d $-7i + 4j \text{ cm s}^{-1}$

Solution:

a Speed $= |3i + 4j| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ m s}^{-1}$

b Speed $= |24i - 7j| = \sqrt{24^2 + (-7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25 \text{ km h}^{-1}$

c Speed $= |5i + 2j| = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29} = 5.39 \text{ m s}^{-1}$ (3 s.f.)

d Speed $= |-7i + 4j| = \sqrt{(-7)^2 + 4^2} = \sqrt{49 + 16} = \sqrt{65} = 8.06 \text{ cm s}^{-1}$ (3 s.f.)

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Vectors
Exercise E, Question 2

Question:

Find the distance moved by a particle which travels for:

a 5 hours at velocity $8i + 6j\,\mathrm{km\,h^{-1}}$

b 10 seconds at velocity $5i - j\,\mathrm{m\,s^{-1}}$

c 45 minutes at velocity $6i + 2j\,\mathrm{km\,h^{-1}}$

d 2 minutes at velocity $-4i - 7j\,\mathrm{cm\,s^{-1}}$.

Solution:

a Distance = speed × time = $\sqrt{8^2 + 6^2} \times 5 = 5 \times \sqrt{64 + 36} = 5 \times \sqrt{100} = 50$ km

b Distance = speed × time = $\sqrt{5^2 + (-1)^2} \times 10 = 10 \times \sqrt{25 + 1} = 10 \times \sqrt{26} = 51.0$ m (3 s.f.)

c Distance = speed × time = $\sqrt{6^2 + 2^2} \times 0.75 = 0.75 \times \sqrt{36 + 4} = 0.75 \times \sqrt{40} = 4.74$ km (3 s.f.)

d Distance = speed × time = $\sqrt{(-4)^2 + (-7)^2} \times 120 = 120 \times \sqrt{16 + 49} = 120 \times \sqrt{65} = 967$ cm (3 s.f.)

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Vectors
Exercise E, Question 3

Question:

Find the speed and the distance travelled by a particle moving with:

a velocity $-3i + 4j \text{ m s}^{-1}$ for 15 seconds

b velocity $2i + 5j \text{ m s}^{-1}$ for 3 seconds

c velocity $5i - 2j \text{ km h}^{-1}$ for 3 hours

d velocity $12i - 5j \text{ km h}^{-1}$ for 30 minutes.

Solution:

a Speed $= \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ m s}^{-1}$,

Distance $= 5 \times 15 = 75$ m

b Speed $= \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} = 5.39 \text{ m s}^{-1}$ (3 s.f.)

Distance $= 3 \times 5.39 = 16.2$ m (3 s.f.)

c Speed $= \sqrt{5^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29} = 5.39 \text{ km h}^{-1}$ (3 s.f.)

Distance $= 3 \times 5.39 = 16.2$ km (3 s.f.)

d Speed $= \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ km h}^{-1}$,

Distance $= 0.5 \times 13 = 6.5$ km

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Vectors
Exercise F, Question 1

Question:

A particle P is moving with constant velocity v m s^{-1}. Initially P is at the point with position vector **r**. Find the position of P
t seconds later if:

a $r_0 = 3j$, $v = 2i$ and $t = 4$,
b $r_0 = 2i - j$, $v = -2j$ and $t = 3$,
c $r_0 = i + 4j$, $v = -3i + 2j$ and $t = 6$,
d $r_0 = -3i + 2j$, $v = 2i - 3j$ and $t = 5$.

Solution:

a Using $r = r_o + vt, r = 3j + 2i \times 4 = 8i + 3j$

b Using $r = r_o + vt, r = (2i - j) + (-2j) \times 3 = 2i - j - 6j = 2i - 7j$

c Using $r = r_o + vt, r = (i + 4j) + (-3i + 2j) \times 6 = i + 4j - 18i + 12j = -17i + 16j$

d Using $r = r_o + vt, r = (-3i + 2j) + (2i - 3j) \times 5 = -3i + 2j + 10i - 15j = 7i - 13j$

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Vectors
Exercise F, Question 2

Question:

A particle P moves with constant velocity **v**. Initially P is at the point with position vector **a**. t seconds later P is at the point with position vector **b**. Find **v** when:

a $a = 2i + 3j$, $b = 6i + 13j$, $t = 2$,

b $a = 4i + j$, $b = 9i + 16j$, $t = 5$,

c $a = 3i - 5j$, $b = 9i + 7j$, $t = 3$,

d $a = -2i + 7j$, $b = 4i - 8j$, $t = 3$,

e $a = -4i + j$, $b = -12i - 19j$, $t = 4$.

Solution:

a Using $r = r_o + vt, \left(6i + 13j \right) = \left(2i + 3j \right) + 2v, 2v = \left(6i + 13j \right) - \left(2i + 3j \right) = 4i + 10j$
$v = 2i + 5j$

b Using $r = r_o + vt, \left(9i + 16j \right) = \left(4i + j \right) + 5v, 5v = \left(9i + 16j \right) - \left(4i + j \right) = 5i + 15j$
$v = i + 3j$

c Using $r = r_o + vt, \left(9i + 7j \right) = \left(3i - 5j \right) + 3v, 3v = \left(9i + 7j \right) - \left(3i - 5j \right) = 6i + 12j$
$v = 2i + 4j$

d Using $r = r_o + vt, \left(4i - 8j \right) = \left(-2i + 7j \right) + 3v, 3v = \left(4i - 8j \right) - \left(-2i + 7j \right) = 6i - 15j$
$v = 2i - 5j$

e Using $r = r_o + vt, \left(-12i - 19j \right) = \left(-4i + j \right) + 4v, 4v = \left(-12i - 19j \right) - \left(-4i + j \right) = -8i - 20j$
$v = -2i - 5j$

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Vectors
Exercise F, Question 3

Question:

A particle moving with speed $v \text{ m s}^{-1}$ in direction **d** has velocity vector **v**. Find **v** for these.

a $v = 10$, $d = 3i - 4j$
b $v = 15$, $d = -4i + 3j$
c $v = 7.5$, $d = -6i + 8j$
d $v = 5\sqrt{2}$, $d = i + j$
e $v = 2\sqrt{13}$, $d = -2i + 3j$
f $v = \sqrt{68}$, $d = 3i - 5j$
g $v = \sqrt{60}$, $d = -4i - 2j$
h $v = 15$, $d = -i + 2j$

Solution:

a $|d| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5, 10 \div 5 = 2, v = 2(3i - 4j) = 6i - 8j$

b $|d| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5, 15 \div 5 = 3, v = 3(-4i + 3j) = -12i + 9j$

c $\left| d \right| = \sqrt{(-6)^2 + 8^2} = \sqrt{100} = 10, 7.5 \div 10 = \frac{3}{4}, v = \frac{3}{4}\left(-6i + 8j\right) = -4.5i + 6j$

d $|d| = \sqrt{1^2 + 1^2} = \sqrt{2}, 5\sqrt{2} \div \sqrt{2} = 5, v = 5(i + j) = 5i + 5j$

e $|d| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}, 2\sqrt{13} \div \sqrt{13} = 2, v = 2(-2i + 3j) = -4i + 6j$

f $|d| = \sqrt{3^2 + (-5)^2} = \sqrt{34}, \sqrt{68} \div \sqrt{34} = \sqrt{2}, v = \sqrt{2}(3i - 5j) = 3\sqrt{2}i - 5\sqrt{2}j$

g $|d| = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}, \sqrt{60} \div \sqrt{20} = \sqrt{3}, v = \sqrt{3}(-4i - 2j) = -4\sqrt{3}i - 2\sqrt{3}j$

h $|d| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}, 15 \div \sqrt{5} = 3\sqrt{5}, v = 3\sqrt{5}(-i + 2j) = -3\sqrt{5}i + 6\sqrt{5}j$

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Vectors
Exercise F, Question 4

Question:

A particle P starts at the point with position vector r_0. P moves with constant velocity $v\,\mathrm{m\,s^{-1}}$. After t seconds, P is at the point with position vector **r**.

a Find **r** if $r_0 = 2i$, $v = i + 3j$, and $t = 4$.

b Find **r** if $r_0 = 3i - j$, $v = -2i + j$, and $t = 5$.

c Find r_0 if $r = 4i + 3j$, $v = 2i - j$, and $t = 3$.

d Find r_0 if $r = -2i + 5j$, $v = -2i + 3j$, and $t = 6$.

e Find **v** if $r_0 = 2i + 2j$, $r = 8i - 7j$, and $t = 3$.

f Find the speed of P if $r_0 = 10i - 5j$, $r = -2i + 9j$, and $t = 4$.

g Find t if $r_0 = 4i + j$, $r = 12i - 11j$, and $v = 2i - 3j$.

h Find t if $r_0 = -2i + 3j$, $r = 6i - 3j$, and the speed of P is $4\,\mathrm{m\,s^{-1}}$.

Solution:

a Using $r = r_o + vt, r = (2i) + (i + 3j) \times 4 = 2i + 4i + 12j = 6i + 12j$

b Using $r = r_o + vt, r = (3i - j) + (-2i + j) \times 5 = 3i - j - 10i + 5j = -7i + 4j$

c Using $r = r_o + vt, (4i + 3j) = r_o + (2i - j) \times 3, r_o = (4i + 3j) - (6i - 3j) = 4i + 3j - 6i + 3j = -2i + 6j$

d Using $r = r_o + vt, (-2i + 5j) = r_o + (-2i + 3j) \times 6, r_o = (-2i + 5j) - (-12i + 18j)$
$= -2i + 5j + 12i - 18j = 10i - 13j$

e Using $r = r_o + vt, (8i - 7j) = (2i + 2j) + v \times 3, 3v = (8i - 7j) - (2i + 2j) = 6i - 9j$
$v = 2i - 3j$

f Using $r = r_o + vt, (-2i + 9j) = (10i - 5j) + v \times 4, 4v = (-2i + 9j) - (10i - 5j) = -12i + 14j \; v = -3i + 3.5j$,

$$\text{speed} = \sqrt{(-3)^2 + 3.5^2} = \sqrt{21.25} \approx 4.61\ \mathrm{m\,s^{-1}}$$

g Using $r = r_o + vt, (12i - 11j) = (4i + j) + (2i - 3j) \times t, (2i - 3j) \times t = (12i - 11j) - (4i + j)$
$= 8i - 12j, t = 4$

h Using $r = r_o + vt, (6i - 3j) = (-2i + 3j) + vt, vt = (6i - 3j) - (-2i + 3j) = 8i - 6j$
$4t = |vt| \; 4t = |8i - 6j| = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10, 4t = 10, 4\; t = 2.5$

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Vectors
Exercise F, Question 5

Question:

The initial velocity of a particle P moving with uniform acceleration a m s^{-2} is u m s^{-1}. Find the velocity and the speed of P after t seconds in these cases.

a $u = 5i$, $a = 3j$, and $t = 4$
b $u = 3i - 2j$, $a = i - j$, and $t = 3$
c $a = 2i - 3j$, $u = -2j + j$, and $t = 2$
d $t = 6$, $u = 3i - 2j$, and $a = -i$
e $a = 2i + j$, $t = 5$, and $u = -3i + 4j$

Solution:

a Using $v = u + at, v = (5i) + (3j) \times 4 = 5i + 12j$
speed $= \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ m s^{-1}

b Using $v = u + at, v = (3i - 2j) + (i - j) \times 3 = 3i - 2j + 3i - 3j = 6i - 5j$
speed $= \sqrt{6^2 + (-5)^2} = \sqrt{36 + 25} = \sqrt{61} \approx 7.81$ m s^{-1}

c Using $v = u + at, v = (-2i + j) + (2i - 3j) \times 2 = -2i + j + 4i - 6j = 2i - 5j$

speed $= \sqrt{2^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.39$ m s^{-1}

d Using $v = u + at, v = (3i - 2j) + (-i) \times 6 = 3i - 2j - 6i = -3i - 2j$

speed $= \sqrt{(-3)^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13} \approx 3.61$ m s^{-1}

e Using $v = u + at, v = (-3i + 4j) + (2i + j) \times 5 = -3i + 4j + 10i + 5j = 7i + 9j$

speed $= \sqrt{7^2 + 9^2} = \sqrt{49 + 81} = \sqrt{130} \approx 11.4$ m s^{-1}

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Vectors
Exercise F, Question 6

Question:

A constant force **F** N acts on a particle of mass 4 kg for 5 seconds. The particle was initially at rest, and after 5 seconds it has velocity $6i - 8j \text{ m s}^{-1}$. Find **F**.

Solution:

Using $v = u + at$, $\left(6i - 8j \right) = a \times 5, a = \frac{1}{5} \left(6i - 8j \right)$

Using $F = ma, F = 4 \times \frac{1}{5} \left(6i - 8j \right) = 4.8i - 6.4j$

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Vectors
Exercise F, Question 7

Question:

A force $2i - j$ N acts on a particle of mass 2 kg. If the initial velocity of the particle is $i + 3j$ m s^{-1}, find how far it moves in the first 3 seconds.

Solution:

Using $F = ma$, $\left(2i - j \right) = 2a, a = i - \frac{1}{2}j$

Using $s = ut + \frac{1}{2}at^2, s = \left(i + 3j \right) \times 3 + \frac{1}{2}\left(i - \frac{1}{2}j \right) \times 3^2 = 3i + 9j + 4\frac{1}{2}i - 2\frac{1}{4}j = 7\frac{1}{2}i + 6\frac{3}{4}j$

$$\text{distance} = \sqrt{\left(7\frac{1}{2} \right)^2 + \left(6\frac{3}{4} \right)^2} = \sqrt{56.25 + 45.5625} = \sqrt{101.8125} \approx 10.1 \text{ m}$$

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Vectors
Exercise F, Question 8

Question:

At time $t = 0$, the particle P is at the point with position vector $4\mathbf{i}$, and moving with constant velocity $i + j \text{ m s}^{-1}$. A second particle Q is at the point with position vector $-3\mathbf{j}$ and moving with velocity $v \text{ m s}^{-1}$. After 8 seconds, the paths of P and Q meet. Find the speed of Q.

Solution:

Using $r = r_o + vt$ for P, $r = (4i) + (i + j) \times 8 = 4i + 8i + 8j = 12i + 8j$

Using $r = r_o + vt$ for Q, $r = (-3j) + v \times 8$

Both at the same point: $12i + 8j = (-3j) + v \times 8, 8v = 12i + 8j + 3j = 12i + 11j$

$$v = \frac{1}{8}\left(12i + 11j\right) = 1.5i + 1.375j$$

$$\text{speed} = |v| = \sqrt{1.5^2 + 1.375^2} = \sqrt{2.25 + 1.890625} \approx 2.03 \text{ m s}^{-1}$$

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Vectors
Exercise F, Question 9

Question:

In questions 9 and 10 the unit vectors **i** and **j** are due east and due north respectively.

At 2 pm the coastguard spots a rowing dinghy 500 m due south of his observation point. The dinghy has constant velocity $(2i + 3j)\ \mathrm{m\,s^{-1}}$.

a Find, in terms of t, the position vector of the dinghy t seconds after 2 pm.

b Find the distance of the dinghy from the observation point at 2.05 pm.

Solution:

a Using $r = r_o + vt, r = -500j + (2i + 3j) \times t = -500j + 2ti + 3tj = 2ti + (-500 + 3t) j$

b 5 minutes = 5 × 60 seconds = 300 seconds, $r = 2 \times 300i + (-500 + 3 \times 300) j = 600i + 400j$

$$\text{distance} = \sqrt{600^2 + 400^2} = \sqrt{360\,000 + 160\,000} = \sqrt{520\,000} \approx 721 \text{ m}$$

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Vectors
Exercise F, Question 10

Question:

At noon a ferry F is 400 m due north of an observation point O moving with constant velocity $(7i + 7j)\ \mathrm{m\,s^{-1}}$, and a speedboat S is 500 m due east of O, moving with constant velocity $(-3i + 15j)\ \mathrm{m\,s^{-1}}$.

a Write down the position vectors of F and S at time t seconds after noon.

b Show that F and S will collide, and find the position vector of the point of collision.

Solution:

a

Using $r = r_o + vt$ for F, $r = 400j + (7i + 7j) \times t = 400j + 7ti + 7tj$
$= 7ti + (400 + 7t)j$

Using $r = r_o + vt$ for S, $r = 500i + (-3i + 15j) \times t = 500i - 3ti + 15tj$
$= (500 - 3t)i + 15tj$

b For F and S to collide, $7ti + (400 + 7t)j = (500 - 3t)i + 15tj$,

i components equal: $7t = 500 - 3t, 10t = 500, t = 50$

j components equal: $400 + 7t = 15t, 400 = 8t, t = 50$

Both conditions give the same value of t, so the two position vectors are equal when $t = 50$, i.e. F and S collide at $r = 7 \times 50i + (400 + 7 \times 50)j = 350i + 750j$.

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Vectors
Exercise F, Question 11

Question:

At 8 am two ships A and B are at $r_A = (i + 3j)$ km and $r_B = (5i - 2j)$ km from a fixed point P. Their velocities are $v_A = (2i - j)$ km h^{-1} and $v_B = (-i + 4j)$ km h^{-1} respectively.

a Write down the position vectors of A and B t hours later.

b Show that t hours after 8 am the position vector of B relative to A is given by $((4 - 3t)i + (-5 + 5t)j)$ km .

c Show that the two ships do not collide.

d Find the distance between A and B at 10 am.

Solution:

a Using $r = r_o + vt$ for $A, r = (i + 3j) + (2i - j) \times t = (1 + 2t)i + (3 - t)j$

Using $r = r_o + vt$ for $B, r = (5i - 2j) + (-i + 4j) \times t = (5 - t)i + (-2 + 4t)j$

b Using $AP + PB, AB = PB - PA$

$$= \{(5 - t)i + (-2 + 4t)j\{ - \{(1 + 2t)i + (3 - t)j\{$$
$$= (5 - t - 1 - 2t)i + (-2 + 4t - 3 + t)j = (4 - 3t)i + (-5 + 5t)j$$

c If A and B collide, the vector $\boldsymbol{AB}$ would be zero,

so $4 - 3t = 0$ and $-5 + 5t = 0$, but these two equations are not consistent ($t = 1$ and $t \neq 1$), so vector $\boldsymbol{AB}$ can never be zero, and A and B will not collide.

d At 10 am, $t = 2, AB = (4 - 3 \times 2)i + (-5 + 5 \times 2)j = -2i + 5j$,

Distance $= \sqrt{(-2)^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.39$ km

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Vectors
Exercise F, Question 12

Question:

A particle A starts at the point with position vector $12i + 12j$. The initial velocity of A is $(-i + j)$ m s^{-1}, and it has constant acceleration $(2i - 4j)$ m s^{-2}. Another particle, B, has initial velocity i m s^{-1} and constant acceleration $2j$ m s^{-2}. After 3 seconds the two particles collide. Find

a the speeds of the two particles when they collide,

b the position vector of the point where the two particles collide,

c the position vector of B's starting point.

Solution:

a Using $v = u + at$ and $t = 3$,

For A:

$$\begin{aligned} v &= (-i + j) + (2i - 4j) \times 3 \\ &= (-1 + 6)i + (1 - 12)j \\ &= 5i - 11j \end{aligned}$$

$$\begin{aligned} \text{Speed} &= \sqrt{5^2 + 11^2} = \sqrt{25 + 121} \\ &= \sqrt{146} = 12.1 \text{ ms}^{-1} \ (3 \text{ s.f.}) \end{aligned}$$

For B:

$$\begin{aligned} v &= i + 2j \times 3 \\ &= i + 6j \end{aligned}$$

$$\begin{aligned} \text{Speed} &= \sqrt{1^2 + 6^2} = \sqrt{1 + 36} \\ &= \sqrt{37} = 6.08 \text{ ms}^{-1} \ (3 \text{ s.f.}) \end{aligned}$$

b Using $s = ut + \frac{1}{2}at^2$ for A, $s = \left(-i + j\right) \times 3 + \frac{1}{2} \times \left(2i - 4j\right) \times 9 = -3i + 3j + 9i - 18j = 6i - 15j$

So at the instant of the collision, A is at the point with position vector

$$r = (12i + 12j) + (6i - 15j) = 18i - 3j$$

c Using $s = ut + \frac{1}{2}at^2$ for B, $s = \left(i\right) \times 3 + \frac{1}{2} \times \left(2j\right) \times 9 = 3i + 9j$, so B's starting point is

$$(18i - 3j) - (3i + 9j) = 15i - 12j$$

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Vectors
Exercise G, Question 1

Question:

A particle is in equilibrium at O under the action of three forces F_1, F_2 and F_3. Find F_3 in these cases.

a $F_1 = (2i + 7j)$ and $F_2 = (-3i + j)$ **b** $F_1 = (3i - 4j)$ and $F_2 = (2i + 3j)$

c $F_1 = (-4i - 2j)$ and $F_2 = (2i - 3j)$ **d** $F_1 = (-i - 3j)$ and $F_2 = (4i + j)$

Solution:

a $F_1 + F_2 + F_3 = 0 \Rightarrow (2i + 7j) + (-3i + j) + F_3 = 0$

$\Rightarrow F_3 = -(2i + 7j) - (-3i + j) = -2i - 7j + 3i - j = i - 8j$

b $F_1 + F_2 + F_3 = 0 \Rightarrow (3i - 4j) + (2i + 3j) + F_3 = 0$

$\Rightarrow F_3 = -(3i - 4j) - (2i + 3j) = -3i + 4j - 2i - 3j = -5i + j$

c $F_1 + F_2 + F_3 = 0 \Rightarrow (-4i - 2j) + (2i - 3j) + F_3 = 0$

$\Rightarrow F_3 = -(-4i - 2j) - (2i - 3j) = 4i + 2j - 2i + 3j = 2i + 5j$

d $F_1 + F_2 + F_3 = 0 \Rightarrow (-i - 3j) + (4i + j) + F_3 = 0$

$\Rightarrow F_3 = -(-i - 3j) - (4i + j) = i + 3j - 4i - j = -3i + 2j$

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Vectors
Exercise G, Question 2

Question:

For each part of Question 1 find the magnitude of F_3 and the angle it makes with the positive x-axis.

Solution:

a $|F_3| = \sqrt{(1)^2 + (-8)^2} = \sqrt{1 + 64} = \sqrt{65} \approx 8.06$

$$\tan \theta = \frac{8}{1} \Rightarrow \theta = 82.9° \quad (3 \text{ s.f.})$$

$$\text{angle with } Ox = 82.9° \text{ below } (3 \text{ s.f.})$$

b $|F_3| = \sqrt{(-5)^2 + (1)^2} = \sqrt{25 + 1} = \sqrt{26} \approx 5.10$

$$\tan \theta = \frac{1}{5} \Rightarrow \theta = 11.3° \quad (3 \text{ s.f.})$$

$$\text{angle with } Ox = 169° \text{ above } (3 \text{ s.f.})$$

c $|F_3| = \sqrt{(2)^2 + (5)^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.39$

$$\tan \theta = \frac{5}{2} \Rightarrow \theta = 68.2° \quad (3 \text{ s.f.})$$

$$\text{angle with } Ox = 68.2° \text{ above } (3 \text{ s.f.})$$

d $|F_3| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9 + 4} = \sqrt{13} \approx 3.61$

$$\tan \theta = \frac{2}{3} \Rightarrow \theta = 33.7° \quad (3 \text{ s.f.})$$

$$\text{angle with } Ox = 146° \text{ above } (3 \text{ s.f.})$$

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Vectors
Exercise G, Question 3

Question:

Forces **P** N, **Q** N and **R** N act on a particle of m kg. Find the resultant force on the particle and the acceleration produced when

a $P = 3i + j$, $Q = 2i - 3j$, $r = i + 2j$ and $m = 2$,

b $P = 4i - 3j$, $Q = -3i + 2j$, $r = 2i - j$ and $m = 3$,

c $P = -3i + 2j$, $Q = 2i - 5j$, $r = 4i + j$ and $m = 4$,

d $P = 2i + j$, $Q = -6i - 4j$, $r = 5i - 3j$ and $m = 2$.

Solution:

a Resultant force $= P + Q + R = (3i + j) + (2i - 3j) + (i + 2j) = 6i$

$\Rightarrow$ using $F = \mathrm{m}a$, $6i = 2 \times a$, $a = 3i \text{ m s}^{-2}$

b Resultant force $= P + Q + R = (4i - 3j) + (-3i + 2j) + (2i - j) = 3i - 2j$

$\Rightarrow$ using $F = \mathrm{m}a$, $3i - 2j = 3 \times a$, $a = \left(i - \frac{2}{3}j \right) \text{ m s}^{-2}$

c Resultant force $= P + Q + R = (-3i + 2j) + (2i - 5j) + (4i + j) = 3i - 2j$

$\Rightarrow$ using $F = \mathrm{m}a$, $3i - 2j = 4 \times a$, $a = \left(\frac{3}{4}i - \frac{1}{2}j \right) \text{ m s}^{-2}$

d Resultant force $= P + Q + R = (2i + j) + (-6i - 4j) + (5i - 3j) = i - 6j$

$\Rightarrow$ using $F = \mathrm{m}a$, $i - 6j = 2 \times a$, $a = \left(\frac{1}{2}i - 3j \right) \text{ m s}^{-2}$

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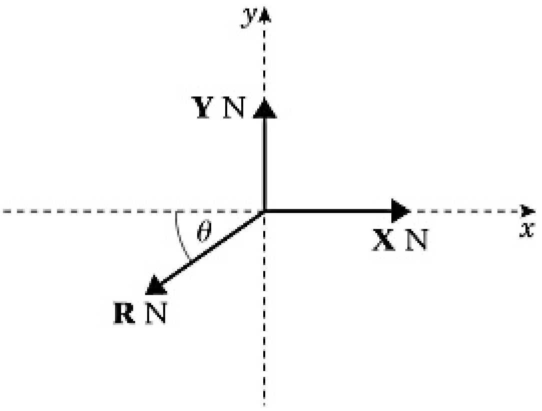
Vectors
Exercise G, Question 4

Question:

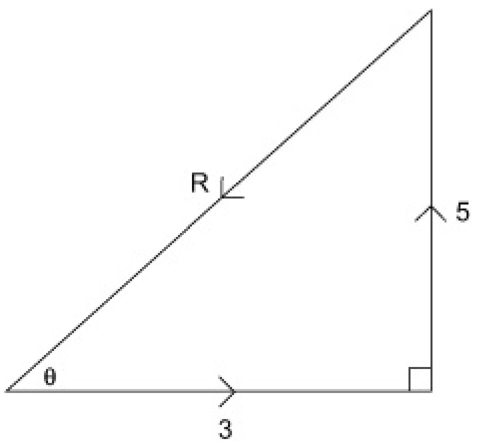


A particle is in equilibrium at O under the action of forces **X**, **Y** and **R**, as shown in the diagram. Use a triangle of forces to find the magnitude of **R** and the value of θ when:

a $|X| = 3\text{N}$, $|Y| = 5\text{N}$, **b** $|X| = 6\text{N}$, $|Y| = 2\text{N}$, **c** $|X| = 5\text{N}$, $|Y| = 4\text{N}$.

Solution:

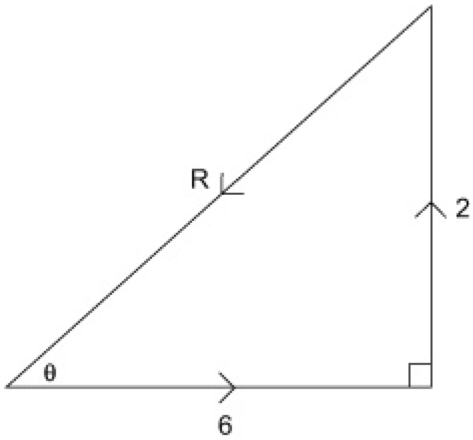
a



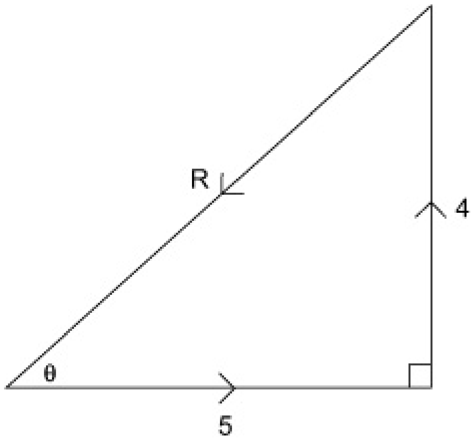
Using Pythagoras' theorem,
$|R| = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} \approx 5.83\text{ N}$ $\tan\theta = \frac{5}{3} \Rightarrow \theta = 59.0°$ (3 s.f.)

b

Using Pythagoras' theorem,
$|R| = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} \approx 6.32\text{ N}$ $\tan\theta = \frac{2}{6} \Rightarrow \theta = 18.4°$ (3 s.f.)



c



Using Pythagoras' theorem, $|R| = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41} = 6.40$ N (3 s.f.) $\tan\theta = \frac{4}{5} \Rightarrow \theta = 38.7°$ (3 s.f.)

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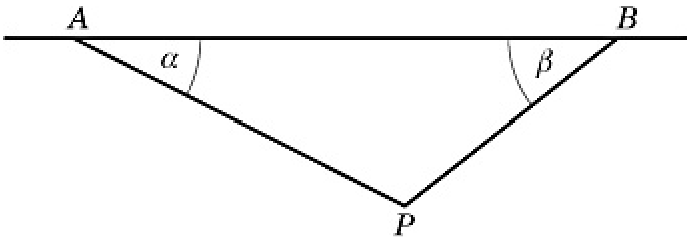
Vectors
Exercise G, Question 5

Question:



The diagram shows two strings attached to a particle *P*, of weight W N, and to two fixed points *A* and *B*. The line *AB* is horizontal and *P* is hanging in equilibrium with $\angle BAP = \alpha$ and $\angle ABP = \beta$. The magnitude of the tension in *AP* is T_AN, and the magnitude of the tension in *BP* is T_BN.

Use a triangle of vectors to find:

a T_A and T_B if W = 5 , $\alpha = 40°$ and $\beta = 50°$,

b T_A and T_B if W = 6 , $\alpha = 30°$ and $\beta = 45°$,

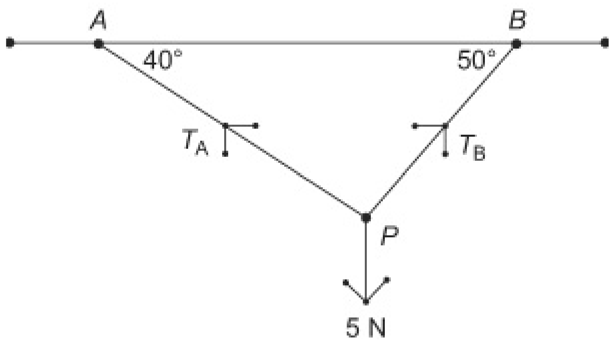
c T_A and W if $T_B = 5$, $\alpha = 40°$ and $\beta = 50°$,

d W and T_B if $T_A = 5$, $\alpha = 30°$ and $\beta = 70°$,

e T_A and α if W = 7 , $T_B = 6$ and $\beta = 50°$.

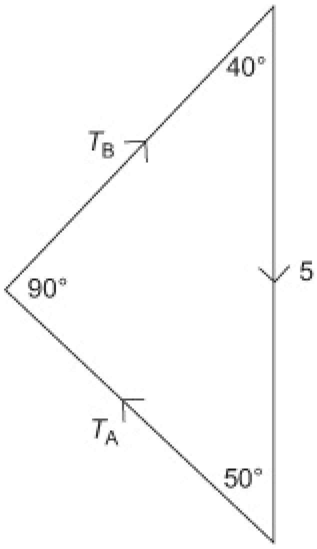
Solution:

a

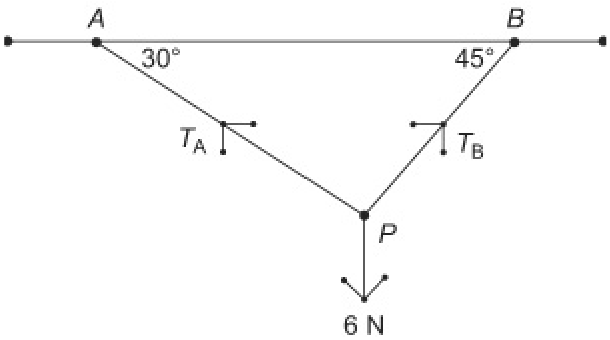


Triangle of forces

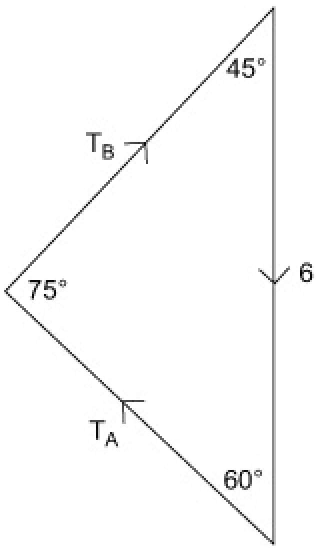
The triangle of forces is a right-angled triangle
$\Rightarrow T_A = 5 \times \sin 40° = 3.21$ N (3 s.f.) and
$T_B = 5 \times \cos 40° = 3.83$ N (3 s.f.)



b



Triangle of forces



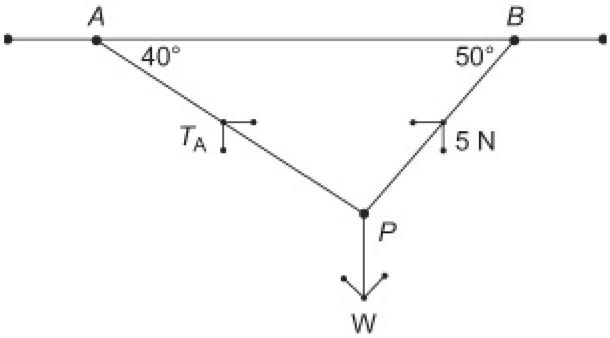
Using the Sine rule,

$$\frac{T_B}{\sin\ 60°} = \frac{T_A}{\sin\ 45°} = \frac{6}{\sin\ 75°}$$

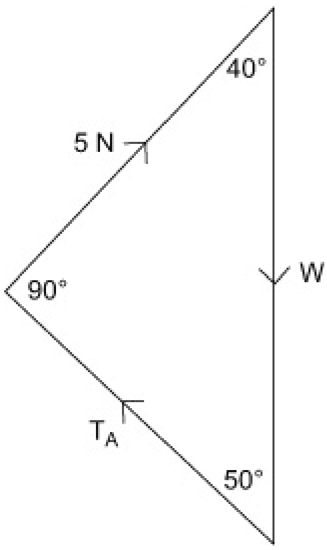
$$T_A = \frac{6 \times \sin\ 45°}{\sin\ 75°} = 4.39\text{ N} \ (3\text{ s.f.})$$

$$T_B = \frac{6 \times \sin\ 60°}{\sin\ 75°} = 5.38\text{ N} \ (3\text{ s.f.})$$

c



Triangle of forces



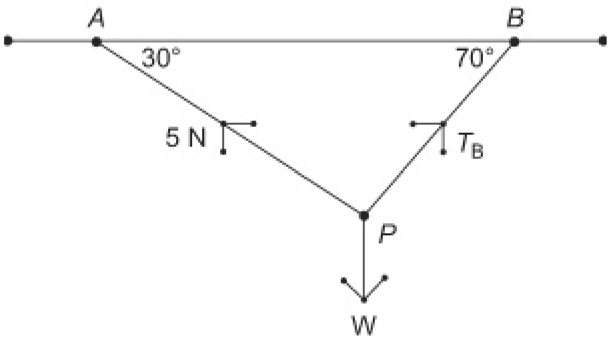
The triangle of forces is a right-angled triangle

$\Rightarrow T_A = 5 \times \tan 40° = 4.20\text{ N} \ (3\text{ s.f.})$

and $5 = W \times \cos 40°$,

$$W = \frac{5}{\cos 40°} = 6.53\text{ N} \ (3\text{ s.f.})$$

d



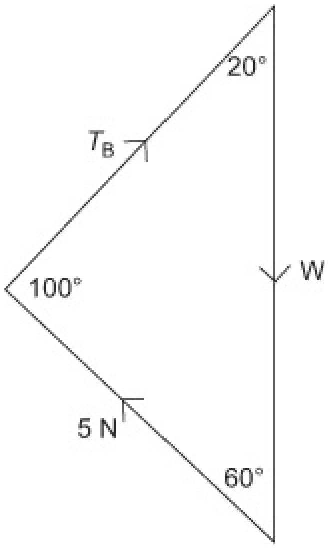
Triangle of forces

Using the Sine rule

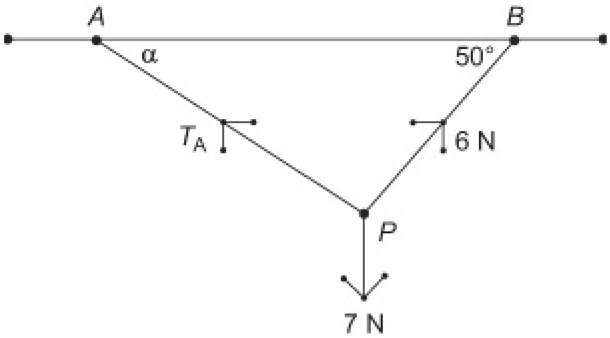
$$\frac{W}{\sin 100°} = \frac{5}{\sin 20°} = \frac{T_B}{\sin 60°}$$

$$W = \frac{5 \times \sin 100°}{\sin 20°} = 14.4\text{ N} \ (3\text{ s.f.})$$

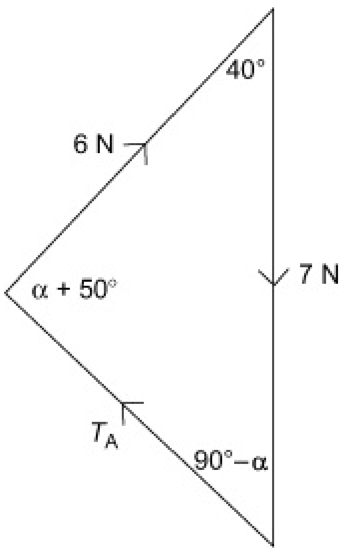
$$T_B = \frac{5 \times \sin 60°}{\sin 20°} = 12.7\text{ N} \ (3\text{ s.f.})$$



e



Triangle of forces



Using the Cosine rule:

$T_A{}^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos\ 40\,° = 20.65\ \ldots$

$T_A = 4.54$ N (3 s.f.)

Using the Sine rule:

$$\frac{\sin\ 40\,°}{T_A} = \frac{\sin(90\,° - \alpha)}{6}$$

$$\sin(90\,° - \alpha) = \frac{6 \times \sin\ 40\,°}{4.54} = 0.848\ \ldots$$

$90\,° - \alpha = 58.1\,°$, $\alpha = 31.9\,°$ (3 s.f.)

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Vectors
Exercise H, Question 1

Question:

Three forces F_1 , F_2 and F_3 act on a particle. $F_1 = (-3i + 7j)$ N , $F_2 = (i - j)$ N and $F_3 = (pi + qj)$ N .

a Given that this particle is in equilibrium, determine the value of p and the value of q. The resultant of the forces F_1 and F_2 is **R**.

b Calculate, in N, the magnitude of **R**.

c Calculate, to the nearest degree, the angle between the line of action of **R** and the vector **j**.

Solution:

a $F_1 + F_2 + F_3 = 0 \Rightarrow (-3i + 7j) + (i - j) + (pi + qj) = 0$

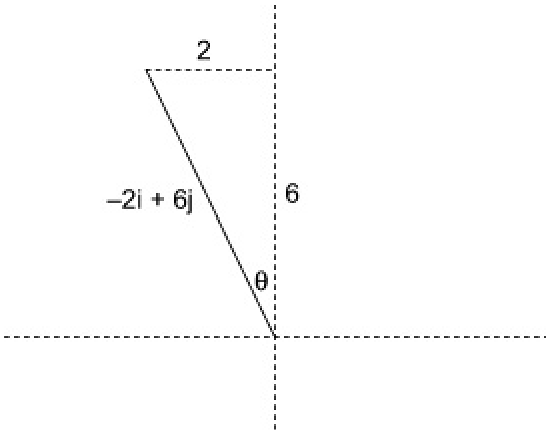
$(-3 + 1 + p) i + (7 - 1 + q) j = 0$

$p = 2, q = -6$

b $R = F_1 + F_2 = (-3i + 7j) + (i - j) = -2i + 6j$

$| R | = \sqrt{(-2)^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} = 6.32$ N

c



$\tan \theta = \frac{2}{6}$

$\theta = 18°$

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Vectors
Exercise H, Question 2

Question:

In this question, the horizontal unit vectors **i** and **j** are directed due east and north respectively.

A coastguard station O monitors the movements of ships in a channel. At noon, the station's radar records two ships moving with constant speed. Ship A is at the point with position vector $(-3i + 10j)$ km relative to O and has velocity $(2i + 2j)$ km h^{-1}. Ship B is at the point with position vector $(6i + j)$ km and has velocity $(-i + 5j)$ km h^{-1}.

a Show that if the two ships maintain these velocities they will collide.

The coastguard radios ship A and orders it to reduce its speed to move with velocity $(i + j)$ km h^{-1}. Given that A obeys this order and maintains this new constant velocity.

b find an expression for the vector AB at time t hours after noon,

c find, to three significant figures, the distance between A and B at 1500 hours,

d find the time at which B will be due north of A.

Solution:

a At time t

$$r_A = (-3 + 2t)i + (10 + 2t)j$$
$$r_B = (6 - t)i + (1 + 5t)j$$

i components equal when $-3 + 2t = 6 - t \Rightarrow 3t = 9, t = 3$

$t = 3 : r_A = 3i + 16j \; ; \; r_B = 3i + 16j \Rightarrow$ collide

b New $r_A = (-3 + t)i + (10 + t)j$

$$\Rightarrow AB = r_B - r_A = (6 - t)i + (1 + 5t)j - \{(-3 + t)i + (10 + t)j\{$$
$$= (6 - t + 3 - t)i + (1 + 5t - 10 - t)j$$
$$= (9 - 2t)i + (-9 + 4t)j$$

c $t = 3 : AB = 3i + 3j, \Rightarrow$ dist. $= \sqrt{(3^2 + 3^2)} \approx 4.24$ km

d B north of $A \Rightarrow$ no ***i*** component $\Rightarrow 9 - 2t = 0 \Rightarrow t = \frac{9}{2} \Rightarrow$ time 1630 hours

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Vectors
Exercise H, Question 3

Question:

Two ships P and Q are moving along straight lines with constant velocities. Initially P is at a point O and the position vector of Q relative to O is $(12i + 6j)$ km, where **i** and **j** are unit vectors directed due east and due north respectively. Ship P is moving with velocity $6i\ \text{km h}^{-1}$ and ship Q is moving with velocity $(-3i + 6j)\ \text{km h}^{-1}$. At time t hours the position vectors of P and Q relative to O are **p** km and **q** km respectively.

a Find **p** and **q** in terms of t.

b Calculate the distance of Q from P when $t = 4$.

c Calculate the value of t when Q is due north of P.

Solution:

a $p = 6ti$

$q = (12i + 6j) + (-3i + 6j)t = (12 - 3t)i + (6 + 6t)j$

b $t = 4 : p = 24i, q = 30j$

$\Rightarrow$ dist. apart $= \sqrt{24^2 + 30^2} = \sqrt{576 + 900} = \sqrt{1476} \approx 38.4$ km

c Q north of $P \Rightarrow i$ components match $\Rightarrow 6t = 12 - 3t \Rightarrow 9t = 12 \Rightarrow t = 1\frac{1}{3}$

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Vectors
Exercise H, Question 4

Question:

A particle P moves with constant acceleration $(-3i + j)$ m s^{-2}. At time t seconds, its velocity is v m s^{-1}. When $t = 0$, $v = 5i - 3j$.

a Find the value of t when P is moving parallel to the vector **i**.

b Find the speed of P when $t = 5$.

c Find the angle between the vector **i** and the direction of motion of P when $t = 5$.

Solution:

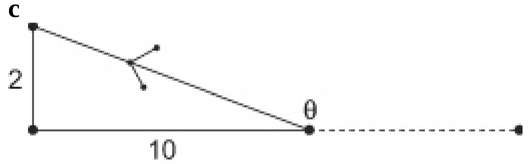
a $v = u + at : v = (5i - 3j) + (-3i + j) \times t = (5 - 3t)i + (-3 + t)j$

v parallel to $i \Rightarrow -3 + t = 0 \Rightarrow t = 3$ s

b $t = 5, v = -10i + 2j$

Speed $= |v| = \sqrt{104} \approx 10.2$ m s^{-1}

c



Angle $= \left(\arctan \frac{10}{2}\right) + 90^\circ = 168.7^\circ \left(1 \text{ s.f.}\right)$

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Vectors
Exercise H, Question 5

Question:

A particle P of mass 5 kg is moving under the action of a constant force **F** newtons. At $t = 0$, P has velocity $\left(5i - 3j \right)$ m s^{-1}. At $t = 4$ s, the velocity of P is $\left(-11i + 5j \right)$ m s^{-1}. Find

a the acceleration of P in terms of **i** and **j**,

b the magnitude of **F**.

At $t = 6$ s, P is at the point A with position vector $\left(28i + 6j \right)$ m relative to a fixed origin O. At this instant the force **F** newtons is removed and P then moves with constant velocity. Two seconds after the force has been removed, P is at the point B.

c Calculate the distance of B from O.

Solution:

a $a = \frac{v - u}{t}$: $a = \frac{1}{4}\left[\left(-11i + 5j \right) - \left(5i - 3j \right) \right] = -4i + 2j \text{ m s}^{-2}$

b $F = m\ a = 5 \times \left(-4i + 2j \right) = -20i + 10j$
$| F | = \sqrt{\left(-20 \right)^2 + 10^2} = \sqrt{500} \approx 22.4$ N

c $t = 6, v = u + at \Rightarrow v = \left(5i - 3j \right) + \left(-4i + 2j \right) \times 6 = 5i - 3j - 24i + 12j = -19i + 9j$
At B : $r = r_0 + vt \Rightarrow r = \left(28i + 6j \right) + \left(-19i + 9j \right) \times 2 = -10i + 24j$

Distance $OB = \sqrt{\left(-10 \right)^2 + 24^2} = \sqrt{100 + 576} = \sqrt{676} = 26$ m

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Vectors
Exercise H, Question 6

Question:

In this question the vectors **i** and **j** are horizontal unit vectors in the directions due east and due north respectively.

Two boats *A* and *B* are moving with constant velocities. Boat *A* moves with velocity $6i \text{ km h}^{-1}$. Boat *B* moves with velocity $(3i + 5j) \text{ km h}^{-1}$.

a Find the bearing on which *B* is moving.

At noon, *A* is at point *O* and *B* is 10 km due south of *O*. At time *t* hours after noon, the position vectors of *A* and *B* relative to *O* are **a** km and **b** km respectively.

b Find expressions for **a** and **b** in terms of *t*, giving your answer in the form $pi + qj$.

c Find the time when *A* is due east of *B*.

At time *t* hours after noon, the distance between *A* and *B* is *d*km. By finding an expression for *AB*,

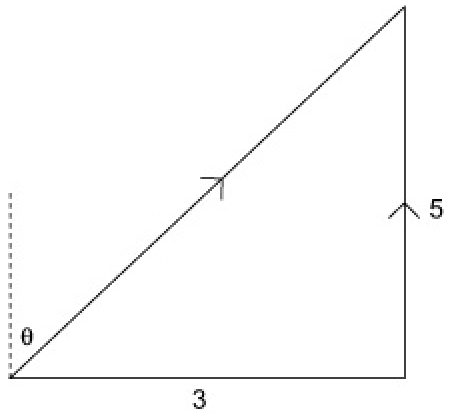
d show that $d^2 = 34t^2 - 100t + 100$.

At noon, the boats are 10 km apart.

e Find the time after noon at which the boats are again 10 km apart.

Solution:

a



$\tan \theta = \frac{3}{5} \Rightarrow \theta = 31^\circ$, bearing is 031°

b

$$a = 6ti$$
$$b = 3ti + (-10 + 5t)j$$

c *A* due east of *B* $\Rightarrow$ *j* components match $\Rightarrow -10 + 5t = 0$

$t = 2 \Rightarrow$ 1400 hours

d

$AB = b - a = \{ 3ti + (-10 + 5t) j \{ -6ti = -3ti + (-10 + 5t) j$

$d^2 = | b - a |^2 = (-3t)^2 + (-10 + 5t)^2 = 9t^2 + 100 - 100t + 25t^2$

$= 34t^2 - 100t + 100$, as required.

e $d = 10 \Rightarrow d^2 = 100 \Rightarrow 34t^2 - 100t = 0$

$\Rightarrow t = 0$ or $\frac{100}{34} = 2.9411 \ldots \ldots$

$\Rightarrow$ time is 1456 hours

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Vectors
Exercise H, Question 7

Question:

A small boat S, drifting in the sea, is modelled as a particle moving in a straight line at constant speed. When first sighted at 0900, S is at a point with position vector $(-2\mathbf{i}-4\mathbf{j})$ km relative to a fixed origin O, where **i** and **j** are unit vectors due east and due north respectively. At 0940, S is at the point with position vector $(4\mathbf{i}-6\mathbf{j})$ km. At time t hours after 0900, S is at the point with position vector **s** km.

a Calculate the bearing on which S is drifting.

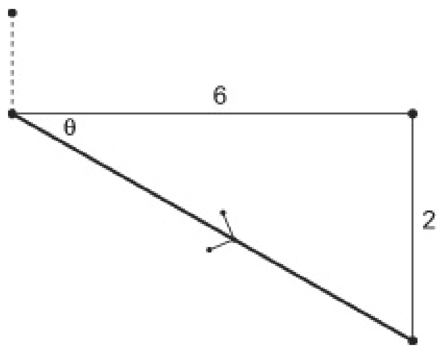
b Find an expression for **s** in terms of t.

At 1100 a motor boat M leaves O and travels with constant velocity $(p\mathbf{i}+q\mathbf{j})$ km h^{-1}.

c Given that M intercepts S at 1130, calculate the value of p and the value of q.

Solution:

a (Direction of v) $=(4i-6j)-(-2i-4j)=6i-2j$



$$\tan\theta=\frac{1}{3}\Rightarrow\theta=18.43\ldots^{\circ}$$

$$\text{Bearing}=90^{\circ}+\theta=108^{\circ}$$

b Expressing v in kmh^{-1}, $v=\left(6i-2j\right)\times\frac{60}{40}=9i-3j$

$r=r_0+vt\Rightarrow s=(-2i-4j)+t(9i-3j)=(-2+9t)i+(-4-3t)j$

c At 1130, $t=2.5$, $s=(-2+9\times2.5)i+(-4-3\times2.5)j=20.5i-11.5j$

M has been travelling for 30 minutes $\Rightarrow m=0.5(pi+qj)$

$s=m\Rightarrow p=41, q=-23$

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Vectors
Exercise H, Question 8

Question:

A particle P moves in a horizontal plane. The acceleration of P is $(-2i+3j)\ \mathrm{m\,s^{-2}}$. At time $t=0$, the velocity of P is $(3i-2j)\ \mathrm{m\,s^{-1}}$.

a Find, to the nearest degree, the angle between the vector **j** and the direction of motion of P when $t=0$.

At time t seconds, the velocity of P is $v\ \mathrm{m\,s^{-1}}$. Find

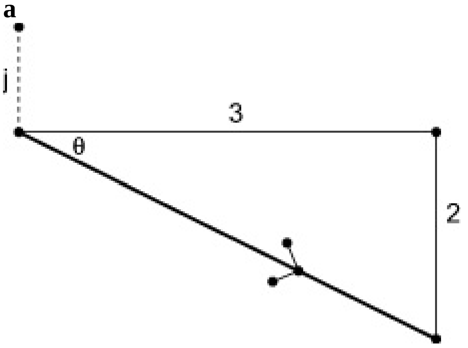
b an expression for **v** in terms of t, in the form $ai+bj$,

c the speed of P when $t=4$,

d the time when P is moving parallel to $i+j$.

Solution:

a



$\tan\theta = \frac{2}{3}\ \left(\theta = 33.7^\circ\right)$

angle between v and $j = 90 + 33.7 \approx 124^\circ$

b $v = u + at : v = 3i - 2j + (-2i+3j)\times t = (3-2t)\,i + (-2+3t)\,j$

c $t=4, v = \underline{(3-2\times 4)\,i} + \underline{(-2+3\times 4)\,j} = -5i + 10j$
speed $= \sqrt{(-5)^2 + 10^2} = \sqrt{25+100} = \sqrt{125} \approx 11.2\ \mathrm{m\,s^{-1}}$

d v parallel to $i+j \Rightarrow i$ and **j** components must be equal $\Rightarrow 3-2t = -2+3t$
$\Rightarrow 5t = 5, t = 1$

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Vectors
Exercise H, Question 9

Question:

In this question, the unit vectors **i** and **j** are horizontal vectors due east and north respectively.

At time $t = 0$ a football player kicks a ball from the point A with position vector $(3i + 2j)$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(4i + 9j)$ m s^{-1}. Find

a the speed of the ball,

b the position vector of the ball after t seconds.

The point B on the field has position vector $(29i + 12j)$ m .

c Find the time when the ball is due north of B.

At time $t = 0$, another player starts running due north from B and moves with constant speed $v\,\text{m s}^{-1}$.

d Given that he intercepts the ball, find the value of v.

Solution:

a $v = 4i + 9j \Rightarrow$ speed of ball $= \sqrt{(4^2 + 9^2)} \approx 9.85\ \text{m s}^{-1}$

b $r = r_0 + vt \Rightarrow$ position vector of the ball after t seconds

$= (3i + 2j) + (4i + 9j) \times t = (3 + 4t)\,i + (2 + 9t)\,j$

c North of B when $\boldsymbol{i}$ components same, i.e. $3 + 4t = 29, 4t = 26, t = 6.5$ s

d When $t = 6.5$, position vector of the ball

$$= (3 + 4 \times 6.5)\,i + (2 + 9 \times 6.5)\,j = 29i + 60.5j$$
$$\Rightarrow j \text{ component} = 60.5$$

Distance travelled by 2nd player $= 60.5 - 12 = 48.5$ m

Speed $= 48.5 \div 6.5 \approx 7.46\ \text{m s}^{-1}$

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Vectors
Exercise H, Question 10

Question:

Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(10i + 15j)$ km relative to a fixed origin O. At the same time, Q is at the point with position vector $(-16i + 26j)$ km. Three hours later, P is at the point with position vector $(25i + 24j)$ km. The ship Q travels with velocity $12i \text{ km h}^{-1}$. At time t hours after midnight, the position vectors of P and Q are **p** km and **q** km respectively. Find

a the velocity of P in terms of **i** and **j**,

b expressions for **p** and **q** in terms of t, **i** and **j**.

At time t hours after midnight, the distance between P and Q is d km.

c By finding an expression for PQ, show that

$d^2 = 58t^2 - 430t + 797$

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 13 km or less.

d Given that when $t = 2$ the lights on Q move into sight of the observer, find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

Solution:

a $r = r_0 + vt \Rightarrow v_P = \{(25i + 24j) - (10i + 15j)\{ / 3 = (5i + 3j) \text{ km h}^{-1}$

b

$r = r_0 + vt \Rightarrow$ after t hours,

$$p = (10i + 15j) + (5i + 3j) \times t = (10 + 5t)i + (15 + 3t)j$$
$$q = (-16i + 26j) + 12i \times t = (-16 + 12t)i + 26j$$

c

$$\begin{aligned} PQ = q - p &= \{(-16 + 12t)i + 26j\{ - \{(10 + 5t)i + (15 + 3t)j\{ \\ &= (-26 + 7t)i + (11 - 3t)j \\ \Rightarrow d^2 &= (-26 + 7t)^2 + (11 - 3t)^2 \\ &= 676 - 364t + 49t^2 + 121 - 66t + 9t^2 \\ &= 58t^2 - 430t + 797\text{, as required} \end{aligned}$$

d $d = 13 \Rightarrow 58t^2 - 430t + 797 = 169, 58t^2 - 430t + 628 = 0$

We are given that $t = 2$ is one solution, so we know that $(t - 2)$ is a factor

$$\Rightarrow (t - 2)(58t - 314) = 0$$
$$\Rightarrow t = \frac{314}{58} \approx 5.41 \text{ hours, so time} \approx 0525 \text{ to the nearest minute}$$

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Edexcel AS and A Level Modular Mathematics

Examination style paper
Exercise A, Question 1

Question:

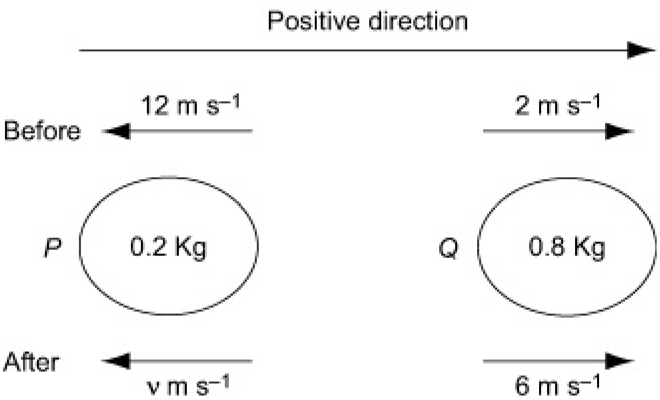
A particle P of mass 0.2 kg is moving along a straight horizontal line with constant speed 12 m s $^{-1}$. Another particle Q of mass 0.8 kg is moving in the same direction as P, along the same straight horizontal line, with constant speed 2m s $^{-1}$. The particles collide. Immediately after the collision, Q is moving with speed 6 m s $^{-1}$.

a Find the speed of P immediately after the collision.

b State whether or not the direction of motion of P is changed by the collision.

c Find the magnitude of the impulse exerted on Q in the collision.

Solution:



a Conservation of linear momentum

$$0.2 \times 12 + 0.8 \times 2 = 0.2 \times v + 0.8 \times 6$$
$$0.2v = -0.8 \Rightarrow v = -4$$

the speed of P immediately after the collision is 4 m s $^{-1}$

b The direction of motion of P has been changed by the collision.

c For Q, $I = 0.8 \times 6 - 0.8 \times 2 = 3.2$

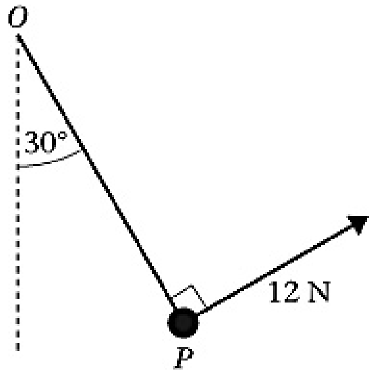
the magnitude of the impulse on Q is 3.2 N s

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Examination style paper
Exercise A, Question 2

Question:

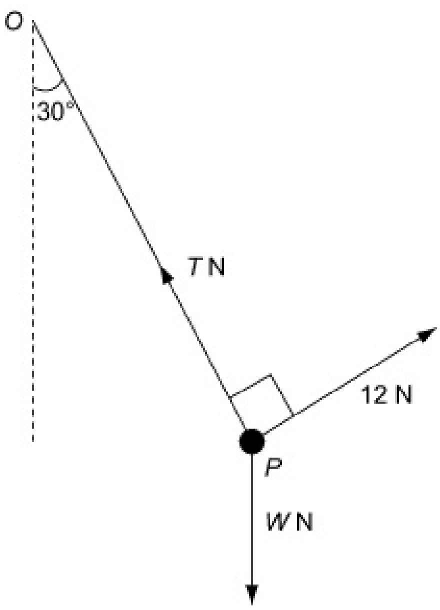
A particle *P* of weight *W* newtons is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point *O*. The string is taut and makes an angle 30° with the vertical. The particle *P* is held in equilibrium under gravity by a force of magnitude 12 N acting in a direction perpendicular to the string, as shown. Find



a the tension in the string,

b the value of *W*.

Solution:



a R (→) $T \cos 60° = 12 \cos 30°$

$T = 12\sqrt{3}\ (\approx 20.8)$

the tension in the string is $12\sqrt{3}$ N

b

R (↑) $W = T \sin 60° + 12 \sin 30°$

$$= 12\sqrt{3} \times \frac{\sqrt{3}}{2} + 12 \times \frac{1}{2} = 24$$

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Examination style paper
Exercise A, Question 3

Question:

A car is moving along a straight horizontal road. At time $t = 0$, the car passes a sign
A with speed 8 m s $^{-1}$ and this speed is maintained for 6 s. The car then accelerates uniformly from 8 m s $^{-1}$ to 12 m s $^{-1}$ in 9 s. The speed of 12 m s $^{-1}$ is then maintained until the car passes a second sign B. The distance between A and B is 390 m.

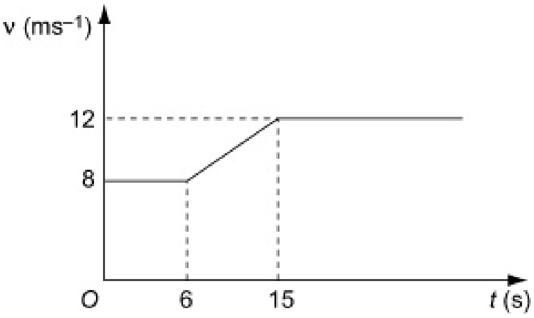
a Sketch a speed-time graph to illustrate the motion of the car as it travels from *A* to *B*.

b Find the time the car takes to travel from *A* to *B*.

c Sketch a distance-time graph to illustrate the motion of the car as it travels from *A* to *B*.

Solution:

a



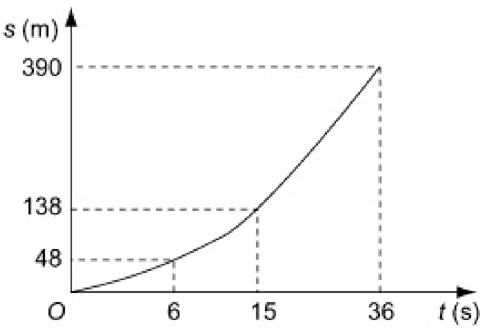
b Let the time travelled at 12 m s $^{-1}$ be T seconds.

$$6 \times 8 + \frac{1}{2}(8 + 12) \times 9 + 12 \times T = 390$$

$$12T = 390 - 48 - 90 = 252 \Rightarrow T = 21$$

the time the car takes to travel from *A* to *B* is 36 *s*

c



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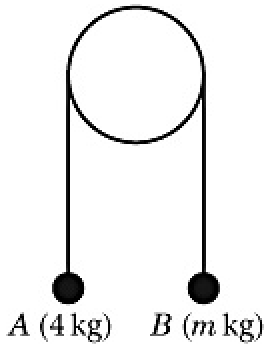
Examination style paper
Exercise A, Question 4

Question:

Two particles *A* and *B* are connected by a light inextensible string which passes over a fixed smooth pulley. The mass of *A* is 4 kg and the mass of *B* is *m*, where $m > 4kg$. The system is released from rest with the string taut and the hanging parts of the string vertical, as shown.

After release, the tension in the string is $\frac{1}{4}mg$.

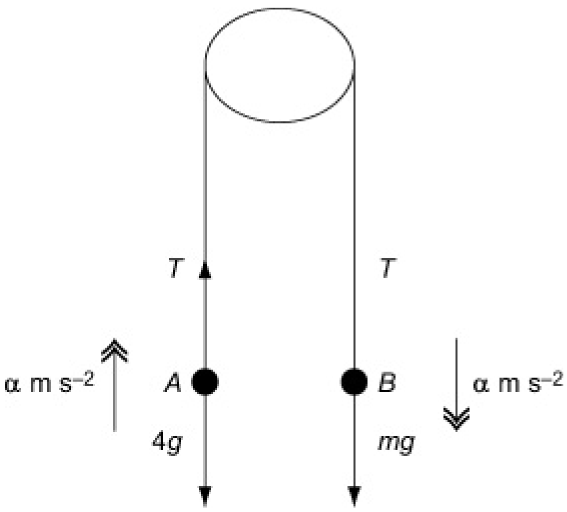


a Find the magnitude of the acceleration of the particles.

b Find the value of *m*.

c State how you have used the fact that the string is inextensible.

Solution:



a For *B*

$$\text{R}(\downarrow) \quad mg - T = ma$$

$$\cancel{m}g - \frac{1}{4}\cancel{m}g = \cancel{m}a$$

$$a = \frac{3}{4}g$$

the magnitude of the acceleration of the particles is $\frac{3}{4}g$

b For *A*

$$\begin{aligned}
&\text{R}(\uparrow) \quad T - 4g &&= 4a \\
&\tfrac{1}{4}mg - 4g &&= 4 \times \tfrac{3}{4}g \\
&m &&= 28
\end{aligned}$$

c the accelerations of the particles have the same magnitude.

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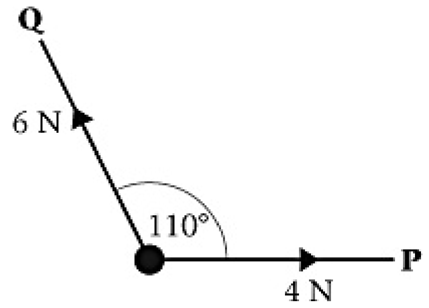
Examination style paper
Exercise A, Question 5

Question:

A particle of mass 0.8 kg is moving under the action of two forces **P** and **Q**. The force **P** has magnitude 4 N and the force **Q** has magnitude 6 N. The angle between **P** and **Q** is 110°, as shown. The resultant of **P** and **Q** is **F**. Find



a the angle between the direction of **F** and the direction of **P**.

b the magnitude of the acceleration of the particle.

Solution:

(i)

$$\text{R}(\rightarrow) \quad X = 4 - 6\cos 70° = 1.947879\ldots$$
$$\text{R}(\uparrow) \quad Y = 6\sin 70° = 5.638155\ldots$$

$$\tan\theta = \frac{Y}{X} = 2.89451\ldots$$
$$\theta = 70.9° \quad (3\text{ s.f.})$$

the angle between the direction of ***F*** and the direction of ***P*** is 70.9 ° (3 s.f.)

(ii)

$$|F|^2 = X^2 + Y^2 = 35.583\ldots$$
$$F = |F| = \sqrt{35.583\ldots} = 5.96515\ldots$$
$$F = ma$$

$5.96515\ldots = 0.8a \Rightarrow a = 7.456\ldots$

the acceleration of P is 7.46 m s^{-2} (3 s.f.)

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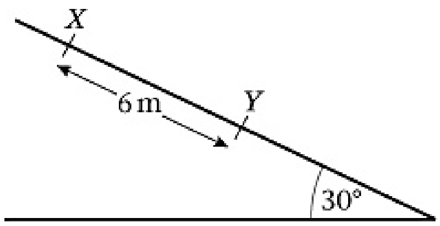
Examination style paper
Exercise A, Question 6

Question:

A small stone, S, of mass 0.3 kg, slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 30° to the horizontal. The stone passes through a point X with speed 1.5 m s^{-1}. Three seconds later it passes through a point Y, where XY = 6 m, as shown. Find.

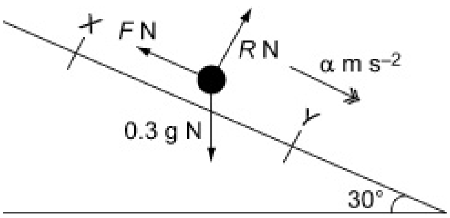


a the acceleration of S,

b the magnitude of the normal reaction of the plane on S,

c the coefficient of friction between S and the plane.

Solution:



a $u = 1.5, t = 3, s = 6, a = ?$

$$s = ut + \frac{1}{2}at^2$$

$$6 = 1.5 \times 3 + \frac{1}{2}a \times 9$$

$$1.5 = 4.5a \Rightarrow a = \frac{1}{3}$$

the acceleration of S is $\frac{1}{3}$ m s^{-2}

b R (↑) $R = 0.3g \cos 30° = 2.546 \ldots$

the magnitude of the normal reaction of the plane on S is 2.5 N (2 s.f.)

c Friction is limiting $F = \mu R = \mu \times 0.3g \cos 30°$

$$R(\nwarrow)\ 0.3g \sin 30° - \mu 0.3g \cos 30° = 0.3 \times \frac{1}{3}$$

$$\mu = \frac{g \sin 30° - \frac{1}{3}}{g \cos 30°} = 0.538 \ldots$$

The coefficient of friction between S and the planeis 0.54 (2 s.f.)

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Examination style paper
Exercise A, Question 7

Question:

In this question the unit vectors **i** *and* **j** *are due east and north respectively and position vectors are given with respect to a fixed origin O.*

A ship S is moving with constant velocity $(2\mathbf{i}-3\mathbf{j})$ km h^{-1} and a ship R is moving with constant velocity $6\mathbf{i}$ km h^{-1}.

a Find the bearing along which S is moving.

At noon S is at the point with position vector $8\mathbf{i}$ km and R is at O. At time t hours after noon, the position vectors of S and T are $\mathbf{s}$ km and $\mathbf{r}$ km respectively.

b Find $\mathbf{s}$ and $\mathbf{r}$, in terms of t.

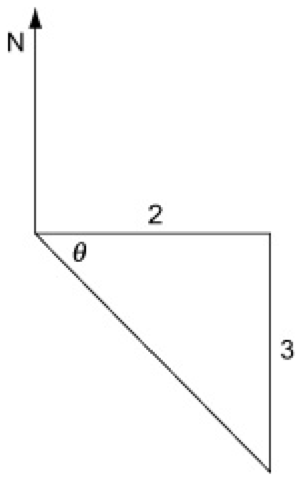
At time T hours, R is due north-east of S. Find

c the value of T,

d the distance between S and R at time T hours.

Solution:

a



$\tan\theta = \frac{3}{2} \Rightarrow \theta \approx 56.3\,°$

the bearing along which S is moving is 146

b

$s = 8i + (2i - 3j)\,t$

$r = 6ti$

c At time $t = T$, $r - s = (4T - 8)\,i + 3Tj$

If S is north-east of R,

$\frac{3T}{4T-8} = 1 \Rightarrow T = 8$

d When $T = 8$

$$r - s = 24i + 24j$$
$$SR^2 = 24^2 + 24^2 \Rightarrow SR = 24\sqrt{2}$$

The distance between S and R is $24\sqrt{2}$km.

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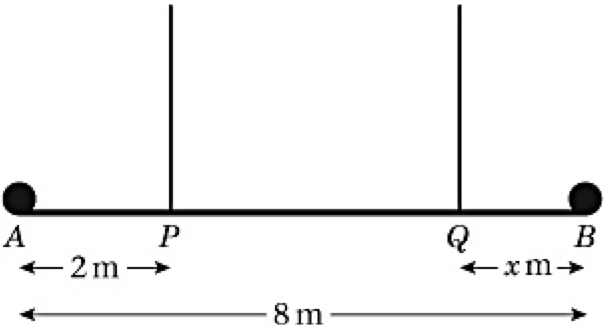
Examination style paper
Exercise A, Question 8

Question:



A uniform steel girder AB has length 8 m and weight 400 N. A load of weight 200 N is attached to the girder at A and a load of weight W newtons is attached to the girder at B. The girder and the loads hang in equilibrium, with the girder horizontal. The girder is held in equilibrium by two cables attached to the girder at P and Q, where AP = 2 m and QB = x m, as shown. The girder is modelled as a uniform rod, the loads as particles and the cables as light inextensible strings.

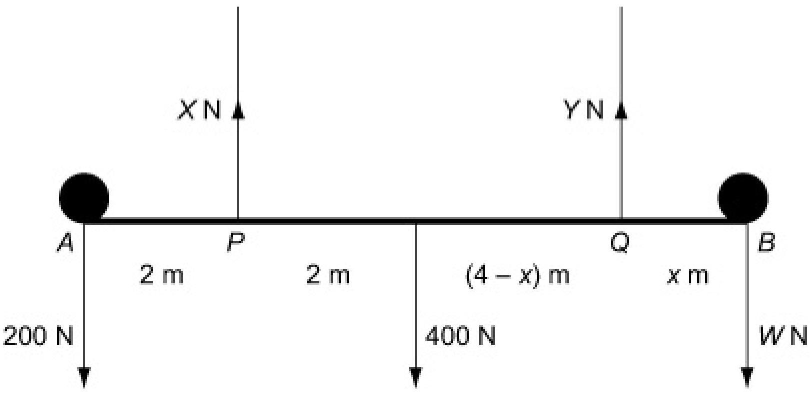
a Show that the tension in the cable at Q is $\left(\frac{400 + 6W}{6 - x}\right)$ N.

Given that the tension in the cable attached at P is five times the tension in the cable attached to Q,

b find W in terms of x,

c deduce that $x < 2$.

Solution:



a

$$M(P)\quad Y(6 - x) + 200 \times 2 = 400 \times 2 + W \times 6$$
$$Y(6 - x) = 800 - 400 + 6W$$

$Y = \frac{400 + 6W}{6 - x}$

the tension in the cable at Q is $\left(\frac{400+6W}{6-x}\right)$ N

b

$$\text{R}(\uparrow) \quad X + Y = 600 + W$$

$$X = 600 + W - \frac{400+6W}{6-x}$$

$$= \frac{3200 - 600x - wx}{6-x}$$

$$X = 5Y \Rightarrow 3200 - 600x - Wx = 5(400 + 6W)$$

$$1200 - 600x = (30 + x)W$$

$$W = \frac{600(2-x)}{30+x}$$

c $W \geq 0 \Rightarrow x \geq 2$

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Review Exercise
Exercise A, Question 1

Question:

A train decelerates uniformly from 35 m s $^{-1}$ to 21 m s $^{-1}$ in a distance of 350 m. Calculate

a the deceleration,

b the total time taken, under this deceleration, to come to rest from a speed of 35 m s $^{-1}$.

Solution:

a $u = 35, v = 21, s = 350, a = ?$

$v^2 = u^2 + 2as$

$21^2 = 35^2 + 2 \times s \times 350$

$a = \dfrac{21^2 - 35^2}{700} = -1.12$

The deceleration of the train is 1.12 m s^{-2}.

There is no t here, so you choose the formula without t, $v^2 = u^2 + 2as$.

b $u = 35, v = 0, a = -1.12, t = ?$

$v = u + at$

$0 = 35 - 1.12t$

$t = \dfrac{35}{1.12} = 31.25$

The total time taken to come to rest is 31.25 s.

You use the value of a you found in part **a** in part **b**. As the train is decelerating, a is negative.

Solutionbank M1
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Review Exercise
Exercise A, Question 2

Question:

In taking off, an aircraft moves on a straight runway *AB* of length 1.2 km. The aircraft moves from *A* with initial speed 2 m s $^{-1}$. It moves with constant acceleration and 20s later it leaves the runway at *C* with speed 74 m s $^{-1}$. Find

a the acceleration of the aircraft,

b the distance *BC*.

Solution:

a $u = 2, t = 20, v = 74, a = ?$

$v = u + at$

$74 = 2 + 20a$

$a = \dfrac{74-2}{20} = 3.6$

The acceleration of the aircraft is 3.6 m s^{-2}.

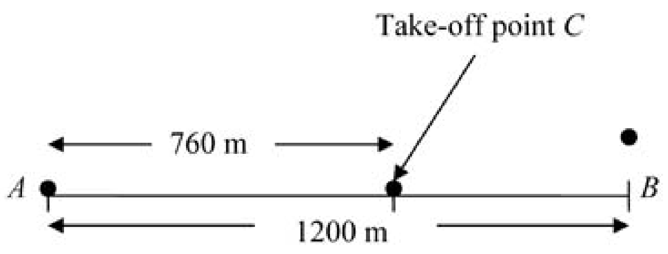
There is no s here, so you choose the formula without s, $v = u + at$.

b $u = 2, t = 20, v = 74, s = ?$

$s = \left(\dfrac{u+v}{2}\right)t$

$= \left(\dfrac{2+74}{2}\right) \times 20 = 760$

You could use the value of a you found in part **a** but, if it can be avoided without causing a lot of extra work, it is safer not to use your answer from **a**. Everyone makes mistakes from time to time!



The distance *AC* is 760 m.
The distance *BC* is $(1200 - 760)\text{ m} = 440 \text{ m}$.

The aircraft takes off 760 m from its starting point *A*. The runway is 1.2 km, which is 1200 m, long. So the aircraft is $(1200 - 760)\text{ m}$ from the end *C* of the runway.

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Review Exercise
Exercise A, Question 3

Question:

A car is moving along a straight horizontal road at a constant speed of 18 m s $^{-1}$. At the instant when the car passes a lay-by, a motorcyclist leaves the lay-by, starting from rest, and moves with constant acceleration 2.5 m s $^{-2}$ in pursuit of the car. Given that the motorcyclist overtakes the car T seconds after leaving the lay-by, calculate

a the value of T,

b the speed of the motorcyclist at the instant of passing the car.

Solution:

a After time t, let the distance moved by the car be s_1 and the distance moved by the motor cycle s_2.

The distance moved by the car is given by

$$s_1 = 18t$$

The car is travelling at a constant speed, so you use distance = speed × time to obtain an expression for the distance moved by the car.

The distance moved by the motor cycle is given by

$$s = ut + \frac{1}{2}at^2, \text{ with } u = 0 \text{ and } a = 2.5$$

$$s_2 = 0 \times t + \frac{1}{2} \times 2.5t^2 = 1.25t^2$$

When $t = T$, $s_1 = s_2$

$$18T = 1.25T^2$$

$$1.25T^2 - 18T = T(1.25T - 18) = 0$$

$$T = \frac{18}{1.25} = 14.4$$

As the car and the motor cycle were level at the lay-by, when the motor cycle overtakes the car, they have travelled the same distance. Equating s_1 to s_2 gives and equation you can solve.

$T = 0$ is a solution of this equation but that is the time at the lay-by and can be ignored.

b $u = 0, a = 2.5, t = 14.4, v = ?$

$$v = u + at$$
$$= 0 + 2.5 \times 14.4 = 36$$

The speed of the motor cycle at the instant of passing the car is 36 m s^{-1}.

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Review Exercise
Exercise A, Question 4

Question:

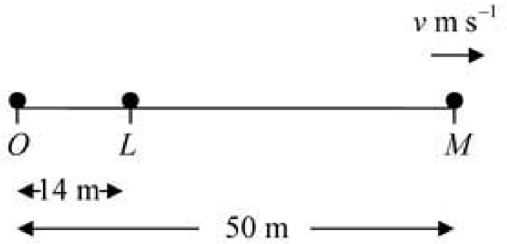
A particle moves with constant acceleration along the straight line *OLM* and passes through the points *O*, *L* and *M* at times 0 s, 4 s and 10 s respectively. Given that *OL* = 14 m and *OM* = 50 m, find

a the acceleration of the particle,

b the speed of the particle at *M*.

Solution:

a



You are asked to find two variables in this question, a and v. You are given information about two variables, t and s. You are neither given any information nor asked about the initial speed u. The most efficient way of solving the question is to use the formula without u, $s = vt - \frac{1}{2}at^2$. This enables you to solve both parts of the question together.

From L to M
The particle takes $(10-4)\,\text{s} = 6\,\text{s}$ to travel a distance $(50-14)\,\text{m} = 36\,\text{m}$.

$t = 6,\ s = 36,\ a = ?,\ v = ?$

$$s = vt - \frac{1}{2}at^2$$

$$36 = 6v - \frac{1}{2}a \times 6^2 \Rightarrow 6v - 18a = 36$$

$$\Rightarrow v - 3a = 6 \ldots\ldots (1)$$

You obtain two linear simultaneous equations from the data. You solve these in the way you learnt for GCSE.

From O to M, $t = 10,\ s = 50,\ a = ?,\ v = ?$

$$s = vt - \frac{1}{2}at^2$$

$$50 = 10v - \frac{1}{2}a \times 10^2 \Rightarrow 10v - 50a = 50$$

$$\Rightarrow v - 5a = 5 \ldots\ (2)$$

(1) − (2)

$$2a = 1 \Rightarrow a = 0.5$$

The acceleration of the particle is $0.5\ \text{m s}^{-2}$.

b Substitute $a = 0.5$ into (1)

$$v - 1.5 = 6 \Rightarrow v = 7.5$$

The speed of the particle at M is $7.5\ \text{m s}^{-1}$.

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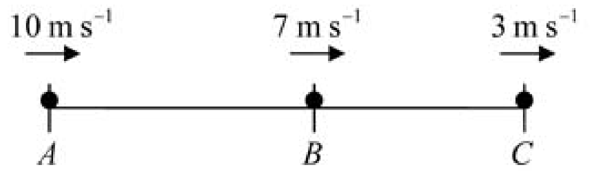
Review Exercise
Exercise A, Question 5

Question:

A particle P moves in a straight line with constant retardation. At the instants when P passes through the points A, B and C, it is moving with speeds 10 m s^{-1}, 7 m s^{-1} and 3 m s^{-1} respectively.

Show that $\frac{AB}{BC} = \frac{51}{40}$.

Solution:



From A to B

$u = 10, v = 7, s = AB$

$v^2 = u^2 + 2as$

$7^2 = 10^2 + 2a \times AB$

$2a \times AB = 49 - 100 = -51 \quad \ldots \ldots (1)$

You are given information about speeds and asked about distances. Time will not come into the question, so it is reasonable to try and make two equations with $v^2 = u^2 + 2as$.

From B to C

$u = 7, v = 3, s = BC$

$v^2 = u^2 + 2as$

$3^2 = 7^2 + 2a \times BC$

$2a \times BC = 9 - 49 = -40 \quad \ldots \ldots (2)$

Dividing equation (1) by equation (2), $2a$ is in both the numerator and denominator of the fraction and so can be "cancelled".

$(1) \div (2)$

$$\frac{\cancel{2a} \times AB}{\cancel{2a} \times BC} = \frac{-51}{-40} = \frac{51}{40}$$

$$\frac{AB}{BC} = \frac{51}{40}, \text{ as required.}$$

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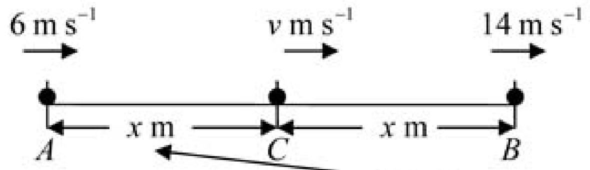
Review Exercise
Exercise A, Question 6

Question:

A car moving with uniform acceleration along a straight level road, passed points *A* and *B* when moving with speed 6 m s $^{-1}$ and 14 m s $^{-1}$ respectively. Find the speed of the car at the instant that it passed *C*, the mid-point of *AB*.

Solution:



You do not know the distances but as C is the mid-point of AB if you let AC be x, then $AB = 2x$. This use of x as the unknown allows you to form equations.

From A to B
$u = 6,\ v = 14,\ s = AB = 2x$
$v^2 = u^2 + 2as$
$14^2 = 6^2 + 2a \times 2x \Rightarrow 196 = 36 + 4ax$
$ax = \dfrac{196 - 36}{4} = 40$

You are given information about speeds and distances.
Time will not come into the question, so it is reasonable to try and make two equations with $v^2 = u^2 + 2as$.

From A to C
$u = 6,\ s = AC = x,\ v = ?$
$v^2 = u^2 + 2as$
$\quad = 6^2 + 2ax = 36 + 2 \times 40 = 116$
$v = \sqrt{116} \approx 10.8$
The speed of the car at C is 10.8 m s^{-1} (3 s.f.).

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Review Exercise
Exercise A, Question 7

Question:

A particle P is moving along the x-axis with constant deceleration 3 m s $^{-2}$. At time t = 0 s, P is passing through the origin O and is moving with speed u m s $^{-1}$ in the direction of x increasing. At time t = 8 s, P is instantaneously at rest at the point A. Find

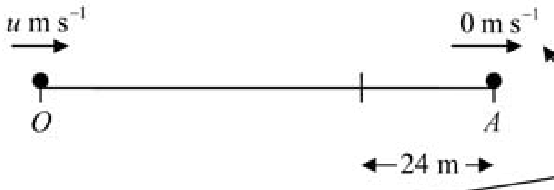
a the value of u,

b the distance OA,

c the times at which P is 24 m from A.

Solution:

a



'Instantaneously at rest' is translated into algebra as $v = 0$.

$a = -3, t = 8, v = 0, u = ?$
$v = u + at$
$0 = u - 3 \times 8 \Rightarrow u = 24$

b $a = -3, t = 8, v = 0, s = ?$

$s = vt - \frac{1}{2}at^2$

$= 0 \times 8 - \frac{1}{2} \times (-3) \times 8^2 = 96$

$OA = 96$ m

You do not need u here, since you can use the formula without u, $s = vt - \frac{1}{2}at^2$.

c 24 m from A is $(96 - 24)$m $= 72$ m from O

$s = 72, a = -3, u = 24, t = ?$

$s = ut + \frac{1}{2}at^2$

$72 = 24t + \frac{1}{2} \times (-3) \times t^2 = 24t - 1.5t^2$

$3t^2 - 48t + 144 = 0$

Multiply the equation by 2 and rearrange.

Dividing throughout by 3

$t^2 - 16t + 48 = (t-4)(t-12) = 0$

$t = 4, 12$

You may use any method to solve the quadratic equation but if you did use the formula you would be expected to obtain exact answers.

P is 24 m for A at times $t = 4$ s and $t = 12$ s.

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Review Exercise
Exercise A, Question 8

Question:

A train moves along a straight track with constant acceleration. Three telegraph poles are set at equal intervals beside the track at points *A*, *B* and *C*, where *AB* = 50 m and *BC* = 50 m. The front of the train passes *A* with speed 22.5 m s^{-1}, and 2 s later it passes *B*. Find

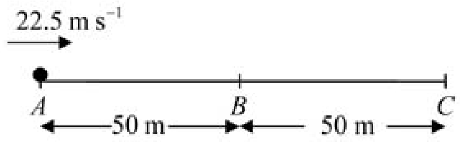
a the acceleration of the train,

b the speed of the front of the train when it passes *C*,

c the time that elapses from the instant the front of the train passes *B* to the instant it passes *C*.

Solution:

a



From A to B

$u = 22.5,\ s = 50,\ t = 2,\ a = ?$

$s = ut + \frac{1}{2}at^2$

There is no v here, so you use the formula without v, $s = ut + \frac{1}{2}at^2$.

$50 = 22.5 \times 2 + \frac{1}{2}a \times 2^2 = 45 + 2a$

$a = \frac{50 - 45}{2} = 2.5$

The acceleration of the train is $2.5\ \text{m s}^{-2}$.

b From A to C

$u = 22.5,\ s = 100,\ a = 2.5,\ v = ?$

$v^2 = u^2 + 2as$

There is no t here, so you choose the formula without t, $v^2 = u^2 + 2as$.

$= 22.5^2 + 2 \times 2.5 \times 100 = 1006.25$

$v = \sqrt{1006.25} = 31.7\ \ldots$

The speed at C is $31.7\ \text{m s}^{-1}$ (3 s.f.).

Where no accuracy is specified, it is reasonable to give your answer to 3 significant figures.

c To find time from A to C

$u = 22.5,\ v = \sqrt{1006.25},\ a = 2.5,\ t = ?$

$v = u + at$

$\sqrt{1006.25} = 22.5 + 2.5t$

$t = \frac{\sqrt{1006.25} - 22.5}{2.5} = 3.688\ \ldots$

The time from B to C is

$(3.69 - 2)\text{s} = 1.69\ \text{s}$ (3 s.f.).

It is possible to find the time directly using $s = vt - \frac{1}{2}at^2$ but this would give an awkward quadratic with difficult coefficients. $v = u + at$ is easy to use and, in this case, it is easy to adjust the time found, from A to C, by subtracting the time taken to go from A to B. This gives the time from B to C as required.

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Review Exercise
Exercise A, Question 9

Question:

A particle *X*, moving along a straight line with constant speed 4 m s $^{-1}$, passes through a fixed point *O*. Two seconds later, another particle *Y*, moving along the same straight line and in the same direction, passes through *O* with speed 6 m s $^{-1}$. The particle *Y* is moving with constant deceleration 2 m s $^{-2}$.

a Write down expressions for the velocity and displacement of each particle *t* seconds after *Y* passed through *O*.

b Find the shortest distance between the particles after they have both passed through *O*.

c Find the value of *t* when the distance between the particles has increased to 23 m.

Solution:

a For *X*, let the velocity at time *t* second be v_x m s^{-1} and the displacement s_x m.

$$v_x = 4$$
$$s_x = 4(t+2)$$

t seconds is the time since *Y* passed through *O*. *X* passed through *O* 2 seconds earlier, so it is $(t+2)$s since *X* passed through *O*.
As *X* is moving with constant speed, distance = speed × time.

For *Y*, let the velocity at time *t* second be v_y m s^{-1} and the displacement s_y m.

$u = 6, a = -2$

$$v_y = u + at$$
$$= 6 - 2t$$
$$s_y = ut + \frac{1}{2}at^2$$
$$= 6t - t^2$$

b The distance between *X* and *Y*, *d* say, is given by

$$d = s_x - s_y = 4(t+2) - (6t - t^2)$$
$$= 8 - 2t + t^2$$
$$= 7 + 1 - 2t + t^2$$
$$= 7 + (t-1)^2 \geqslant 7$$

All real numbers squared are non-negative. So $(t-1)^2 \geqslant 0$ and it follows that $7 + (t-1)^2 \geqslant 7 + 0 = 7$.
There are alternative solutions using differentiation.

The shortest distance is 7 m.

c

$$7 + (t-1)^2 = 23$$
$$(t-1)^2 = 16$$
$$t - 1 = 4$$
$$t = 5$$

The negative square root of 16, -4, gives $t = -3$. This can be ignored as the time would be before *Y* passed through *O*.

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Review Exercise
Exercise A, Question 10

Question:

A stone is projected vertically upwards from a point *A* with initial speed u m s^{-1}. It takes 3.5 s to reach its maximum height above *A*. Find

a the value of u,

b the maximum height of the stone above *A*.

Solution:

a Taking the upwards direction as positive.

At the greatest height the stone is instantaneously at rest. In algebra, $v = 0$.

$v = 0, t = 3.5, a = -9.8, u = ?$
$v = u + at$
$0 = u - 9.8 \times 3.5$
$u = 34.3$

$u = 34$ (2 s.f.)

When you model an object moving in a vertically as a particle moving with constant acceleration of magnitude 9.8 m s^{-2}, you should give your answers corrected to 2 significant figures. You should, however, show your working to at least 3 significant figures.

b $v = 0, t = 3.5, a = -9.8, s = ?$

$$s = vt - \frac{1}{2}at^2$$
$$= 0 - \frac{1}{2} \times (-9.8) \times 3.5^2 = 60.025$$

The maximum height above A is 60 m (2 s.f.).

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Review Exercise
Exercise A, Question 11

Question:

A small ball is projected vertically upwards from a point *A*. The greatest height reached by the ball is 40 m above *A*. Calculate

a the speed of projection,

b the time between the instant that the ball is projected and the instant it returns to *A*.

Solution:

a Taking the upwards direction as positive.

$s = 40,\ v = 0,\ a = -9.8,\ u = ?$

At the greatest height the velocity of the ball is instantaneously zero.

$v^2 = u^2 + 2as$

$0^2 = u^2 - 2 \times 9.8 \times 40$

$u^2 = 784 \Rightarrow u = 28\ \ldots$

The speed of projection is $28\ \text{m s}^{-1}$.

When the ball returns to *A*, its displacement from *A* is zero.

b $s = 0,\ u = 28,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 28t - 4.9t^2 = t(28 - 4.9t)$

$t = 0,\ t = \frac{28}{4.9} = 5.714\ \ldots$

The solution $t = 0$ represents the time of projection. You choose the other solution for the time of return.

The time taken to return to *A* is 5.7 s, (2 s.f.).

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Review Exercise
Exercise A, Question 12

Question:

A ball is projected vertically upwards and takes 3 seconds to reach its highest point. At time t seconds, the ball is 39.2 m above its point of projection. Find the possible values of t.

Solution:

Find the speed of projection.
Taking the upwards direction as positive.

$v = 0, t = 3, a = -9.8, u = ?$

$v = u + at$

$0 = u - 9.8 \times 3 \Rightarrow u = 29.4$

There are two values of t, one as the ball ascends and one as it descends. To find two values of t, you will need to form a quadratic equation using $s = ut + \frac{1}{2}at^2$.
Before you can do this, you will need to use the information, that the ball reaches its greatest height in 3 s, to find u.

$s = 39.2, u = 29.4, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$39.2 = 29.4t - 4.9t^2$

$4.9t^2 - 29.4t + 39.2 = 0$

You may solve the quadratic by any exact method. Dividing all the terms by 4.9 gives a quadratic equation with integer coefficients, this equation factorises easily.

Dividing all terms by 4.9

$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2, 4$

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Review Exercise
Exercise A, Question 13

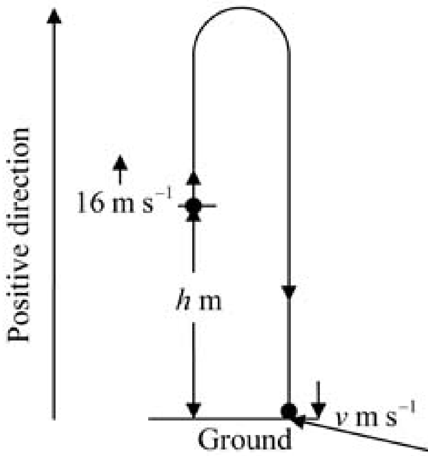
Question:

A stone is thrown vertically upwards with speed 16 m s^{-1} from a point *h* metres above the ground. The stone hits the ground 4 s later. Find

a the value of *h*,

b the speed of the stone as it hits the ground.

Solution:



When the stone hits the ground, it is h metres **below** the point of projection and so the displacement, s, is "minus h".

a $u = 16,\ a = -9.8,\ t = 4,\ s = -h$

$$s = ut + \frac{1}{2}at^2$$
$$-h = 16 \times 4 - 4.9 \times 4^2$$
$$= -14.4$$
$$h = 14 \ (2 \text{ s.f.}).$$

b $u = 16,\ a = -9.8,\ t = 4,\ v = ?$

$$v = u + at$$
$$= 16 - 9.8 \times 4 = -23.2$$

The speed of the stone as it hits the ground is 23 m s^{-1} (2 s.f.).

The velocity, $v \text{ m s}^{-1}$, is -23.2 m s^{-1} because when the stone hits the ground it is moving downwards and the positive direction has been taken upwards. However, you have been asked for the speed of the stone as it hits the ground and speed is the magnitude of the velocity, 23 m s^{-1} (2 s.f.).

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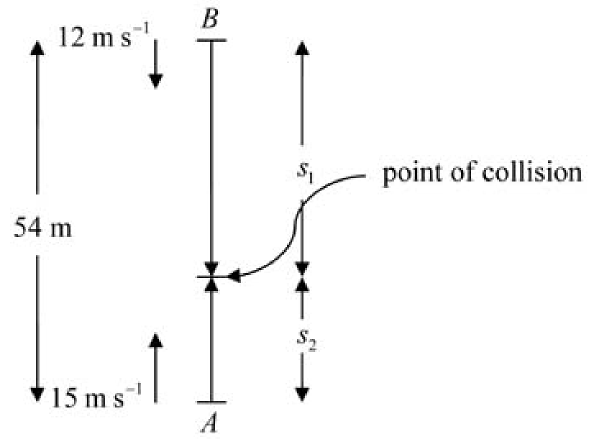
Review Exercise
Exercise A, Question 14

Question:

Two balls are projected simultaneously from two points *A* and *B*. The point *A* is 54 m vertically below *B*. Initially one ball is projected from *A* towards *B* with speed 15 m s^{-1}. At the same time the other ball is projected from *B* towards *A* with speed 12 m s^{-1}.

Find the distance between *A* and the point where the balls collide.

Solution:



From *B*, take the downwards direction as positive
$u = 12,\ a = 9.8$

$$s_1 = ut + \frac{1}{2}at^2$$
$$= 12t + 4.9t^2 \quad \ldots\ldots (1)$$

From *A*, take the upwards direction as positive
$u = 15,\ a = -9.8$

$$s_2 = ut + \frac{1}{2}at^2$$
$$= 15t - 4.9t^2 \quad \ldots\ldots (2)$$

(1) + (2)
$$s_1 + s_2 = 12t + 4.9t^2 + 15t - 4.9t^2$$
$$54 = 27t$$
$$t = 2$$

You can form an equation in *t* using the relation that, at the point of collision, the displacement downwards of the ball projected from *B* added to the displacement upwards of the ball projected from *A* is 54 m, the distance between *A* and *B*.

Substitute $t = 2$ into (2)
$$s_2 = 15 \times 2 - 4.9 \times 2^2 = 10.4$$

The balls collide at a point 10 m (2 s.f.) above *A*.

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Review Exercise
Exercise A, Question 15

Question:

A particle is projected vertically upwards from a point A with speed u m s^{-1}. The particle takes $2\frac{6}{7}$ s to reach its greatest height above A. Find

a the value of u,

b the total time for which the particle is more than $17\frac{1}{2}$ m above A.

Solution:

a Taking upwards as the positive direction

$v = 0, a = -9.8, t = 2\frac{6}{7} = \frac{20}{7}, u = ?$

$v = u + at$

$0 = u - 9.8 \times \frac{20}{7}$

$u = 9.8 \times \frac{20}{7} = 28$

b $s = 17.5, u = 28, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$17.5 = 28t - 4.9t^2$

$4.9t^2 - 28t + 17.5 = 0$

Divide all terms by 0.7

$7t^2 - 40t + 25 = 0$

$(7t - 5)(t - 5) = 0$

$t = \frac{5}{7}, 5$

The time for which the particle is more than $17\frac{1}{2}$ m above A is $\left(5 - \frac{5}{7}\right)$s $= 4\frac{2}{7}$s.

The particle passes through the point $17\frac{1}{2}$ m above A twice. The first time is as it ascends. The second, as it descends. The first stage of the solution to part (b) is to find the two values of t when these happen.

The difference between these times is the total time for which the particle is more than $17\frac{1}{2}$ m above A.

This is an exact answer and can be left as it is. 4.3 s would also be an acceptable answer.

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Review Exercise
Exercise A, Question 16

Question:

A particle moves along a horizontal straight line. The particle starts from rest, accelerates at 2 m s $^{-2}$ for 3 seconds, and then decelerates at a constant rate coming to rest in a further 8 seconds.

a Sketch a speed–time graph to illustrate the motion of the particle.

b Find the total distance travelled by the particle during these 11 seconds.

Solution:

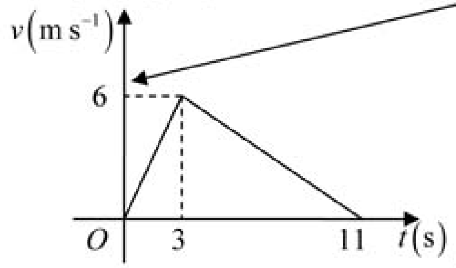
a To find the velocity after 3 seconds

$u = 0, a = 2, t = 3, v = ?$

$v = u + at$

$v = 0 + 2 \times 3 = 6$

You need to find the speed after 3 seconds, both to draw the graph in part (a) and to calculate the area of the triangle in part (b). You can do this either by using $v = u + at$, as is shown here, or by using the key point that the gradient of a speed-time graph represents the acceleration.



b Area $= \frac{1}{2}$ base $\times$ height

$s = \frac{1}{2} 11 \times 6 = 33$

The area of the triangle between the speed-time graph and the axis represents the distance travelled.

The total distance travelled by the particle is 33 m.

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Review Exercise
Exercise A, Question 17

Question:

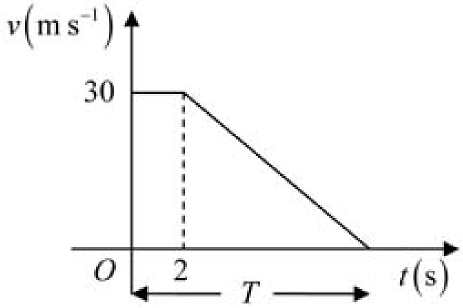
A man is driving a car on a straight horizontal road. He sees a junction *S* ahead, at which he must stop. When the car is at the point *P*, 300 m from *S*, its speed is 30 m s $^{-1}$. The car continues at this constant speed for 2 s after passing *P*. The man then applies the brakes so that the car has constant deceleration and comes to rest at *S*.

a Sketch a speed-time graph to illustrate the motion of the car in moving from *P* to *S*.

b Find the time taken by the car to travel from *P* to *S*.

Solution:

a



b Let T seconds be the time taken to travel from A to B.

$$s=\frac{1}{2}(a+b)h$$

The area of the trapezium between the speed-time graph and the axis represents the distance travelled.

$$300=\frac{1}{2}(2+T)\times 30$$
$$=15(2+T)=30+15T$$
$$T=\frac{300-30}{15}=18$$

You know that the distance is 300 m. The shorter of the parallel sides of the trapezium represents 2 s and the longer side T seconds. You then form an equation using the formula for the area and solve it for T.

The car takes 18 s to travel from A to B.

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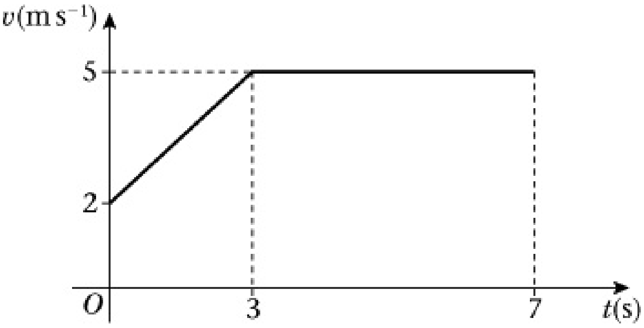
Review Exercise
Exercise A, Question 18

Question:



The figure shows the speed–time graph of a cyclist moving on a straight road over a 7 s period. The sections of the graph from $t = 0$ to $t = 3$, and from $t = 3$ to $t = 7$, are straight lines. The section from $t = 3$ to $t = 7$ is parallel to the t-axis.

State what can be deduced about the motion of the cyclist from the fact that

a the graph from $t = 0$ to $t = 3$ is a straight line,

b the graph from $t = 3$ to $t = 7$ is parallel to the t-axis.

c Find the distance travelled by the cyclist during this 7 s period.

Solution:

a For the first 3 s the cyclist is moving with constant acceleration.

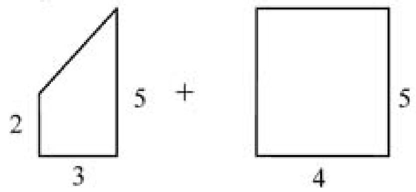
b For the remaining 4 s the cyclist is moving with constant speed.

c area = trapezium + rectangle

$$s = \frac{1}{2}(2+5)\times 3 + 5\times 4$$
$$= 10.5 + 20 = 30.5$$

The distance travelled by the cyclist is 30.5 m.

The area, representing the distance travelled, is made up of two parts.



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Review Exercise
Exercise A, Question 19

Question:

A train stops at two stations 7.5 km apart. Between the stations it takes 75 s to accelerate uniformly to a speed 24 m s $^{-1}$, then travels at this speed for a time *T* seconds before decelerating uniformly for the final 0.6 km.

a Draw a speed–time graph to illustrate this journey.

Hence, or otherwise, find

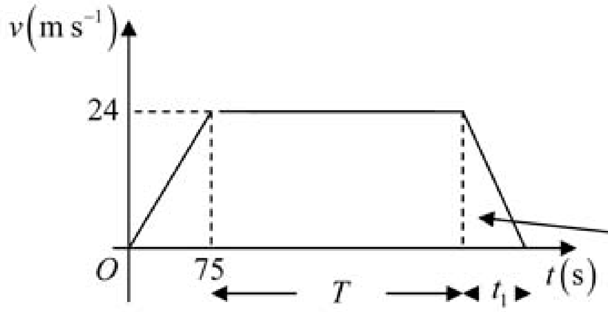
b the deceleration of the train during the final 0.6 km,

c the value of *T*,

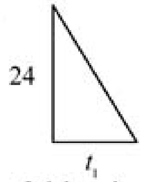
d the total time for the journey.

Solution:

a



The motion of the train while decelerating is represented by this triangle .



The area of this triangle represents the distance travelled while decelerating, which is given to be 0.6 km = 600m.

b Let time for which the train decelerates be t_1 s .
While decelerating

$$\text{area} = \frac{1}{2}\text{base} \times \text{height}$$

$$600 = \frac{1}{2}t_1 \times 24$$

$$t_1 = \frac{1200}{24} = 50$$

Acceleration is represented by the gradient.

$$a = \frac{-24}{t_1} = -\frac{24}{50} = -0.48$$

The deceleration is 0.48 m s^{-2}.

The shorter of the parallel sides of the trapezium represents T seconds.
The longer of the parallel sides of the trapezium represents

$$(75 + T + t_1)\text{s} = (75 + T + 50)\text{s}$$
$$= (T + 125)\text{s}$$

c For the whole journey

$$s = \frac{1}{2}(a + b)h$$

$$7500 = \frac{1}{2}(T + T + 125) \times 24$$

$$= 12(2T + 125) = 24T + 1500$$

$$T = \frac{7500 - 1500}{24} = 250$$

d Total time is $(75 + T + t_1)\text{s} = (75 + 250 + 50)\text{s} = 375\text{ s}$.

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Review Exercise
Exercise A, Question 20

Question:

A car accelerates uniformly from rest to a speed of 20 m s^{-1} in T seconds. The car then travels at a constant speed of 20 m s^{-1} for $4T$ seconds and finally decelerates uniformly to rest in a further 50 s.

a Sketch a speed–time graph to show the motion of the car.

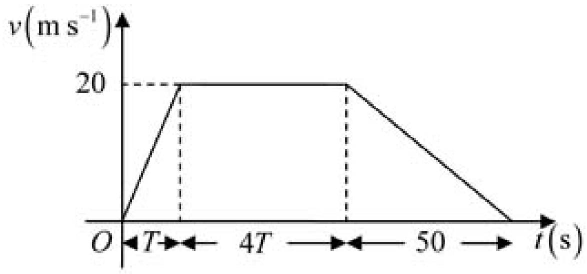
The total distance travelled by the car is 1220 m. Find

b the value of T,

c the initial acceleration of the car.

Solution:

a



b $s = \frac{1}{2}(a+b)h$

$1220 = \frac{1}{2}(4T + 5T + 50) \times 20$

$= 10(9T + 50) = 90T + 500$

$T = \frac{1220 - 500}{90} = 8$

The shorter of the parallel sides of the trapezium represents $4T$ seconds.
The longer of the parallel sides of the trapezium represents $(T + 4T + 50)\text{s} = (5T + 50)\text{s}$.

c $a = \frac{20}{T} = \frac{20}{8} = 2.5$

The initial acceleration of the car is 2.5 m s^{-2}.

The gradient of the line for the first T seconds represents the initial acceleration.
The gradient is $\frac{\text{the increase in velocity}}{\text{the increase in time}}$.

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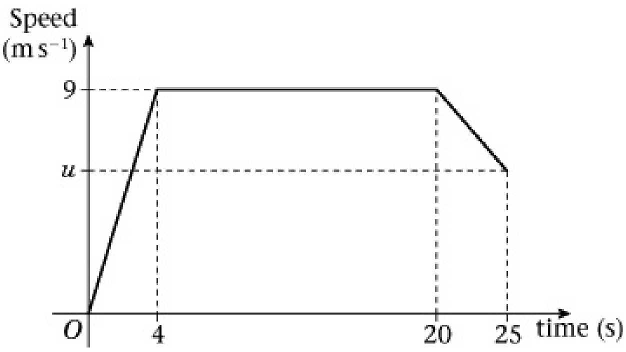
Review Exercise
Exercise A, Question 21

Question:



A sprinter runs a race of 200 m. Her total time for running the race is 25 s. The figure is a sketch of the speed-time graph for the motion of the sprinter. She starts from rest and accelerates uniformly to a speed of 9 m s $^{-1}$ in 4 s. The speed of 9 m s $^{-1}$ is maintained for 16 s and she then decelerates uniformly to a speed of u m s $^{-1}$ at the end of the race. Calculate

a the distance covered by the sprinter in the first 20 s of the race,

b the value of u,

c the deceleration of the sprinter in the last 5 s of the race.

Solution:

a For first 20 s

$$s = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(16+20)\times 9 = 162$$

The distance covered in the first 20 s is 162 m.

b The distance covered in the last 5 s is

$(200-162)\text{m} = 38\text{ m}$.

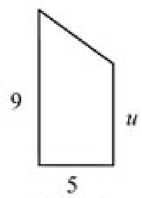
$$s = \frac{1}{2}(a+b)h$$

$$38 = \frac{1}{2}(u+9)\times 5$$

$$u+9 = \frac{38\times 2}{5} = 15.2$$

$$u = 6.2$$

The trapezium representing the distance travelled in the last 5 s has shape



The area of this trapezium must equal 38 and you can form an equation in u.

c $$a = \frac{u-9}{5} = \frac{6.2-9}{5} = \frac{-2.8}{5} = -0.56$$

The deceleration in the last 5 s is 0.56 m s^{-2}.

The gradient of the line representing the last 5 s of motion is negative. v decreases by 2.8 as t increases by 5.

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Review Exercise
Exercise A, Question 22

Question:

An electric train starts from rest at a station *A* and moves along a straight level track. The train accelerates uniformly at 0.4 m s^{-2} to a speed of 16 m s^{-1}. The speed is then maintained for a distance of 2000 m. Finally the train retards uniformly for 20 s before coming to rest at a station *B*. For this journey from *A* to *B*,

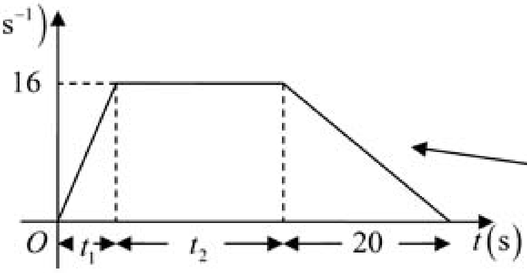
a find the total time taken,

b find the distance from *A* to *B*,

c sketch the *distance*–time graph, showing clearly the shape of the graph for each stage of the journey.

Solution:

a



You were not asked to sketch a speed-time graph but it helps you to visualise the problem and you will be able to work out part **b** using the area of the trapezium.

Let the time for which the train accelerates be t_1 s and the time for which it travels at a constant speed be t_2 s.

During acceleration

$v = u + at$

$16 = 0 + 0.4t_1 \Rightarrow t_1 = \frac{16}{0.4} = 40$

At constant speed

At constant speed, distance = speed × time.

$2000 = 16 \times t_2 \Rightarrow t_2 = \frac{2000}{16} = 125$

The total time is $\left(t_1 + t_2 + 20\right)\text{s} = \left(40 + 125 + 20\right)\text{s} = 185\text{ s}$.

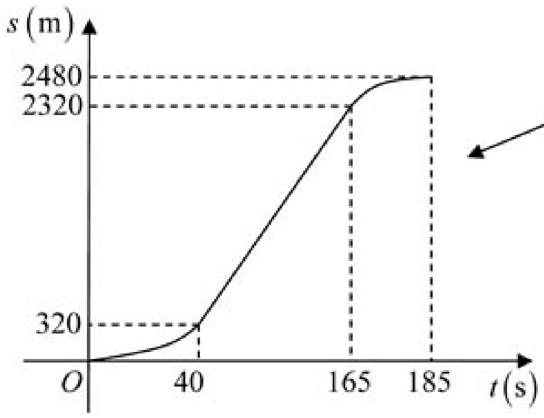
b $s = \frac{1}{2}\left(a + b\right)h$

$= \frac{1}{2}\left(125 + 185\right) \times 16 = 2480$

$AB = 2480$ m

c The distance moved in the first 40 s of motion is given by

$s = \frac{1}{2} \times 40 \times 16 = 320$



A distance-time graph is a straight line when the object is moving at a constant speed and a curve when the object is accelerating or decelerating.

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Review Exercise
Exercise A, Question 23

Question:

A car starts from rest at a point *S* on straight racetrack. The car moves with constant acceleration for 20 s, reaching a speed of 25 m s^{-1}. The car then travels at a constant speed of 25 m s^{-1} for 120 s. Finally it moves with constant deceleration, coming to rest at a point *F*.

a Sketch a speed–time graph to illustrate the motion of the car.

The distance between *S* and *F* is 4 km.

b Calculate the total time the car takes to travel from *S* to *F*.

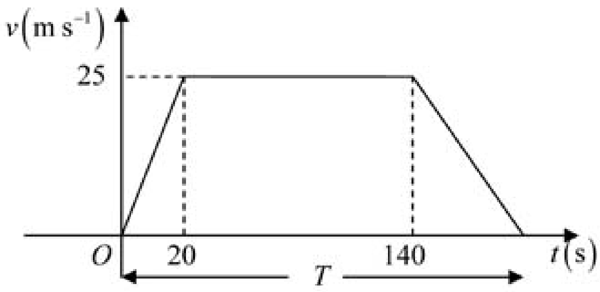
A motorcycle starts at *S*, 10 s after the car has left *S*. The motorcycle moves with constant acceleration from rest and passes the car at a point *P* which is 1.5 km from *S*. When the motorcycle passes the car, the motorcycle is still accelerating and the car is moving at a constant speed. Calculate

c the time the motorcycle takes to travel from *S* to *P*,

d the speed of the motorcycle at *P*.

Solution:

a



b Let the total time be T seconds.

$$s=\frac{1}{2}(a+b)h$$

$$4000=\frac{1}{2}(120+T)\times 25$$

$$120+T=\frac{4000\times 2}{25}=320 \Rightarrow T=200$$

The total time the car takes to travel from S to F is 200 s.

c The distance the car travels while accelerating is given by

$$s=\frac{1}{2}\times 20\times 25=250\,(\mathrm{m})$$

The car travels a further $(1500-250)\mathrm{m}=1250\mathrm{m}$ at a constant speed. The time it takes to do this is given by

$1250=25t \Rightarrow t=50$.

The car takes 70 s to reach P.

Hence the motorcycle takes 60 s to reach P.

In part **c**, you first have to find the time the car takes to get to P. There are two stages to this – the time for which the car accelerates (20 s, given) and the time for which it travels at a constant speed, which needs to be calculated.
The motorcycle then takes 10 s less than the sum of these two times.

d For the motorcycle

$u=0,\ s=1500,\ t=60,\ v=?$

$$s=\left(\frac{u+v}{2}\right)t$$

$$1500=\left(\frac{0+v}{2}\right)60=30v \Rightarrow v=\frac{1500}{30}=50$$

The speed of the motorcycle at P is $50\ \mathrm{m\,s^{-1}}$.

This solution to part **d** makes no reference to a speed-time graph. It is not uncommon for some parts of a question to be better done using the properties of graphs and other parts to be better done using the one or more of the 5 kinematics formulae.

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Review Exercise
Exercise A, Question 24

Question:

Two cars *A* and *B* are travelling in the same direction along a motorway. They pass a warning sign at the same instant and, subsequently, arrive at a toll booth at the same instant.

Car *A* passes the warning sign at speed 24 m s^{-1}, continues at this speed for one minute, then decelerates uniformly, coming to rest at the toll booth.

Car *B* passes the warning sign at speed 30 m s^{-1}, continues at this speed for *T* seconds, then decelerates uniformly, coming to rest at the toll booth.

a On the same diagram, draw a speed–time graph to illustrate the motion of each car.

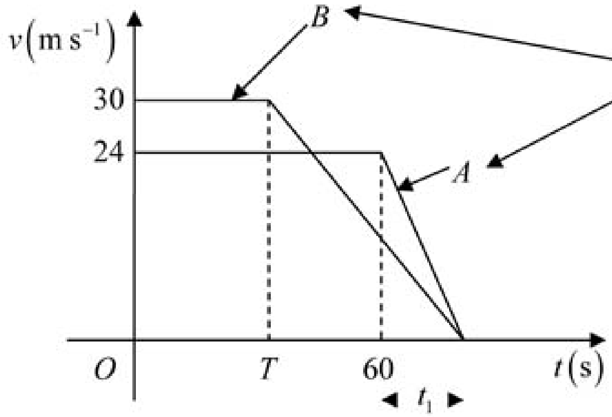
The distance between the warning sign and the toll booth is 1.56 km.

b Find the length of time for which *A* is decelerating.

c Find the value of *T*.

Solution:

a



You should label the graphs so that it is clear which represents car A and which represents car B.

b Let the time for which A is decelerating be t_1 s.

For car A

$$s = \frac{1}{2}(a+b)h$$

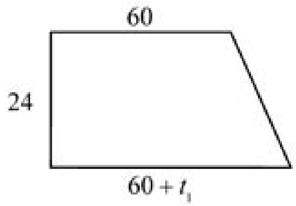
$$1560 = \frac{1}{2}(60+60+t_1) \times 24$$

$$= 12(120+t_1) = 1440 + 12t_1$$

$$t_1 = \frac{1560-1440}{12} = 10$$

The time for which A is decelerating is 10s.

The trapezium representing the distance moved by car A is



Putting the area of this trapezium equal to the distance travelled in metres, 1560, gives an equation which you can solve for t_1.

c Both cars take 70 s to move from the warning sign to the toll booth.

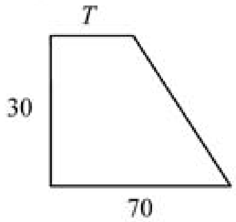
For Car B

$$s = \frac{1}{2}(a+b)h$$

$$1560 = \frac{1}{2}(T+70) \times 30 = 15T + 1050$$

$$T = \frac{1560-1050}{15} = 34$$

The trapezium representing the distance moved by car B is



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Review Exercise
Exercise A, Question 25

Question:

A train is travelling at 10 m s $^{-1}$ on a straight horizontal track. The driver sees a red signal 135 m ahead and immediately applies the brakes. The train immediately decelerates with constant deceleration for 12s, reducing its speed to 3 m s $^{-1}$. The driver then releases the brakes and allows the train to travel at a constant speed of 3 m s $^{-1}$ for a further 15 s. He then applies the brakes again and the train slows down with constant deceleration, coming to rest as it reaches the signal.

a Sketch a speed–time graph illustrating the motion of the train.

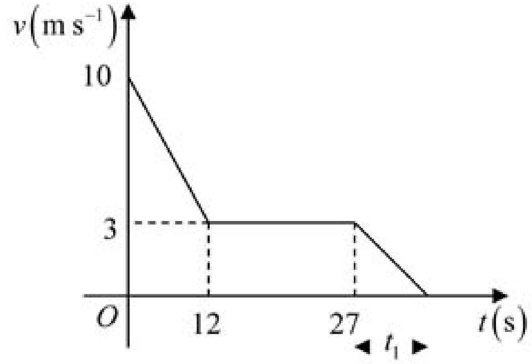
b Find the distance travelled by the train from the moment when the brakes are first applied to the moment when its speed first reaches 3 m s $^{-1}$.

c Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest.

d Sketch an acceleration–time graph illustrating the motion of the train.

Solution:

a

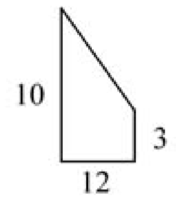


b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(3+10)\times 12 = 78$

The required distance is 78 m.

The trapezium representing the distance moved by the train during the initial deceleration is



c The distance travelled at constant speed 3 m s^{-1} is given by

$s = 3\times 15 = 45$

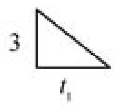
The distance travelled under the final deceleration is $(135 - 78 - 45)\text{m} = 12 \text{ m}$.

Let the time taken for the final deceleration be t_1 s.

$12 = \frac{1}{2}t_1 \times 3 \Rightarrow t_1 = 8$

The total time is $(12+15+8)\text{s} = 35 \text{ s}$.

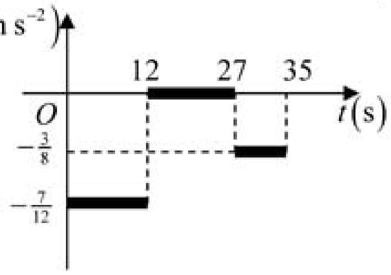
The distance travelled during the final deceleration is represented by the triangle



This distance is 12 m and use of the usual formula for the area of a triangle enables you to find t_1.

d The initial deceleration is $\frac{7}{12} \text{ m s}^{-2}$.

The final deceleration is $\frac{3}{8} \text{ m s}^{-2}$.



The decelerations can be written down using the key point that the gradient of a speed-time graph represents the acceleration.

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Review Exercise
Exercise A, Question 26

Question:

A straight stretch of railway line passes over a viaduct which is 600 m long. An express train on this stretch of line normally travels at a speed of 50 m s^{-1}. Some structural weakness in the viaduct is detected and engineers specify that all trains passing over the viaduct must do so at a speed of no more than 10 m s^{-1}. Approaching the viaduct, the train therefore reduces its speed from 50 m s^{-1} with constant deceleration 0.5 m s^{-2}, reaching a speed of precisely 10 m s^{-1} just as it reaches the viaduct. It then passes over the viaduct at a constant speed of 10 m s^{-1}. As soon as it reaches the other side, it accelerates to its normal speed of 50 m s^{-1} with constant acceleration 0.5 m s^{-2}.

a Sketch a speed–time graph to show the motion of the train during the period from the time when it starts to reduce speed to the time when it is running at full speed again.

b Find the total distance travelled by the train while its speed is less than 50 m s^{-1}.

c Find the extra time taken by the train for the journey due to the speed restriction on the viaduct.

Solution:

a To find time taken to decelerate

$u = 50,\ v = 10,\ a = -0.5,\ t = ?$

$v = u + at$

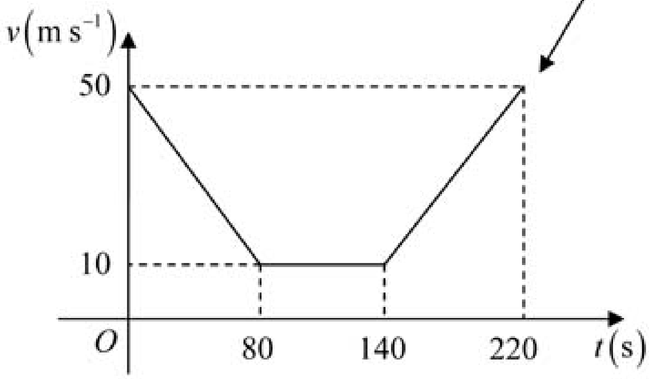
$10 = 50 - 0.5t \Rightarrow t = \dfrac{50-10}{0.5} = 80$

The acceleration is symmetric with the deceleration and takes 80 s.

At the constant speed of $10\ \text{m s}^{-1}$

$s = ut$

$600 = 10 \times t \Rightarrow t = 60$

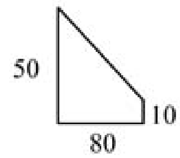


b $s = \frac{1}{2}(10+50)\times 80 + 60\times 10 + \frac{1}{2}(10+50)\times 80$

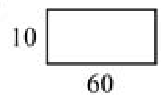
$= 2400 + 600 + 2400 = 5400$

The distance travelled by the train is 5400 m.

The total distance travelled is represented by two trapezia with dimensions



plus the rectangle



c At a constant speed of $50\ \text{m s}^{-1}$

distance = speed × time

$5400 = 50t \Rightarrow t = \dfrac{5400}{50} = 108$

The extra time is $(220 - 108)\,\text{s} = 112\ \text{s}$.

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Review Exercise
Exercise A, Question 27

Question:

A bus and a cyclist are moving along a straight horizontal road in the same direction. The bus starts at a bus stop *O* and moves with constant acceleration of 2 m s $^{-2}$ until it reaches a maximum speed of 12 m s $^{-1}$. It then maintains this constant speed. The cyclist travels with a constant speed of 8 m s $^{-1}$. The cyclist passes *O* just as the bus starts to move. The bus later overtakes the cyclist at the point *A*.

a Show that the bus does not overtake the cyclist before it reaches its maximum speed.

b Sketch, on the same diagram, speed–time graphs to represent the motion of the bus and the cyclist as they move from *O* to *A*.

c Find the time taken for the bus and the cyclist to move from *O* to *A*.

d Find the distance *OA*.

Solution:

a Find the time for the bus to reach its maximum speed.

$u = 0, v = 12, a = 2, t = ?$

$v = u + at$

$12 = 0 + 2t \Rightarrow t = 6$

Find the distance travelled by the bus in reaching its maximum speed.

$u = 0, v = 12, a = 2, s = ?$

$v^2 = u^2 + 2as$

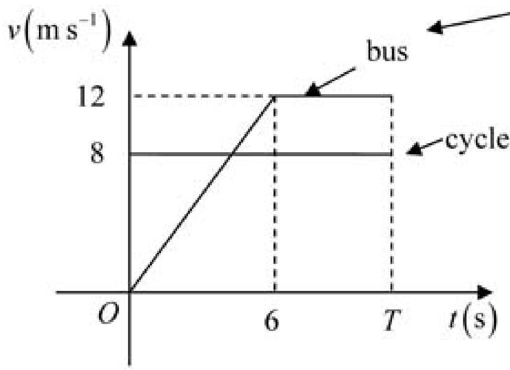
$12^2 = 0^2 + 4s \Rightarrow s = 36 \ (\text{m})$

In 6 s, the distance travelled by the cyclist is given by distance = speed × time.

$$= 8 \times 6 = 48 \,(\text{m})$$

As 36 m is less than 48 m the bus has not overtaken the cyclist.

b



You should label the graphs so that it is clear which represents the bus and which represents the cycle.

c Let T seconds be the time for the bus and the cyclist to move from O to A.

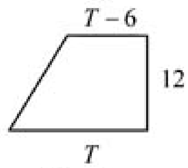
$\frac{1}{2}(T - 6 + T) \times 12 = 8T$

$6(2T - 6) = 12T - 36 = 8T$

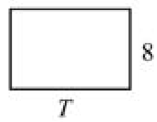
$4T = 36 \Rightarrow T = 9$

The time taken is 9 s.

The area of the trapezium representing the distance travelled by the bus



must equal the rectangle representing the distance travelled by the cycle



d For the cycle, distance = speed × time

$$= 8 \times 9 \text{ m} = 72 \text{ m}$$

$OA = 72$ m

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Review Exercise
Exercise A, Question 28

Question:

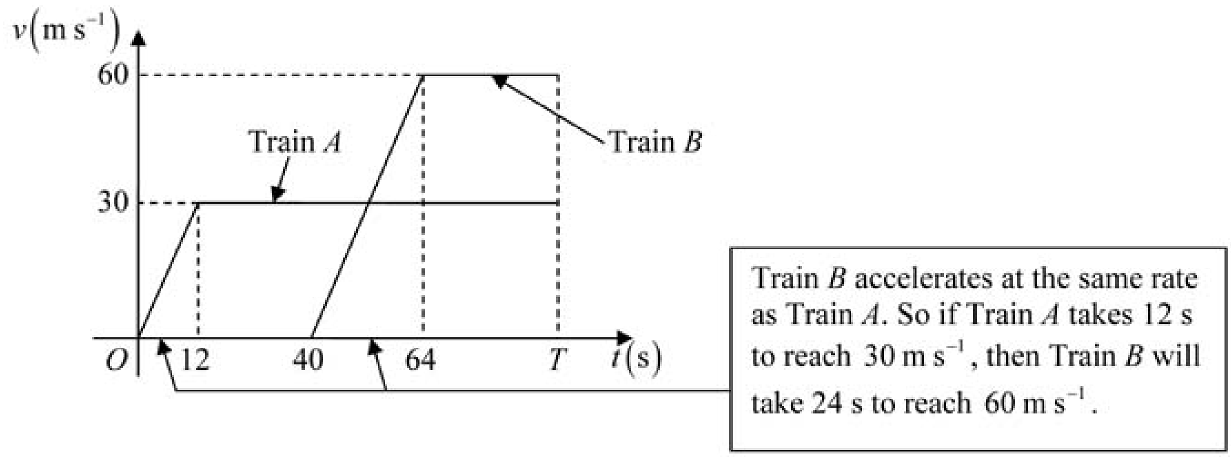
Two trains A and B run on parallel straight tracks. Initially both are at rest in a station and level with each other. At time $t = 0$, A starts to move. It moves with constant acceleration for 12 s up to a speed of 30 m s^{-1}, and then moves at a constant speed of 30 m s^{-1}. Train B starts to move in the same direction as A when $t = 40$, where t is measured in seconds. It accelerates with the same initial acceleration as A, up to a speed of 60 m s^{-1}. It then moves at a constant speed of 60 m s^{-1}. Train B overtakes A after both trains have reached their maximum speed. Train B overtakes A when $t = T$.

a Sketch, on the same diagram, the speed–time graphs of both trains for $0 \le t \le T$.

b Find the value of T.

Solution:

a



b The distance travelled by Train A in T seconds is given by

$$s = \frac{1}{2}(T - 12 + T) \times 30 = 15(2T - 12)$$

The distance travelled by Train B in T seconds is given by

$$s = \frac{1}{2}(T - 64 + T - 40) \times 60 = 20(2T - 104)$$

At the point of overtaking the distances are equal.

$$15(2T - 12) = 30(2T - 104)$$

$$30T - 180 = 60T - 3120$$

$$30T = 2940$$

$$T = \frac{2940}{30} = 98$$

You use the fact that the two trapezia have the same area to form an equation in T, which you solve.

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Review Exercise
Exercise A, Question 29

Question:

A train starts from rest at a station, accelerates uniformly to its maximum speed of 15 m s^{-1}, travels at this speed for a time, and then decelerates uniformly to rest at the next station. The distance from station to station is 1260 m, and the time spent travelling at the maximum speed is three-quarters of the total journey time.

a Sketch a speed–time graph to illustrate this information.

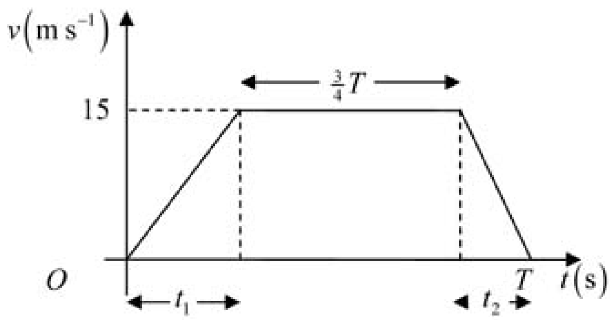
b Find the total journey time.

Given also that the magnitude of the deceleration is twice the magnitude of the acceleration,

c find the magnitude of the acceleration.

Solution:

a



b Let the total time for the journey be T seconds.

$$s = \frac{1}{2}(a+b)h$$

$$1260 = \frac{1}{2}\left(\frac{3}{4}T + T\right) \times 15$$

$$\frac{7}{4}T = \frac{2 \times 1260}{15} = 168$$

$$T = \frac{4 \times 168}{7} = 96$$

The total time for the journey is 96 s.

c Let the time taken accelerating be t_1 seconds.

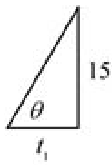
Let the time taken decelerating be t_2 seconds.

$$t_1 + t_2 = \frac{1}{4}T = 24 \quad \ldots \ldots (1)$$

The acceleration is $\frac{15}{t_1}$ s

The acceleration is represented by the gradient of the line .

Gradient $= \frac{15}{t_1}$



The deceleration is $\frac{15}{t_2}$ s

You get an expression for the deceleration in a similar way to that used for the acceleration.

The magnitude of the deceleration is twice the magnitude of the acceleration.

$$\frac{15}{t_2} = 2 \times \frac{15}{t_1} \Rightarrow t_2 = \frac{1}{2}t_1 \quad \ldots \ldots (2)$$

Substitute (2) into (1)

$$t_1 + \frac{1}{2}t_1 = \frac{3}{2}t_1 = 24 \Rightarrow t_1 = \frac{2}{3} \times 24 = 16$$

The acceleration is $\frac{15}{t_1} = \frac{15}{16}$ m s^{-2} .

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Review Exercise
Exercise A, Question 30

Question:

The brakes of a train, which is travelling at 108 km h $^{-1}$, are applied as the train passes a point *A*. The brakes produce a retardation of magnitude $3f$ m s $^{-2}$ until the speed of the train is reduced to 36 km h $^{-1}$. The train travels at this speed for a distance and is then uniformly accelerated at f m s $^{-2}$ until it again reaches the speed 108 km h $^{-1}$ as it passes point *B*. The time taken by the train in travelling from *A* to *B*, a distance of 4 km, is 4 minutes.

a Sketch a speed–time graph to illustrate the motion of the train from *A* to *B*.

b Find the value of *f*.

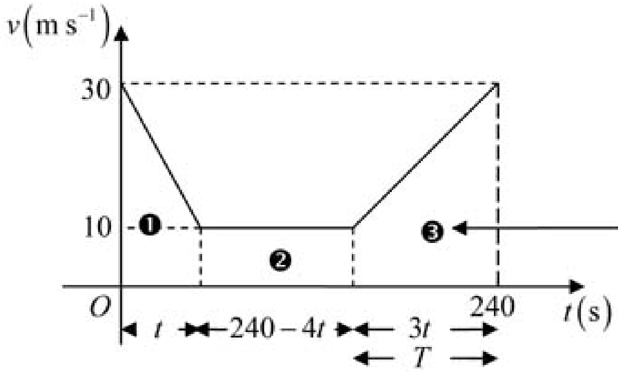
c Find the distance travelled at 36 km h $^{-1}$.

Solution:

a $108 \text{ km h}^{-1} = \dfrac{108 \times 1000}{3600} \text{ m s}^{-1} = 30 \text{ m s}^{-1}$

$36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}$

You must work in a consistent set of units. The speeds are in km and hours, the accelerations in m and s. Reduce everything to m and s.



b Let t seconds be the time the train takes to retard.

Then $3f = \dfrac{20}{t} \Rightarrow f = \dfrac{20}{3t}$ ... ... (1)

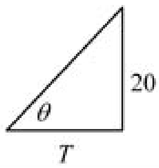
Let T seconds be the time takes to accelerate.

Then $f = \dfrac{20}{T}$ ... ... (2)

Accelerations and deceleration are the gradients of line. Here, while accelerating ,

$f =$ gradient

$= \dfrac{20}{T}$



From (1) and (2) $T = 3t$ and the time travelled at constant speed is $240 - 4t$ seconds.

$$\frac{1}{2}(10+30)t + 10(240-4t) + \frac{1}{2}(10+30)3t = 4000$$

The areas marked ❶, ❷ and ❸ in the speed-time diagram added together represent the distance travelled. ❶ and ❸ are trapezia; ❷ is a rectangle.

$20t + 2400 - 40t + 60t = 4000$

$40t = 1600 \Rightarrow t = 40$

From (2) above

$f = \dfrac{20}{T} = \dfrac{20}{3t} = \dfrac{20}{120} = \dfrac{1}{6}$

c At constant speed, distance = speed × time

$s = 10 \times (240 - 4t) = 10 \times (240 - 4 \times 40)$

$= 10 \times 80 = 800$

The distance travelled at 36 km h^{-1} is 800 m.

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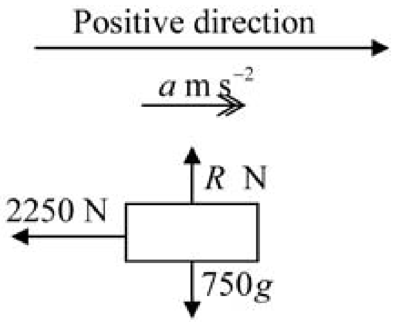
Review Exercise
Exercise A, Question 31

Question:

A car of mass 750 kg, moving along a level straight road, has its speed reduced from 25 m s $^{-1}$ to 15 m s $^{-1}$ by brakes which produce a constant retarding force of 2250 N. Calculate the distance travelled by the car as its speed is reduced from 25 m s $^{-1}$ to 15 m s $^{-1}$.

Solution:



$\mathbf{F} = m\mathbf{a}$

$R(\rightarrow) \quad -2250 = 750a$

The retarding force is slowing the car down and is in the negative direction. So, in the positive direction, the force is -2250.

$$a = -\frac{2250}{750} = -3$$

$u = 25,\ v = 15,\ a = -3,\ s = ?$

$v^2 = u^2 + 2as$

There is no t here, so you choose the formula without t, $v^2 = u^2 + 2as$.

$15^2 = 25^2 - 6s$

$$s = \frac{25^2 - 15^2}{6} = \frac{400}{6} = 66\frac{2}{3}$$

The distance travelled by the car as its speed is reduced is $66\frac{2}{3}$ m s^{-1}.

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Review Exercise
Exercise A, Question 32

Question:

A particle *P* is moving with constant acceleration along a straight horizontal line *ABC*, where *AC* = 24 m. Initially *P* is at *A* and is moving with speed 5 m s^{-1} in the direction *AB*. After 1.5 s, the direction of motion of *P* is unchanged and *P* is at *B* with speed 9.5 m s^{-1}.

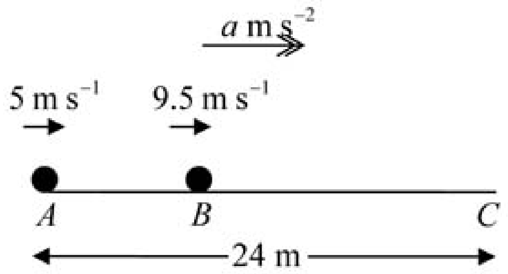
a Show that the speed of *P* at *C* is 13 m s^{-1}.

The mass of *P* is 2 kg. When *P* reaches *C,* an impulse of magnitude 30 Ns is applied to *P* in the direction *CB.*

b Find the velocity of *P* immediately after the impulse has been applied, stating clearly the direction of motion of *P* at this instant.

Solution:

a



From A to B

$u = 5, v = 9.5, t = 1.5, a = ?$

$v = u + at$

$9.5 = 5 + 1.5a$

$a = \frac{9.5 - 5}{1.5} = 3$

From A to C

$u = 5, s = 24, a = 3, v = ?$

$v^2 = u^2 + 2as$

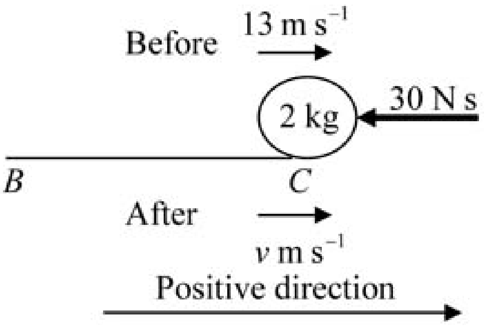
$= 5^2 + 2 \times 3 \times 24 = 169$

$v = \sqrt{169} = 13$

The speed of P at C is 13 m s^{-1}, as required.

You need to find the acceleration, a, first by considering the motion from A to B. Then you can proceed to find the speed at C by considering the motion from A to C.

b



$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$-30 = 2 \times v - 2 \times 13$

$2v = -30 + 26 = -4$

$v = -2$

The velocity of P immediately after the impulse has been applied is 2 m s^{-1} in the direction $\overrightarrow{CB}$.

Impulse is a vector quantity and you must always consider its direction. If the direction from B to C is taken as the positive direction then an impulse of 30 N s in the direction from C to B must be taken as -30.

The answer $v = -2$ shows that the velocity is in the negative direction, that is from C to B.

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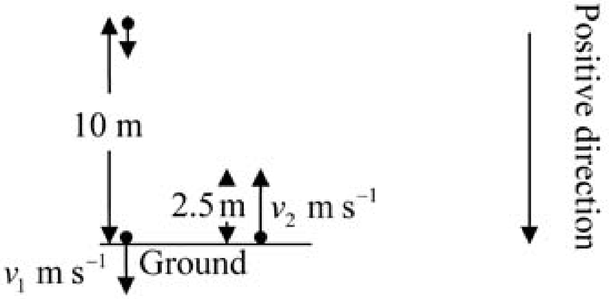
Review Exercise
Exercise A, Question 33

Question:

A ball of mass 0.3 kg is released at rest from a point at a height of 10 m above horizontal ground. After hitting the ground the ball rebounds to a height of 2.5 m.

Calculate the magnitude of the impulse exerted by the ground on the ball.

Solution:



As ball descends

$u = 0,\ a = 9.8,\ s = 10,\ v = v_1$

$v^2 = u^2 + 2as$

$v_1^2 = 0^2 + 2 \times 10 \times 9.8 = 196$

$v_1 = \sqrt{196} = 14$

The ball is released from rest 10 m above the ground. The first step is to calculate the speed with which the ball strikes the ground.

After rebound

$v = 0,\ a = 9.8,\ s = -2.5,\ u = v_2$

You must then use the fact that the ball reaches a maximum height of 2.5 m to find the velocity with which it rebounds from the ground.

$v^2 = u^2 + 2as$

$0^2 = v_2^2 + 2 \times 9.8 \times (-2.5) \Rightarrow v_2^2 = 49$

$v_2 = -\sqrt{49} = -7$

As it rebounds from the ground, the ball is moving upwards. That is in the negative direction. You must take the negative square root of 49, which is -7.

$I = mv_2 - mv_1$

$= 0.3 \times (-7) - 0.3 \times 14 = -6.9$

The magnitude of the impulse is 6.9 N.

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Review Exercise
Exercise A, Question 34

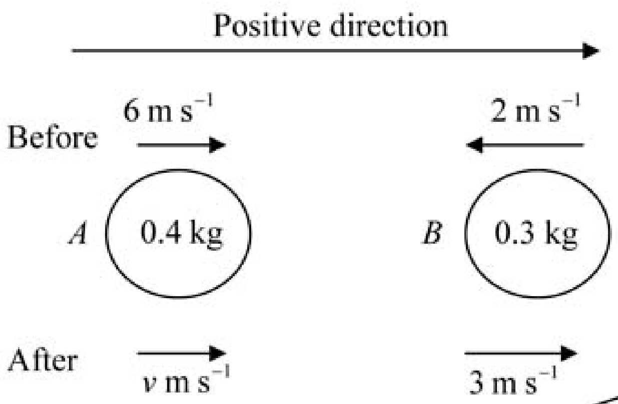
Question:

Two particles *A* and *B* have mass 0.4 kg and 0.3 kg respectively. They are moving in opposite directions on a smooth horizontal table and collide directly. Immediately before the collision, the speed of *A* is 6 m s $^{-1}$ and the speed of *B* is 2 m s $^{-1}$. As a result of the collision, the direction of motion of *B* is reversed and its speed immediately after the collision is 3 m s $^{-1}$. Find

a the speed of *A* immediately after the collision, stating clearly whether the direction of motion of *A* is changed by the collision,

b the magnitude of the impulse exerted on *B* in the collision, stating clearly the units in which your answer is given.

Solution:



The total linear momentum before impact must equal the total linear momentum after impact.

a Conservation of linear momentum

The velocity of *B* before impact is in the negative direction so it must be entered as -2 in any equations involving linear momentum.

$$0.4\times6+0.3\times(-2)=0.4\times v+0.3\times3$$
$$2.4-0.6=0.4v+0.9$$
$$0.4v=2.4-0.6-0.9=0.9$$
$$v=\frac{0.9}{0.4}=2.25$$

The speed of *A* after the collision is 2.25 m s^{-1}.
The direction of motion of *A* is unchanged.

The velocity of *A* is positive (2.25 m s^{-1}) after impact and it was positive (6 m s^{-1}) before impact. So the direction of motion of *A* is unchanged.

b For *B*, $\mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$I=0.3\times3-0.3\times(-2)$$
$$=0.9+0.6=1.5$$

The magnitude of the impulse exerted on *B* is 1.5 N s.

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Review Exercise
Exercise A, Question 35

Question:

A railway truck *S* of mass 2000 kg is travelling due east along a straight horizontal track with constant speed 12 m s^{-1}. The truck *S* collides with a truck *T* which is travelling due west along the same track as *S* with constant speed 6 m s^{-1}. The magnitude of the impulse of *T* on *S* is 28 800 Ns.

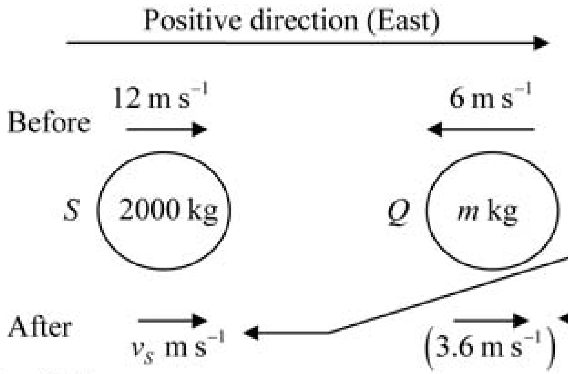
a Calculate the speed of *S* immediately after the collision.

b State the direction of motion of *S* immediately after the collision.

Given that, immediately after the collision, the speed of *T* is 3.6 m s^{-1}, and that *T* and *S* are moving in opposite directions,

c calculate the mass of *T*.

Solution:



You do not know which direction S will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction After you have worked out v, the sign of v will tell you the direction S is moving in.

You may not be sure, when starting the question, which direction I is moving in after the impact. You can leave it blank at this stage a fill it in after answering part **b**.

a For S, $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$$-28\,800 = 2000 \times v_S - 2000 \times 12$$

$$2000 v_S = -28\,800 + 24\,000 = -4800$$

$$v_S = -\frac{4800}{2000} = -2.4$$

The speed of S immediately after the collision is 2.4 m s^{-1}.

b Immediately after the collision S is moving due west.

The sign of v is negative, so S is moving in the negative direction. In this solution, the positive direction has been taken as east, so S is now moving west.

c Conservation of linear momentum

$$2000 \times 12 + m \times (-6) = 2000 \times (-2.4) + m \times 3.6$$

$$9.6m = 24\,000 + 4800 = 28\,800 \Rightarrow m = \frac{28\,800}{9.6} = 3000$$

The mass of T is 3000 kg.

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Review Exercise
Exercise A, Question 36

Question:

Two particles *A* and *B*, of mass 0.5 kg and 0.4 kg respectively, are travelling in the same straight line on a smooth horizontal table. Particle *A*, moving with speed 3 m s $^{-1}$, strikes particle *B*, which is moving with speed 2 m s $^{-1}$ in the same direction. After the collision *A* and *B* are moving in the same direction and the speed of *B* is 0.8 m s $^{-1}$ greater than the speed of *A*.

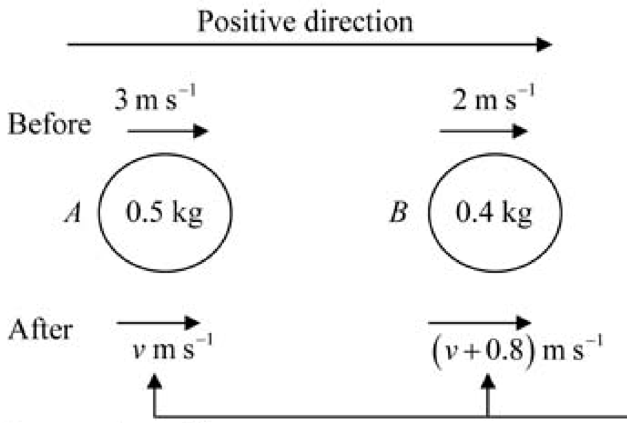
a Find the speed of *A* and the speed of *B* after the collision.

b Show that *A* loses momentum 0.4 N s in the collision.

Particle *B* later hits an obstacle on the table and rebounds in the opposite direction with speed 1 m s $^{-1}$.

c Find the magnitude of the impulse received by *B* in this second impact.

Solution:



You need to translate the statement that 'the speed of B is 0.8 m s^{-1} greater than the speed of A' into algebra. If the speed of A after the collision is $v\text{ m s}^{-1}$ then the speed of B is 0.8 m s^{-1} greater; that is $(v+0.8)\text{m s}^{-1}$.

a Conservation of linear momentum

$$0.5\times3+0.4\times2=0.5\times v+0.4(v+0.8)$$

$$1.5+0.8=0.5v+0.4v+0.32$$

$$0.9v=1.5+0.8-0.32=1.98$$

$$v=\frac{1.98}{0.9}=2.2$$

All velocities in this part are in the positive direction.

The speed of A after the collision is 2.2 m s^{-1}.

The speed of B after the collision is $(2.2+0.8)\text{ m s}^{-1}=3\text{ m s}^{-1}$.

To find the speed of B add 0.8 m s^{-1} to the speed of A.

b The momentum of A before the collision is given by

$$mu=0.5\times3\text{ N s}=1.5\text{ N s}$$

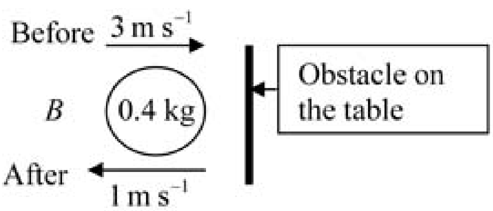
The momentum of A after the collision is given by

$$mv=0.5\times2.2\text{ N s}=1.1\text{ N s}$$

A loses momentum $(1.5-1.1)\text{N s}=0.4\text{ N s}$, as required.

The momentum of a particle is its mass times its velocity. Momentum is a vector quantity.

c



For B, before and after the second impact

$$\mathbf{I}=m\mathbf{v}-m\mathbf{u}$$

$$=0.4\times(-1)-0.4\times3$$

$$=-1.6$$

The magnitude of the impulse received by B in this second impact is 1.6 N s.

Left to right has been taken as the positive direction throughout the question. The impulse on B is negative as, as the situation is drawn here, the impulse on B is in the direction from right to left.

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Review Exercise
Exercise A, Question 37

Question:

Two particles *A* and *B*, of mass 3 kg and 2 kg respectively, are moving in the same direction on a smooth horizontal table when they collide directly. Immediately before the collision, the speed of *A* is 4 m s^{-1} and the speed of *B* is 1.5 m s^{-1}. In the collision, the particles join to form a single particle *C*.

a Find the speed of *C* immediately after the collision.

Two particles *P* and *Q* have mass 3 kg and *m* kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table. Each particle has speed 4 m s^{-1}, when they collide directly. In this collision, the direction of motion of each particle is reversed. The speed of *P* immediately after the collision is 2 m s^{-1} and the speed of *Q* is 1 m s^{-1}

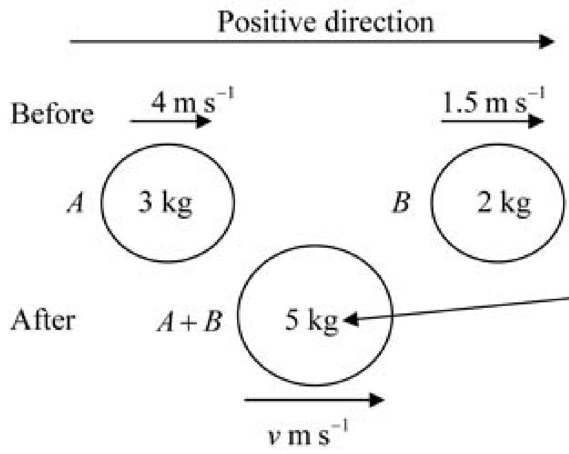
b Find

i the value of *m*,

ii the magnitude of the impulse exerted on *Q* in the collision.

Solution:

a



After the collision A (of mass 3 kg) and B (of mass 2 kg) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, 5 kg.

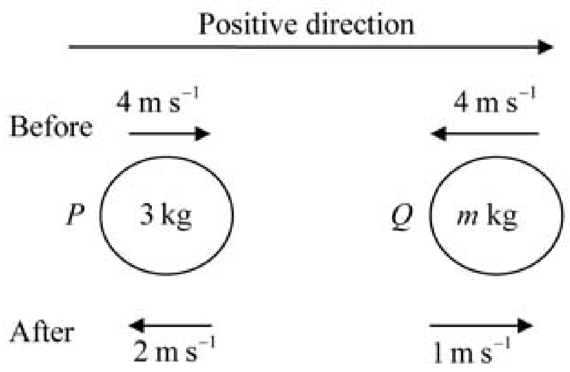
Conservation of linear momentum

$$4 \times 3 + 2 \times 1.5 = 5 \times v$$

$$12 + 3 = 5v \Rightarrow v = \frac{15}{5} = 3$$

The speed of C immediately after the collision is 3 m s^{-1}.

b



In the equation for the conservation of momentum, you must give the velocities in the negative direction a negative sign,

(i) Conservation of linear momentum

$$3 \times 4 + m \times (-4) = 3 \times (-2) + m \times 1$$

$$12 - 4m = -6 + m \Rightarrow 5m = 18$$

$$m = \frac{18}{5} = 3.6$$

(ii) For Q, $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$$I = 3.6 \times 1 - 3.6 \times (-4)$$

$$= 3.6 + 14.4 = 18$$

The magnitude of the impulse exerted on Q in the collision is 18 N s.

As the magnitude of the impulse exerted on P is the same as the magnitude of the impulse exerted on Q, you could equally correctly work out the change in linear momentum of P. The working then would be $I = 3 \times (-2) - 3 \times 4 = -18$, which gives the same magnitude, 18 N s..

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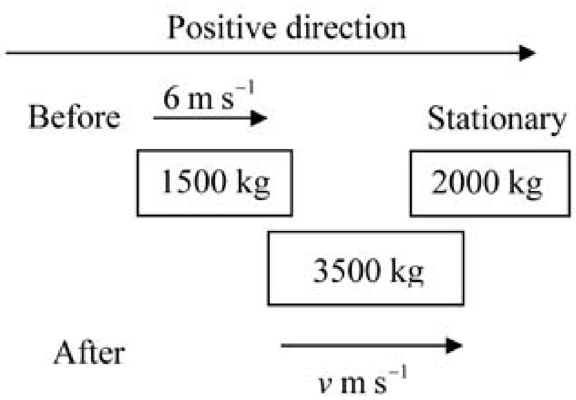
Review Exercise
Exercise A, Question 38

Question:

A railway truck, of mass 1500 kg and travelling with a speed 6 m s $^{-1}$ along a horizontal track, collides with a stationary truck of mass 2000 kg. After the collision the two trucks move on together, coming to rest after 12 seconds. Calculate the magnitude of the constant force resisting their motion after the collision.

Solution:



Conservation of linear momentum

$1500 \times 6 = 3500v$

$$v = \frac{1500 \times 6}{3500} = \frac{18}{7}$$

To find deceleration

$u = \frac{18}{7}, v = 0, t = 12, a = ?$

$v = u + at$

$$0 = \frac{18}{7} + 12a \Rightarrow a = -\frac{18}{7} \times \frac{1}{12} = -\frac{3}{14}$$

To find the magnitude of the resisting force, F, say

$\mathbf{F} = m\mathbf{a}$

$$F = 3500 \times -\frac{3}{14} = -750$$

The magnitude of the resisting force is 750 N.

You need to have a plan when tackling this sort of question. The question has not broken the problem down into separate parts and you need to make your own plan. You have to find the three steps for yourself.
1. Use conservation of linear momentum to find the velocity of the trucks after the collision.
2. Use a kinematics formula to find the deceleration.
3. Use Newton's Second law with your result from step 2 to find the force.

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Review Exercise
Exercise A, Question 39

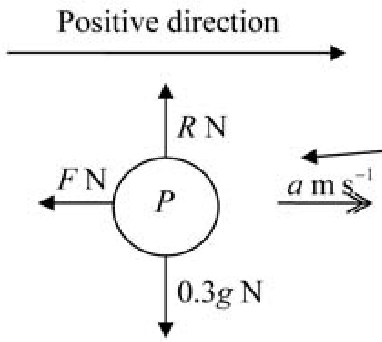
Question:

A particle P of mass 0.3 kg is moving in a straight line on a rough horizontal plane. The speed of P decreases from 7.5 m s^{-1} to 4 m s^{-1} in time T seconds. Given the coefficient of friction between P and the plane is $\frac{1}{7}$, find

a the magnitude of the frictional force opposing the motion of P,

b the value of T.

Solution:



You should begin by drawing a diagram which shows all of the forces acting on P and its acceleration.

a $R(\uparrow)$ $R - 0.3g = 0 \Rightarrow R = 0.3g$

$$F = \mu R$$
$$= \frac{1}{7} \times 0.3 \times 9.8 = 0.42$$

The magnitude of the frictional force opposing the motion of P is 0.42 N.

b $\mathbf{F} = m\mathbf{a}$

$R(\rightarrow)$ $-F = 0.3a$

Using the result of part **a**

$-0.42 = 3a \Rightarrow a = -\frac{0.42}{0.3} = -1.4$

You are asked to find T but before you can use $v = u + at$, you have to find the value of a, using Newton's second law. As the particle is slowing down, a is negative.

$u = 7.5, v = 4, a = -1.4, T = ?$

$v = u + at$

$4 = 7.5 - 1.4T$

$$T = \frac{7.5 - 4}{1.4} = 2.5$$

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 40

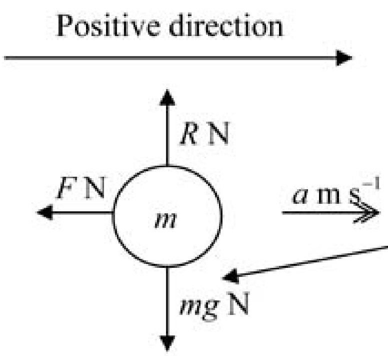
Question:

A small stone moves horizontally in a straight line across the surface of an ice rink. The stone is given an initial speed of 7 m s $^{-1}$. It comes to rest after moving a distance of 10 m. Find

a the deceleration of the stone while it is moving,

b the coefficient of friction between the stone and the ice.

Solution:



You are given no value for mass of the small stone and you will need to have an expression for the weight of the stone. Let the mass of the stone be m kg, then the weight of the stone is mg N.

a $u = 7,\ v = 0,\ s = 10,\ a = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 7^2 + 2 \times a \times 10$$
$$a = -\frac{49}{20} = -2.45$$

The deceleration of the stone is 2.45 m s^{-1}.

b $R(\uparrow)\quad R - mg = 0 \Rightarrow R = mg$

$$F = \mu R = \mu mg$$
$$\mathbf{F} = m\mathbf{a}$$

$R(\rightarrow)\quad -F = ma$

$$-\mu mg = m \times (-2.45)$$
$$\mu = \frac{2.45\cancel{m}}{9.8\cancel{m}} = 0.25$$

The m 'cancels' at the end of the question. This result would be the same with a small stone of any mass.

The coefficient of friction between the stone and the ice is 0.25.

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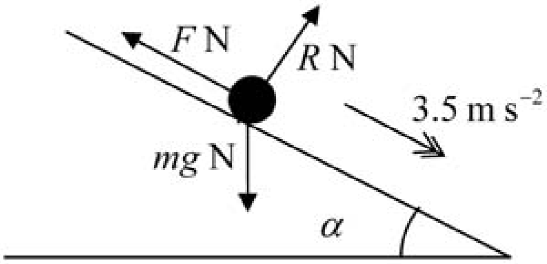
Review Exercise
Exercise A, Question 41

Question:

A rough plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. A particle slides with acceleration 3.5 m s^{-1} down a line of greatest slope of this plane. Calculate the coefficient of friction between the particle and the plane.

Solution:



$R(\nearrow)$ $\quad R - mg\cos\alpha = 0$

$$R = mg\cos\alpha = \tfrac{4}{5}mg$$

Friction is limiting

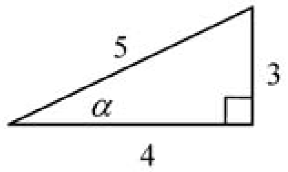
$$F = \mu R = \tfrac{4}{5}\mu mg$$

$R(\searrow)$ $\quad mg\sin\alpha - F = ma$

$$\tfrac{3}{5}\cancel{m}g - \tfrac{4}{5}\mu\cancel{m}g = \cancel{m}3.5$$

$$\mu = \frac{\frac{3}{5}\times 9.8 - 3.5}{\frac{4}{5}\times 9.8} = 0.3035\ldots$$

The coefficient of friction between the particle and the plane is 0.30 (2 s.f.).



You can see from the above diagram that if $\tan\alpha = \frac{3}{4}$, then $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

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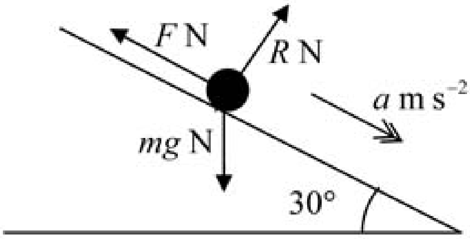
Review Exercise
Exercise A, Question 42

Question:

A particle moves down a line of greatest slope of a rough plane which is inclined at 30 ° to the horizontal. The particle starts from rest and moves 3.5 m in time 2 s. Find the coefficient of friction between the particle and the plane.

Solution:



$u = 0, s = 3.5, t = 2, a = ?$

$s = ut + \frac{1}{2}at^2$

$3.5 = 0 + \frac{1}{2}a \times 2^2 = 2a$

$a = \frac{3.5}{2} = 1.75$

$R(\nearrow)\quad R - mg\cos 30° = 0 \Rightarrow R = mg\cos 30°$

Friction is limiting

$F = \mu R = \mu mg\cos 30°$

$R(\searrow)\quad mg\sin 30° - F = ma$

$\cancel{m}g\sin 30° - \mu \cancel{m}g\cos 30° = \cancel{m} \times 1.75$

$\mu = \frac{9.8\sin 30° - 1.75}{9.8\cos 30°} = 0.3711\ \ldots$

The coefficient of friction between the particle and the plane is 0.37 (2 s.f.).

As often happens, the m which you had to introduce at the beginning, "cancels" because it is a common factor of all of the terms in the equation.

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Review Exercise
Exercise A, Question 43

Question:

A stone S is sliding on ice. The stone is moving along a straight line ABC, where $AB = 24$ m and $AC = 30$ m. The stone is subject to a constant resistance to motion of magnitude 0.3 N. At A the speed of S is 20 m s^{-1}, and at B the speed of S is 16 m s^{-1}. Calculate

a the deceleration of S,

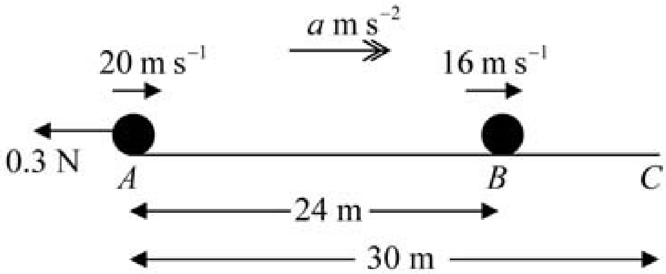
b the speed of S at C.

c Show that the mass of S is 0.1 kg.

At C, the stone S hits a vertical wall, rebounds from the wall and then slides back along the line CA. The magnitude of the impulse of the wall on S is 2.4 N s and the stone continues to move against a constant resistance of 0.3 N.

d Calculate the time between the instant that S rebounds from the wall and the instant that S comes to rest.

Solution:



a From A to B

$u = 20, v = 16, s = 24, a = ?$

$v^2 = u^2 + 2as$

$16^2 = 20^2 + 48s$

$s = \dfrac{16^2 - 20^2}{48} = -3$

The deceleration of S is 3 m s^{-2}.

b From A to C

$u = 20, s = 30, a = -3, v = ?$

$v^2 = u^2 + 2as$

$= 20^2 + 2 \times -3 \times 30 = 400 - 180 = 220$

$v = \sqrt{220} \approx 14.8$

The speed of S at C is 14.8 m s^{-1} (3 s.f.).

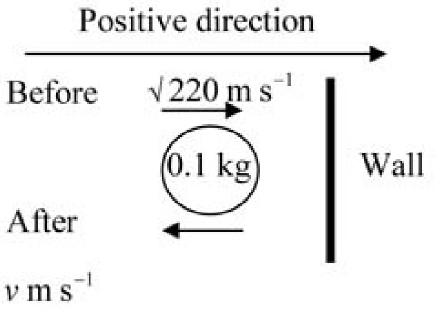
No accuracy is specified in this question and no numerical value of g is used. Where there is no exact answer, it is reasonable for you to give your answers to 3 significant figures.

c $\mathbf{F} = m\mathbf{a}$

$-0.3 = m \times -3 \Rightarrow m = \dfrac{0.3}{3} = 0.1$

The mass of S is 0.1 kg, as required.

d



$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$-2.4 = 0.1 \times (-v) - 0.1 \times \sqrt{220}$

$0.1v = 2.4 - 0.1 \times \sqrt{220} = 0.916\,76\ldots$

$v = 9.167\,6\ldots$

It is quite common for the final velocity in one calculation, here the velocity after the impact with the wall, to become the initial velocity in the next part of the question, here the deceleration. So v in one calculation is u in the next.

S now slows down with deceleration 3 m s^{-2}.

$u = 9.167\,6\ldots,\ v = 0,\ a = -3,\ t = ?$

The equation of motion has not changed since part (c), except that it has reversed direction. The resistance is still 0.3 N and the mass is, obviously, unchanged.

$v = u + at \Rightarrow 0 = 9.167\,6\ldots - 3t$

$t = 3.055\ldots$

The time taken to slow to rest is 3.06 s (3 s.f).

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Review Exercise
Exercise A, Question 44

Question:

A railway truck P of mass 1500 kg is moving on a straight horizontal track. The truck P collides with a truck Q of 2500 kg at a point A. Immediately before the collision, P and Q are moving in the same direction with speeds 10 m s $^{-1}$ and 5 m s $^{-1}$ respectively. Immediately after the collision, the direction of motion of P is unchanged and its speed is 4 m s $^{-1}$. By modelling the trucks as particles,

a show that the speed of Q immediately after the collision is 8.6 m s $^{-1}$.

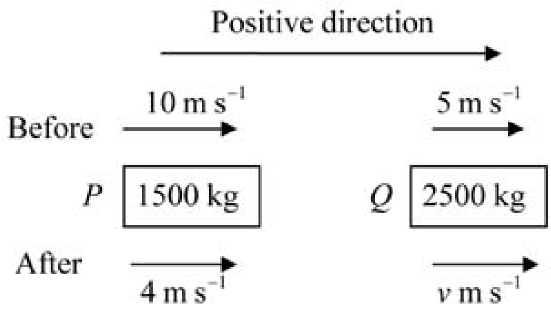
After the collision at A, the truck P is acted upon by a constant braking force of magnitude 500 N. The truck P comes to rest at the point B.

b Find the distance AB.

After the collision Q continues to move with constant speed 8.6 m s $^{-1}$.

c Find the distance between P and Q at the instant when P comes to rest.

Solution:



a Conservation of linear momentum

$1500\times10+2500\times5=1500\times4+2500v$

$15\,000+12\,500=6000+2500v$

$$v=\frac{15\,000+12\,500-6000}{2500}=\frac{21\,500}{2500}=8.6$$

The speed of Q immediately after the collision is 8.6 m s^{-1}, as required.

b For P

$\mathbf{F}=m\mathbf{a}$

$R(\rightarrow)\ -500=1500a \Rightarrow a=-\frac{1}{3}$

The only force acting on P in the horizontal direction is the braking force of 500 N.

$u=4, v=0, a=-\frac{1}{3}, s=?$

$v^2=u^2+2as$

$0^2=4^2-\frac{2}{3}s \Rightarrow s=\frac{3}{2}\times16=24$

The distance AB is 24 m.

c The time taken for P to come to rest is given by

$u=4, v=0, a=-\frac{1}{3}, t=?$

$v=u+at$

$0=4-\frac{1}{3}t \Rightarrow t=12$

Before you can find the distance travelled by truck Q as truck P comes to rest, you will have to find the time taken by P to come to rest. As Q is travelling at a constant speed, the distance it travels is found using distance = speed × time .

The distance travelled by Q is given by distance = speed × time

$s=8.6\times12=103.2$

The distance between P and Q at the instant when P comes to rest is $(103.2-24)\text{m}=79.2\text{ m}$.

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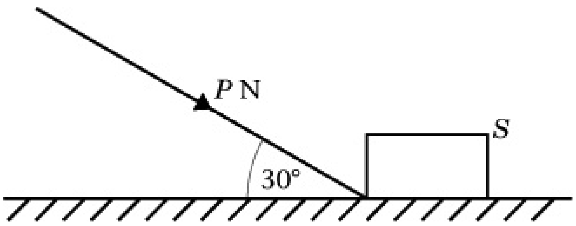
Review Exercise
Exercise A, Question 45

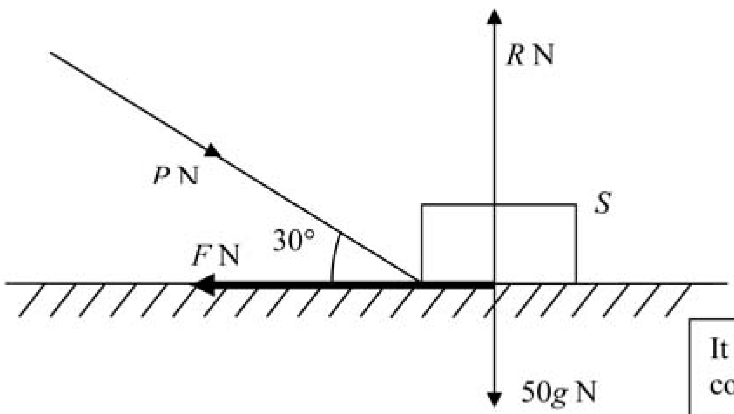
Question:



A heavy suitcase S of mass 50 kg is moving along a horizontal floor under the action of a force of magnitude P newtons. The force acts at 30 ° to the floor, as shown in the figure, and S moves in a straight line at constant speed. The suitcase is modelled as a particle and the floor as a rough horizontal plane. The coefficient of friction between S and the floor is $\frac{3}{5}$.

Calculate the value of P.

Solution:



$R(\uparrow)\quad R - 50g - P\sin 30° = 0$

$$R = 50g + P\sin 30°$$

Friction is limiting

$$F = \mu R = \tfrac{3}{5}(50g + P\sin 30°)$$

As S is moving horizontally at a constant speed, it has zero acceleration.

$R(\rightarrow)\quad P\cos 30° - F = 50 \times 0$

$$P\cos 30° - 0.6(50g + P\sin 30°) = 0$$

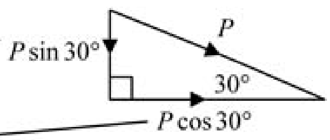
$$P\cos 30° - 30g + 0.6P\sin 30° = 0$$

$$P(\cos 30° + 0.6\sin 30°) = 30g$$

$$P = \frac{30 \times 9.8}{\cos 30° + 0.6\sin 30°} = 252.1\ \ldots$$

$$= 250\ (2\text{ s.f.})$$

It is a common error to omit the component of P when resolving vertically. A force at 30° to the horizontal has components in both the vertical and horizontal directions.



Collect the terms in P on one side of the equation and the constant terms (there is only one here, $30g$) on the other.

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Review Exercise
Exercise A, Question 46

Question:

An engine of mass 25 tonnes pulls a truck of mass 10 tonnes along a railway line. The frictional resistances to the motion of the engine and the truck are modelled as constant and of magnitude 50 N per tonne. When the train is travelling horizontally the tractive force exerted by the engine is 26 kN. Modelling the engine and the truck as particles and the coupling between the engine and the truck as a light horizontal rod, calculate

a the acceleration of the engine and the truck,

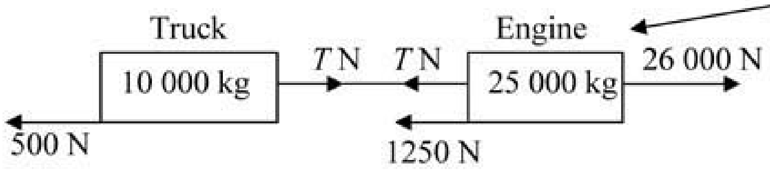
b the tension in the coupling.

The engine and the truck now climb a slope which is modelled as a plane inclined at angle α to the horizontal, where $\sin \alpha = \frac{1}{70}$.

The engine and the truck are moving up the slope with an acceleration of magnitude 0.6 m s $^{-2}$. The frictional resistances to motion are modelled as before.

c Calculate the tractive force exerted by the engine. Give your answer in kN. (1 tonne = 1000 kg)

Solution:



1 tonne = 1000 kg. It is recommended that you work with metres, kilograms and seconds in all questions using Newton's Second Law.

a The resistance on the engine is $25 \times 50 = 1250$ N

The resistance on the truck is $10 \times 50 = 500$ N

For the whole system, the engine and truck

$R(\rightarrow)$ $\quad\quad$ $\mathbf{F} = m\mathbf{a}$

$$26\,000 - 1250 - 500 = 35\,000a$$

$$a = \frac{26\,000 - 1250 - 500}{35\,000} = \frac{97}{140} = 0.6928\ \ldots$$

The acceleration of the engine and truck is $0.693\ \text{m s}^{-2}$ (3 s.f).

There is 50 N of resistance for each tonne, so an engine of 25 tonnes is resisted by a force of 50×25 N.

When you consider the whole system, the tension in the coupling acting on the truck and the tension in the coupling acting on the engine are of equal magnitude and in opposite directions. When you resolve horizontally, the tensions cancel out.

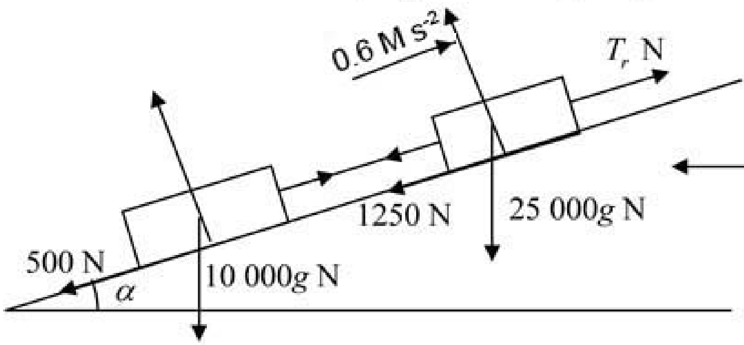
b For the truck alone

$\mathbf{F} = m\mathbf{a}$

$T - 5000 = 10\,000a$

$T = 500 + 10\,000 \times 0.6928\ldots = 7428.57\ \ldots$

The tension in the coupling is 7430 N (3 s.f.).



In this diagram, only the forces which are relevant when resolving up the plane have been labelled. The tractive force has been labelled T_r N.

c For the whole system, the engine and truck

$R(\nearrow)$

$$T_r - 2500 - 500 - 25\,000g \sin\alpha - 10\,000g \sin\alpha = 35\,000a$$

$$T_r = 1750 + 35\,000 \times 9.8 \times \frac{1}{70} + 35\,000 \times 0.6$$

$$= 27\,650$$

The tractive force of the engine is 28 kN (2 s.f.).

There are 5 forces acting parallel to the plane. The tractive force, two resistances and two components of weight.

1 kN = 1000 N

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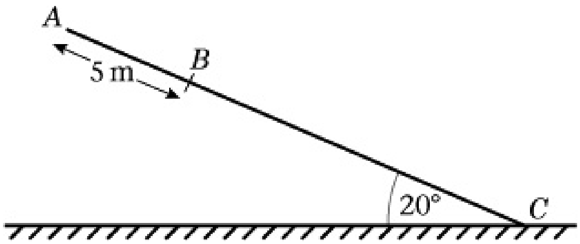
Review Exercise
Exercise A, Question 47

Question:



A suitcase of mass 10 kg slides down a ramp which is inclined at an angle of 20° to the horizontal. The suitcase is modelled as a particle and the ramp as a rough plane. The top of the plane is *A*. The bottom of the plane is *C* and *AC* is a line of greatest slope, as shown in the figure above. The point *B* is on *AC* with *AB* = 5 m. The suitcase leaves *A* with a speed of 10 m s $^{-1}$ and passes *B* with a speed of 8 m s $^{-1}$. Find

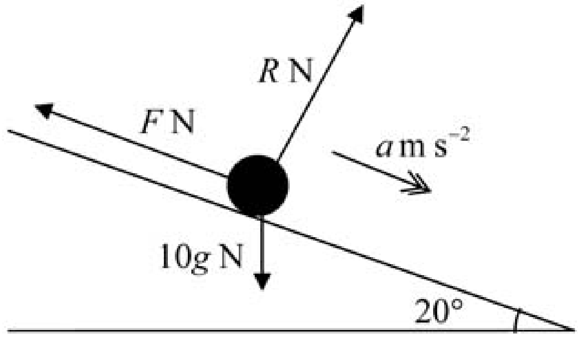
a the deceleration of the suitcase,

b the coefficient of friction between the suitcase and the ramp.

The suitcase reaches the bottom of the ramp.

c Find the greatest possible length of *AC*.

Solution:



a From A to B

$u = 10,\ v = 8,\ s = 5,\ a = ?$

$v^2 = u^2 + 2as$

$8^2 = 10^2 + 2 \times a \times 5$

$a = \dfrac{8^2 - 10^2}{10} = -3.6$

The deceleration of the suitcase is 3.6 m s^{-2}.

b $R(\nearrow)$ $R - 10g\cos 20° = 0 \Rightarrow R = 10g\cos 20°$

Friction is limiting

$F = \mu R = \mu 10g\cos 20°$

$R(\searrow)$ $10g\sin 20° - F = 10a$

$\cancel{10}g\sin 20° - \mu\cancel{10}g\cos 20° = \cancel{10} \times (-3.6)$

$\mu = \dfrac{9.8\sin 20° + 3.6}{9.8\cos 20°} = 0.7548\ \ldots$

As the numerical value $g = 9.8$ has been taken, you should give your final answer for μ, corrected to 2 significant figures.

The coefficient of friction between the suitcase and the ramp is 0.75 (2 s.f.).

c From A to C

$u = 10,\ v = 0,\ a = -3.6,\ s = ?$

To reach the bottom of the ramp, the suitcase must not stop before it reaches the lowest point of the ramp C. The limiting case is that the suitcase has zero speed at C and this is taken to find the greatest possible length of AC.

$v^2 = u^2 + 2as$

$0^2 = 10^2 + 2 \times (-3.6) \times s$

$s = \dfrac{10^2}{7.2} = 13.\dot{8}$

The greatest possible length of AC is 14 m (2 s.f.).

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Review Exercise
Exercise A, Question 48

Question:

A slipway for launching boats consists of a rough straight track inclined at an angle of 10 ° to the horizontal. A boat of mass 300 kg is pulled down the slipway by means of a rope which is parallel to the slipway. When the tension in the rope is 500 N, the boat moves down the slipway with constant speed.

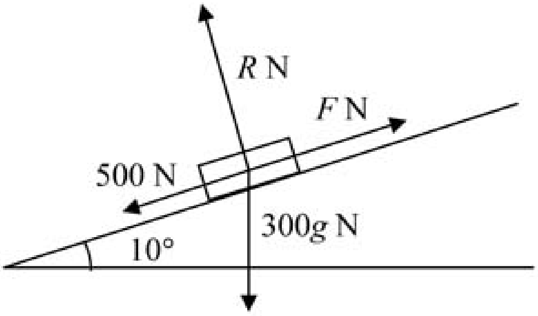
a Find, to two significant figures, the coefficient of friction between the boat and the slipway.

Later the boat returns to the slipway. It is now pulled up the slipway at constant speed by the rope which is again parallel to the slipway.

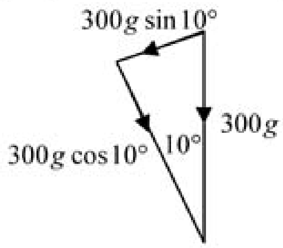
b Give a brief reason why the magnitude of the frictional force is the same as when the boat is pulled down the slope.

c Find, to two significant figures, the tension in the rope.

Solution:



The weight has components parallel and perpendicular to the plane



a $R(\nwarrow)$ $R - 300g\cos 10° = 0 \Rightarrow R = 300g\cos 10°$

Friction is limiting

$$F = \mu R = \mu 300g\cos 10°$$

$R(\swarrow)$ $500 + 300g\sin 10° - F = 300 \times 0$

The boat is moving at a constant speed and, hence, its acceleration is zero.

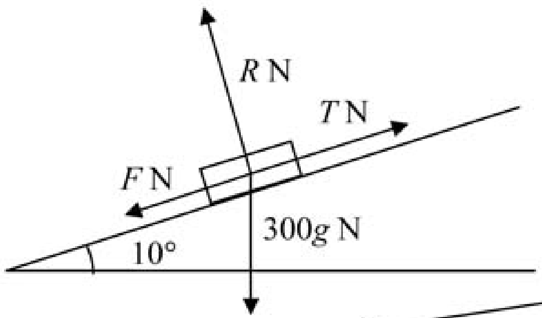
$$F = \mu 300g\cos 10° = 500 + 300g\sin 10°$$

$$\mu = \frac{500 + 300g\sin 10°}{300g\cos 10°} = 0.349\ \ldots$$

The coefficient of friction between the boat and the slipway is 0.35 (2 s.f).

b The normal reaction $(R = 300g\cos 10°)$ is unchanged and, hence, the friction force $(F = \mu R)$ is unchanged.

c



F is unchanged in magnitude but, as friction opposes motion, it has reversed direction and is now acting down the plane.

$R(\nearrow)$ $T - 300g\sin 10° - F = 300 \times 0$

$$T = 300g\sin 10° + F$$
$$= 600g\sin 10° + 500$$
$$= 1521\ \ldots$$

$F = 500 + 300g\sin 10°$ from the working in part **a**.

The tension in the rope is 1500 N (2 s.f.).

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Review Exercise
Exercise A, Question 49

Question:

A tent peg is driven into soft ground by a blow from a hammer. The tent peg has mass 0.2 kg and the hammer has mass 3 kg. The hammer strikes the peg vertically.

Immediately before the impact, the speed of the hammer is 16 m s^{-1}. It is assumed that, immediately after the impact, the hammer and the peg move together vertically downwards.

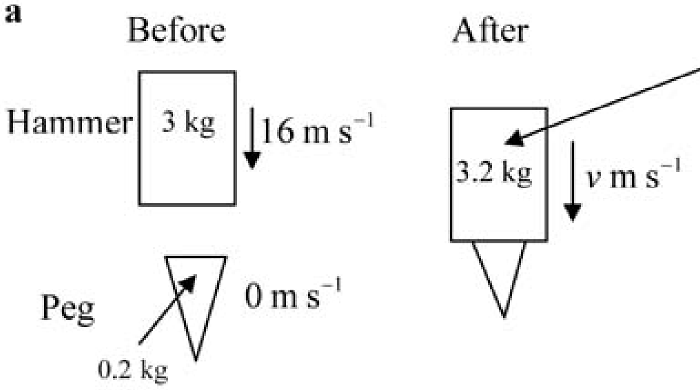
a Find the common speed of the peg and the hammer immediately after the impact.

Until the peg and hammer come to rest, the resistance exerted by the ground is assumed to be constant and of magnitude *R* newtons. The hammer and peg are brought to rest 0.05 s after the impact.

b Find, to three significant figures, the value of *R*.

Solution:

a



After impact, the hammer and the peg move together. You model the hammer and peg as a single particle of mass 3.2 kg

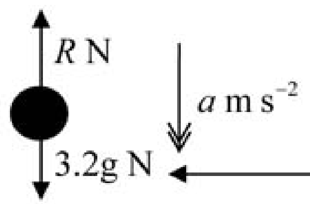
Conservation of linear momentum.

$$3 \times 16 + 0.2 \times 0 = 3.2 \times v$$

$$v = \frac{3 \times 16}{3.2} = 15$$

The common speed of the peg and the hammer immediately after the impact is 15 m s^{-1}.

b



$u = 15, v = 0, t = 0.05, a = ?$

$v = u + at$

$$0 = 15 + a \times 0.05 \Rightarrow a = -\frac{15}{0.05} = -300$$

$\mathbf{F} = m\mathbf{a}$

$3.2g - R = 3.2 \times (-300)$

$R = 3.2 \times 300 + 3.2 \times 9.8 = 991.36$

$= 991$ (3 s.f.)

You need to find R using $\mathbf{F} = m\mathbf{a}$. Before this can be used, you will need to find the deceleration of the hammer and the peg. As in part **a**, you need to consider the hammer and the peg as a single particle of mass 3.2 kg.

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Review Exercise
Exercise A, Question 50

Question:

A ball is projected vertically upwards with a speed u m s^{-1} from a point A which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above A.

a Show that $u = 22.4$.

The ball reaches the ground T seconds after it has been projected from A.

b Find, to two decimal places, the value of T.

The ground is soft and the ball sinks 2.5 cm into the ground before coming to rest. The mass of the ball is 0.6 kg. The ground is assumed to exert a constant resistive force of magnitude F newtons.

c Find, to three significant figures, the value of F.

d State one physical factor which could be taken into account to make the model used in this question more realistic.

Solution:

a From A to the greatest height, taking upwards as positive.

$v = 0,\ a = -9.8,\ s = 25.6,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

The ball reaches the ground at a point which is 1.5 m lower than the point of projection A. So you must take $s = -1.5$.

b $u = 22.4,\ s = -1.5,\ a = -9.8,\ t = T$

$s = ut + \frac{1}{2}at^2$

$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

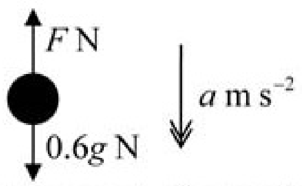
$4.9T^2 - 22.4T - 1.5 = 0$

$T = \dfrac{22.4 + \sqrt{(22.4^2 - 4 \times 4.9 \times -1.5)}}{2 \times 9.8}$

$= 4.637\ \ldots = 4.64$ (3 s.f.).

You can ignore the negative solution of the quadratic equation. That would represent a time before the ball was projected.

c



To find the speed of the ball as it reaches the ground.

$u = 22.4,\ s = -1.5,\ a = -9.8,\ v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

As, at the next stage, you will use the velocity squared, you need not find the square root of 531.16. The final velocity of the motion under gravity becomes the initial velocity of the motion as the ball sinks into the ground.

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16,\ v = 0,\ s = 0.025,\ a = ?$

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

$a = -\dfrac{531.16}{0.05} = -10623.2$

You need to use metres, kilograms and seconds consistently, so 2.5 cm must be converted to 0.025 m.

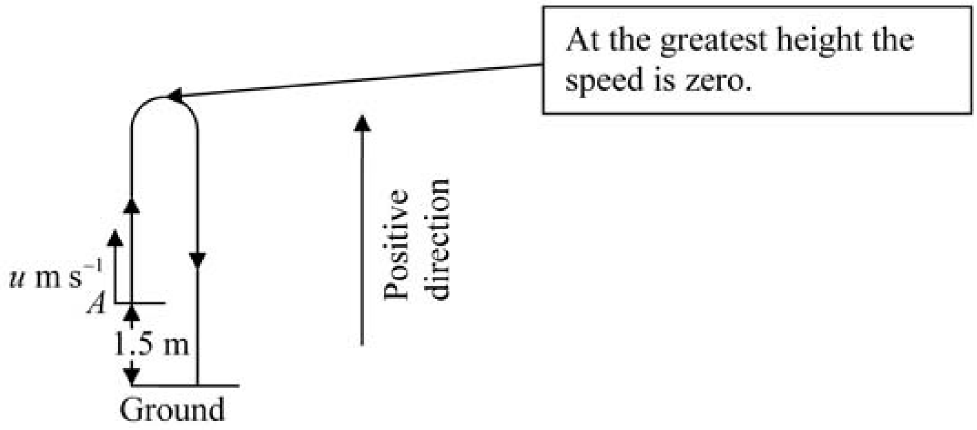
$\mathbf{F} = m\mathbf{a}$

$0.6g - F = 0.6 \times (-10\,623.2)$

$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.).

d Consider air resistance during motion under gravity.

To use a variable F, as resisting forces usually vary with speed, would also be a good answer.


At the greatest height the speed is zero.
u m s^{-1}
A
1.5 m
Ground
Positive direction

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Review Exercise
Exercise A, Question 51

Question:

A particle A, of mass 0.8 kg, resting on a smooth horizontal table, is connected to a particle B, of mass 0.6 kg, which is 1 m from the ground, by a light inextensible string passing over a small pulley at the edge of the table. The particle A is more than 1 m from the edge of the table. The system is released from rest with the horizontal part of the string perpendicular to the edge of the table, the hanging parts vertical and the string taut. Calculate

a the acceleration of A,

b the tension in the string,

c the speed of B when it hits the ground,

c the time taken for B to reach the ground.

Solution:

a From A to the greatest height, taking upwards as positive.

$v = 0,\ a = -9.8,\ s = 25.6,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

The ball reaches the ground at a point which is 1.5 m lower than the point of projection A. So you must take $s = -1.5$.

b $u = 22.4,\ s = -1.5,\ a = -9.8,\ t = T$

$s = ut + \frac{1}{2}at^2$

$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

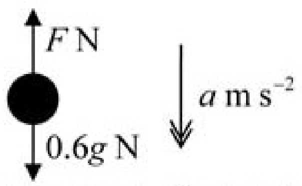
$4.9T^2 - 22.4T - 1.5 = 0$

You can ignore the negative solution of the quadratic equation. That would represent a time before the ball was projected.

$$T = \frac{22.4 + \sqrt{(22.4^2 - 4 \times 4.9 \times -1.5)}}{2 \times 9.8}$$

$= 4.637 \ \ldots = 4.64$ (3 s.f.).

c



To find the speed of the ball as it reaches the ground.

$u = 22.4,\ s = -1.5,\ a = -9.8,\ v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

As, at the next stage, you will use the velocity squared, you need not find the square root of 531.16. The final velocity of the motion under gravity becomes the initial velocity of the motion as the ball sinks into the ground.

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16,\ v = 0,\ s = 0.025,\ a = ?$

You need to use metres, kilograms and seconds consistently, so 2.5 cm must be converted to 0.025 m.

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

$a = -\frac{531.16}{0.05} = -10623.2$

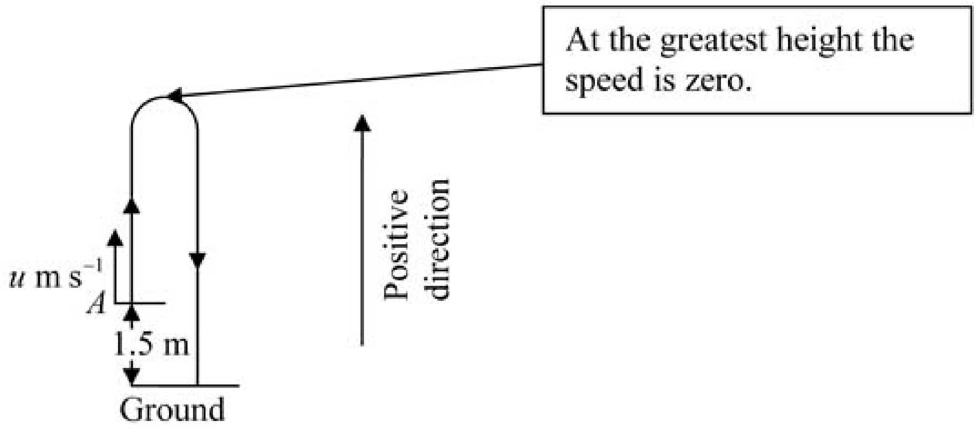
$\mathbf{F} = m\mathbf{a}$

$0.6g - F = 0.6 \times (-10\,623.2)$

$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.).

d Consider air resistance during motion under gravity.

To use a variable F, as resisting forces usually vary with speed, would also be a good answer.


At the greatest height the speed is zero.
Positive direction
u m s^{-1}
A
1.5 m
Ground

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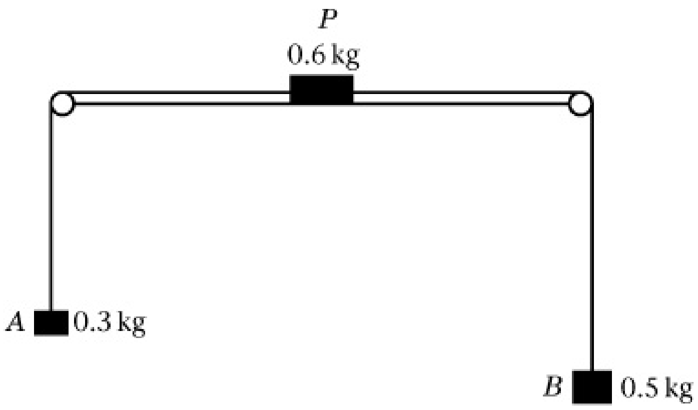
Review Exercise
Exercise A, Question 52

Question:

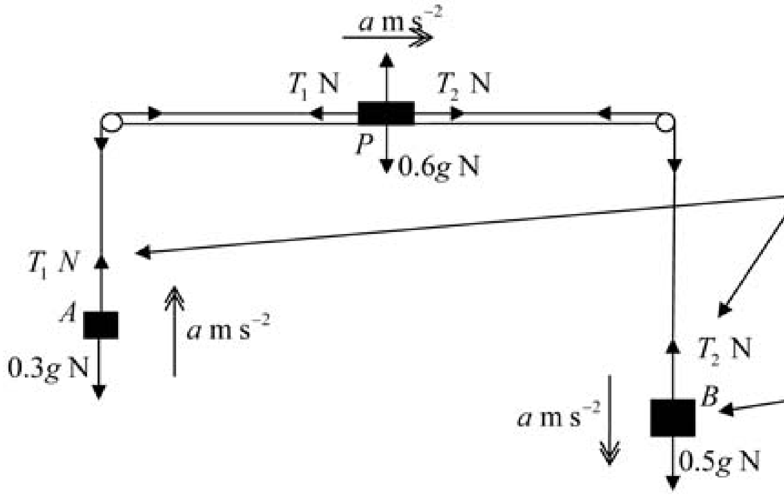


The figure shows a block *P* of mass 0.6 kg resting on the smooth surface of a horizontal table. Inextensible light strings connect *P* to blocks *A* and *B* which hang freely over light smooth pulleys placed at opposite parallel edges of the table. The masses of *A* and *B* are 0.3 kg and 0.5 kg respectively. All portions of the string are taut and perpendicular to their respective edges of the table. The system is released from rest. Calculate

a the common magnitude of the accelerations of the blocks,

b the tensions in the strings.

Solution:



In this question, there are two separate strings; one joining A to P and one joining B to P. These will have different tensions; here labelled T_1 N and T_2 N.

The heavier of the two blocks A and B will fall.

a For A

$R(\uparrow)\qquad T_1 - 0.3g = 0.3a \;\ldots\ldots\; (1)$

For P

$R(\rightarrow)\qquad T_2 - T_1 = 0.6a \;\ldots\ldots\; (2)$

For B

$R(\downarrow)\qquad 0.5g - T_2 = 0.5a \;\ldots\ldots\; (3)$

(1) + (2) + (3)

$$0.2g = 1.4a$$

$$a = \frac{0.2 \times 9.8}{1.4} = 1.4$$

The common magnitude of the accelerations of the blocks is $1.4\ \text{m s}^{-2}$.

3 equations in 3 unknowns can be difficult to solve but, in this case, if you add the 3 equations together the T_1 in (1) cancels with the $-T_1$ in (2) and the T_2 in (2) cancels out with the $-T_2$ in (3), leaving an equation in a alone.

b Substitute the result of part **a** into (1)

$$T_1 - 0.3g = 0.3 \times 1.4$$

$$T_1 = 0.3 \times 1.4 + 0.3 \times 9.8 = 3.36 \;\ldots\; (4)$$

Substitute the result of part **a** and (4) into (2)

$$T_2 - 3.36 = 0.6 \times 1.4$$

$$T_2 = 3.36 + 0.6 \times 1.4 = 4.2$$

The tension in the string joining A to P is 3.4 N (2 s.f.).
The tension in the string joining B to P is 4.2 N.

Alternatively, you can find T_2 by substituting for a in (3).

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Review Exercise
Exercise A, Question 53

Question:

A trailer of mass 600 kg is attached to a car of mass 900 kg by means of a light inextensible tow-bar. The car tows the trailer along a horizontal road. The resistances to motion of the car and trailer are 300 N and 150 N respectively.

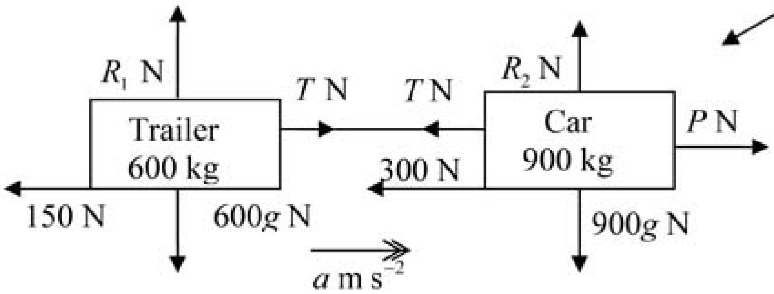
a Given that the acceleration of the car and trailer is 0.4 m s^{-2}, calculate

i the tractive force exerted by the engine of the car,

ii the tension in the tow bar.

b Given that the magnitude of the force in the tow-bar must not exceed 1650N, calculate the greatest possible deceleration of the car.

Solution:



The tractive force exerted by the engine of the car has been called P N. This only acts on the car. It does not act directly on the trailer. The only force moving the trailer forward is the tension in the tow bar.

a(i) For the whole system

$$\mathbf{F} = m\mathbf{a}$$

$$R(\rightarrow) \quad P - 300 - 150 = 1500 \times 0.4$$

$$P = 1050$$

The tractive force exerted by the engine of the car is 1050 N.

When you consider the whole system, the tension in the tow bar acting on the truck and the tension in the tow bar acting on the engine are of equal magnitude and in opposite directions. When you resolve horizontally, the tensions cancel out .

(ii)For the trailer alone

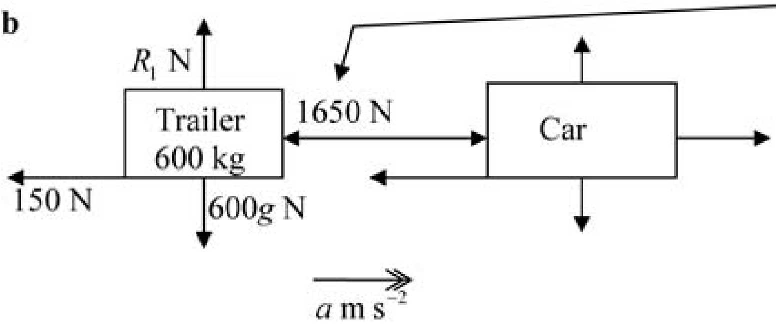
$$\mathbf{F} = m\mathbf{a}$$

$$R(\rightarrow) \qquad T - 150 = 600 \times 0.4$$

$$T = 390$$

The tension in the tow bar is 390 N.

b



When decelerating the force in the tow bar becomes a thrust. The question gives that the greatest magnitude of the thrust is 1650 N. To solve part **b**, you need only the horizontal forces on the trailer.

For the trailer alone

$$\mathbf{F} = m\mathbf{a}$$

$$R(\rightarrow) \qquad -1650 - 150 = 600a$$

$$a = -\frac{1800}{600} = -3$$

The greatest possible deceleration of the car is 3 m s^{-2}.

The deceleration of the trailer and the car are the same. Although the question asks for the deceleration of the car, you could not directly find this, as it would involve an unknown braking force on the car.

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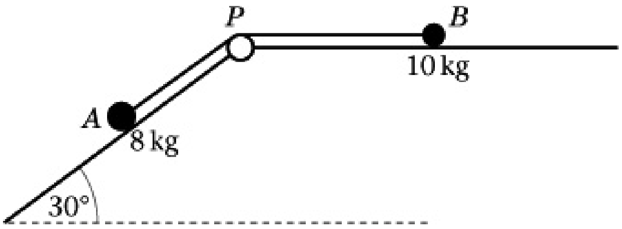
Review Exercise
Exercise A, Question 54

Question:



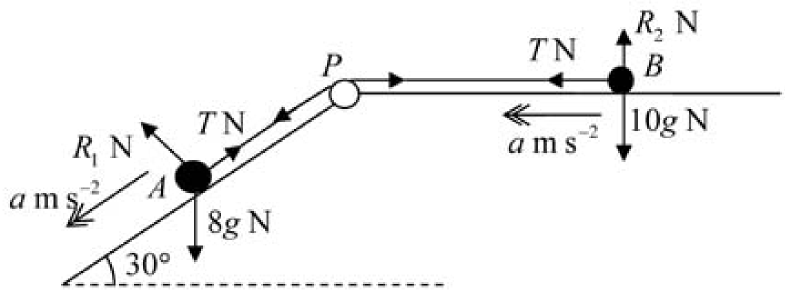
Two particles *A* and *B*, of mass 8 kg and 10 kg respectively, are connected by a light inextensible string which passes over a light smooth pulley *P*. Particle *B* rests on a smooth horizontal table and *A* rests on a smooth plane inclined at 30 ° to the horizontal with the string taut and perpendicular to the line of intersection of the table and the plane as shown in the figure. The system is released from rest. Find

a the magnitude of the acceleration of *B*,

b the tension in the string,

c the distance covered by *B* in the first two seconds of motion, given that *B* does not reach the pulley.

Solution:



a For B

$R(\leftarrow)$ $\qquad T = 10a \quad \dots \dots (1)$

For A

$R(\swarrow)$ $\quad 8g\sin 30° - T = 8a \quad \dots \dots (2)$

(1) + (2)

$$8g\sin 30° = 18a$$

$$a = \frac{8g\sin 30°}{18} = \frac{2}{9}g = 2.1\dot{7}$$

The magnitude of the acceleration of B is 2.2 m s^{-2} (2 s.f.).

(1) and (2) are a pair of linear simultaneous equations in T and a. The methods you can use to solve these is essentially the same as you learnt for GCSE. Either elimination or substitution can be used.

$$\sin 30° = \frac{1}{2}$$

b Substitute $a = \frac{2}{9}g$ into (1)

$$T = \frac{20}{9}g = 21.\dot{7}$$

The tension in the string is 22 N (2 s.f.).

c $u = 0, a = \frac{2}{9}g, t = 2, s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{g}{9} \times 4 = \frac{9.8 \times 4}{9} = 4.3\dot{5}$$

The distance covered in the first 2 seconds is 4.4 m (2 s.f.).

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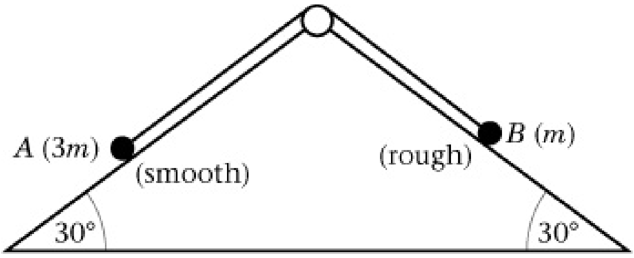
Review Exercise

Exercise A, Question 55

Question:



A fixed wedge has two plane faces, each inclined at 30 ° to the horizontal. Two particles *A* and *B*, of mass 3*m* and *m* respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a smooth light pulley fixed at the top of the wedge. The face on which *A* moves is smooth. The face on which *B* moves is rough. The coefficient of friction between *B* and this face is μ. Particle *A* is held at rest with the string taut. The string lies in the same vertical plane as lines of greatest slope on each plane face of the wedge, as shown in the figure.

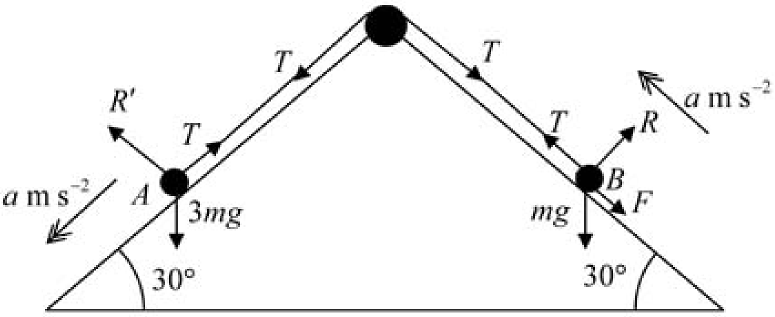
The particles are released from rest and start to move. Particle *A* moves downwards and particle *B* moves upwards. The acceleration of *A* and *B* each have magnitude $\frac{1}{10}g$.

a By considering the motion of *A*, find, in terms of *m* and *g*, the tension in the string.

b By considering the motion of *B*, find the value of μ.

c Find the resultant force exerted by the string on the pulley, giving its magnitude and direction.

Solution:



a For A $\quad\quad$ $\mathbf{F} = m\mathbf{a}$

$R(\swarrow)$ $3mg\sin 30° - T = 3m \times \frac{1}{10} g$

$T = \frac{3}{2}mg - \frac{3}{10}mg = \frac{6}{5}mg$

The tension in the string is $\frac{6}{5}mg$.

$\sin 30° = \frac{1}{2}$

b For B

$R(\nearrow)$ $\quad R - mg\cos 30° = 0 \Rightarrow R = mg\cos 30°$

Friction is limiting

$F = \mu R = \mu mg\cos 30°$

$R(\nwarrow)$ $\quad T - F - mg\sin 30° = ma$

$\frac{6}{5}\cancel{mg} - \mu\cancel{mg}\cos 30° - \cancel{mg}\sin 30° = \frac{1}{10}\cancel{mg}$

$\mu\cos 30° = \frac{6}{5} - \frac{1}{2} - \frac{1}{10} = \frac{3}{5}$

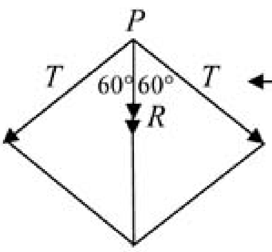
$\mu = \dfrac{3}{5\cos 30°} = 0.692\,82\ \ldots$

$= 0.693$ (3 s.f.).

mg is common to all 4 terms in this equation and can be 'cancelled' leaving an equation in μ alone.

$\cos 30° = \dfrac{\sqrt{3}}{2}$ is the exact value and the exact answer for $\mu\left(= \dfrac{2\sqrt{3}}{5}\right)$ would be acceptable.

c



At P

$R(\downarrow)$ $\quad R = 2T\cos 60°$

$= 2 \times \frac{6}{5}mg \times \frac{1}{2} = \frac{6}{5}mg$

The resultant force exerted by the string on the pulley has magnitude $\frac{6}{5}mg$ and acts vertically downwards.

At P, the string exerts two forces, each of magnitude $\frac{6}{5}mg$, on the pulley both at an angle of $60°$ to the vertical. By symmetry, the resultant force acts vertically downwards. Its magnitude can be found by resolving the two forces vertically downwards.

$\cos 60° = \frac{1}{2}$ is the exact value.

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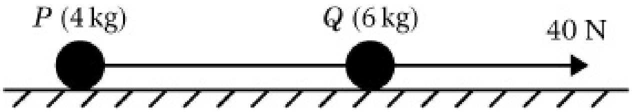
Review Exercise
Exercise A, Question 56

Question:



Two particles P and Q, of mass 4 kg and 6 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. The coefficient of friction between each particle and the plane is $\frac{2}{7}$. A constant force of magnitude 40 N is then applied to Q in the direction PQ, as shown in the figure.

a Show that the acceleration of Q is 1.2 m s^{-2}.

b Calculate the tension in the string when the system is moving.

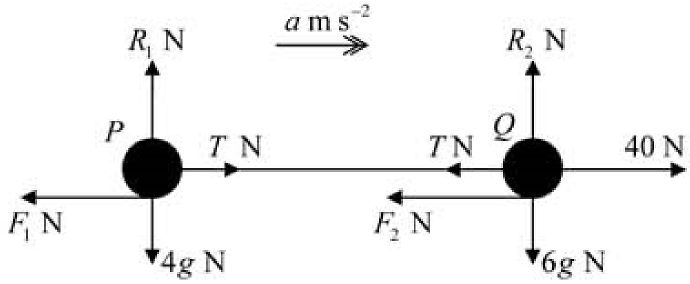
c State how you have used the information that the string is inextensible.

After the particles have been moving for 7 s, the string breaks. The particle Q remains under the action of the force of magnitude 40N.

d Show that P continues to move for a further 3 seconds.

e Calculate the speed of Q at the instant when P comes to rest.

Solution:



a For P, $R(\uparrow)$ $R_1 - 4g = 0 \Rightarrow R_1 = 4g$

Friction is limiting

$$F_1 = \mu R_1 = \tfrac{2}{7} \times 4g = \tfrac{8}{7}g$$

For Q, $R(\uparrow)$ $R_2 - 6g = 0 \Rightarrow R_2 = 6g$

Friction is limiting

$$F_2 = \mu R_2 = \tfrac{2}{7} \times 6g = \tfrac{12}{7}g$$

For the whole system

$R(\rightarrow)$ $\quad 40 - F_1 - F_2 = 10a$

$$40 - \tfrac{8}{7}g - \tfrac{12}{7}g = 10a$$

$$10a = 40 - \tfrac{20}{7} \times 9.8 = 12 \Rightarrow a = 1.2$$

The acceleration of Q is 1.2 m s^{-2}, as required.

When a question asks you to show that a quantity has a value – here that the acceleration is 1.2 m s^{-1} - you must get exactly that value and not approximate during the calculation.

b For P

$R(\rightarrow)$ $\quad T - F_1 = 4a$

$$T - \tfrac{8}{7}g = 4 \times 1.2$$

$$T = 4 \times 1.2 + \tfrac{8}{7} \times 9.8 = 16$$

The tension in the string is 16 N.

c The information that the string is inextensible has been used in assuming that the accelerations of P and Q, and hence of the whole system, are the same.

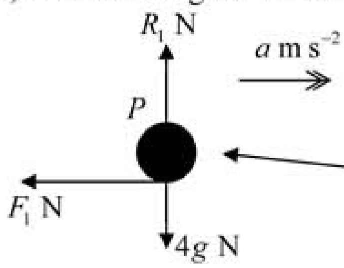
d To find the speed the particles are travelling at when the string breaks.

$u = 0, a = 1.2, t = 7, v = ?$

$v = u + at = 0 + 1.2 \times 7 = 8.4$

The final speed for the part of the motion when the string is taut will be the initial speed of both particles after the string breaks.

For P, after the string has broken



After the string has broken it no longer exerts a tension on P. The forces acting on P are shown in the diagram. The equation obtained by resolving vertically is unchanged and so the normal reaction and the friction force at P are unchanged.

$R(\rightarrow)\quad -F_1 = 4a \Rightarrow -\frac{8}{7}g = 4a \Rightarrow a = -\frac{2}{7}g$

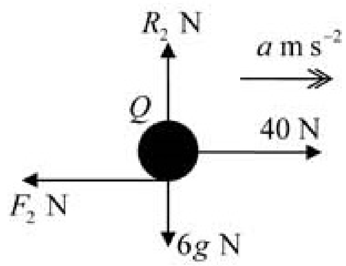
$u = 8.4,\ v = 0,\ a = -\frac{2}{7}g,\ t = ?$

$v = u + at$

$0 = 8.4 - \frac{2}{7}gt \Rightarrow t = \dfrac{8.4\times 7}{2\times 9.8} = 3$

P continues to move for a further 3 s, as required.

e



After the string has broken it no longer exerts a tension on Q. The forces acting on Q are shown in the diagram. The equation obtained by resolving vertically is unchanged and so the normal reaction and the friction force at Q are unchanged.

For Q, after the string has broken.

$R(\rightarrow)\quad 40 - F_2 = 6a$

$40 - \frac{12}{7}g = 6a$

$6a = 40 - \frac{12}{7}\times 9.8 = 23.2$

$a = \dfrac{23.2}{6} = \dfrac{58}{15} = 3.8\dot{6}$

$u = 8.4,\ a = \dfrac{58}{15},\ t = 3,\ v = ?$

$v = u + at = 8.4 + \dfrac{58}{15}\times 3 = 20$

The speed of Q at the instant when P comes to rest is 20 m s^{-1}.

P came to rest 3 seconds after the string had broken. So you have been asked to find the speed of Q after these 3 seconds. First you need to find acceleration of Q. As P is not now attached to Q, Q will accelerate more quickly.

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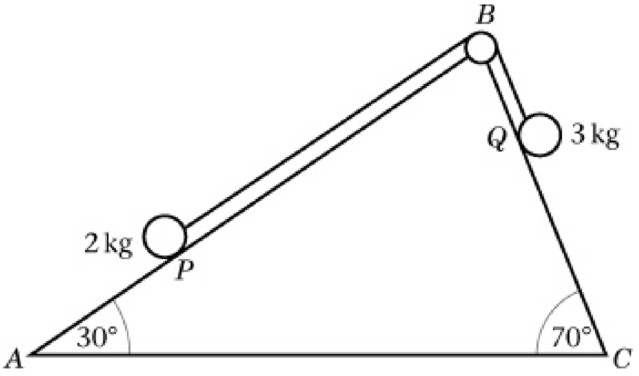
Review Exercise
Exercise A, Question 57

Question:



A fixed wedge whose smooth faces are inclined at 30° and 70° to the horizontal has a small smooth pulley fixed on the top edge at *B*. A light inextensible string, passing over the pulley, has particles *P* and *Q* of mass 2 kg and 3 kg respectively attached at its ends. The figure shows a vertical cross-section of the wedge where *AB* and *AC* are lines of greatest slope of the faces along which *P* and *Q* respectively can slide. The particles are released from rest at time $t = 0$ with the string taut. Assuming that *P* has not reached *B* and that *Q* has not reached *C*, find

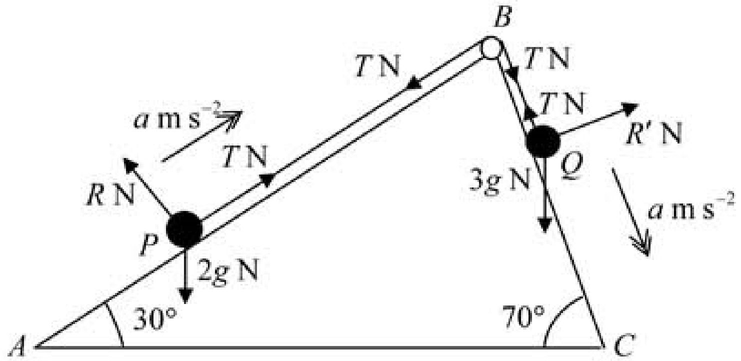
a the distance through which each particle has moved when $t = 0.75$ s,

b the tension in the string,

c the magnitude and direction of the resultant force exerted on the pulley by the string.

When $t = 0.75$ s the string breaks and in the subsequent motion P come to instantaneous rest at time t_1. Assuming that *P* has not reached *B*, **d** calculate *t*.

Solution:



a For P

$R(\nearrow)$ $T - 2g\sin 30° = 2a$ … … (1)

For Q

$R(\searrow)$ $3g\sin 70° - T = 3a$ … … (2)

(1) + (2)

$3g\sin 70° - 2g\sin 30° = 5a$

$a = \dfrac{3g\sin 70° - 2g\sin 30°}{5} = 3.565\,392\ldots$

$u = 0, a = 3.565\ \ldots, t = 0.75, s = ?$

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 3.565\ \ldots \times 0.75^2 = 1.002\ \ldots$

The distance the particles have moved is 1.0 m (2 s.f.).

(1) and (2) are a pair of linear simultaneous equations. You need to solve them for a to solve part **a**. You need to solve them for T in part **b**.

You need the value of a several times in the question and it is sensible to store this value of a in your calculator so you can recall it when needed.

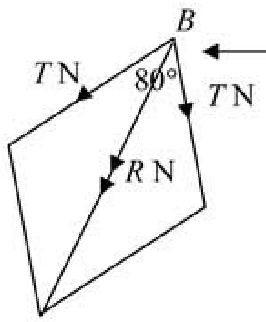
b From (1)

$T = 2g\sin 30° + 2a$

$= 2g\sin 30° + 2 \times 3.565\ \ldots = 16.93\ \ldots$

The tension in the string is 17 N (2 s.f.).

c



At B, the string exerts two forces, each of magnitude T. The resultant force bisects the angle ABC, which is $80°$. Its magnitude can be found by resolving the two forces along the diagonal of the rhombus.

$R = 2T\cos 40° = 2 \times 16.93\ \ldots \times \cos 40° = 25.939\ \ldots$

The resultant force exerted on the pulley by the string has magnitude 26 N (2 s.f.), and acts in the direction bisecting $\angle ABC$, as shown in the diagram above.

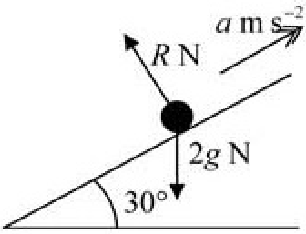
d To find the speed of P immediately before the string breaks

$u = 0, t = 0.75, a = 3.565 \ldots, v = ?$

$v = u + at = 0 + 0.75 \times 3.565 = 2.674 \ldots$

The final speed for the part of the motion when the string is taut will be the initial speed of P after the string breaks.

For P, after the string breaks



$R(\nearrow)$ $\quad -2g\sin 30° = 2a$

$$a = -g\sin 30° = -\frac{1}{2}g$$

After the string has broken, the only force acting on P in the direction parallel to the plane is the component of the weight acting down the plane.

$u = 2.674 \ldots, v = 0, a = -\frac{1}{2}g, t = t_1$

$v = u + at$

$0 = 2.674 \ldots - \frac{1}{2}gt_1$

$\boxed{\sin 30° = \frac{1}{2}}$

$$t_1 = \frac{2 \times 2.674 \ldots}{9.8} = 0.545\,7 \ldots$$

$$= 0.55 \text{ (2 s.f.)}.$$

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Review Exercise
Exercise A, Question 58

Question:

A car of total mass 1200 kg is moving along a straight horizontal road at a speed of 40 m s^{-1}, when the driver makes an emergency stop. When the brakes are fully applied, they exert a constant force and the car comes to rest after travelling a distance of 80 m. The resistance to motion from all factors other than the brakes is assumed to be constant and of magnitude 500 N.

a Find the magnitude of the force by the brakes when fully applied.

A trailer, with no brakes, is now attached to the car by means of a tow-bar. The mass of the trailer is 600 kg, and when the trailer is moving, it experiences a constant resistance to motion of magnitude 420 N. The tow-bar may be assumed to be a light rigid rod which remains parallel to the road during motion. The car and the trailer come to a straight hill, inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{14}$. They move together down the hill. The driver again makes an emergency stop, the brakes applying the same force as when the car was moving along level ground.

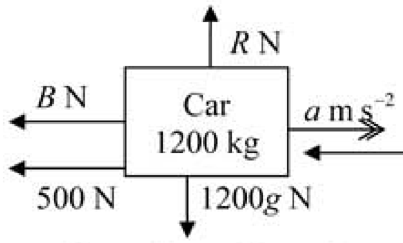
b Find the deceleration of the car and the trailer when the brakes are fully applied.

c Find the magnitude of the force exerted on the car by the trailer when the brakes are fully applied.

d Find the maximum speed at which the car and trailer should travel down the hill to ensure that, when the brakes are fully applied, they can stop within 80m.

Solution:

a



The force exerted by the brakes is labelled as B N in this diagram. During braking both the braking force and the resistance act to slow the car down.

$u = 40,\ v = 0,\ s = 80,\ a = ?$

$v^2 = u^2 + 2as$

$0^2 = 40^2 + 2 \times a \times 80$

$a = -\dfrac{40^2}{2 \times 80} = -10$

To find the braking force, you will have to use $\mathbf{F} = m\mathbf{a}$ and so the first step in the question is to find the deceleration.

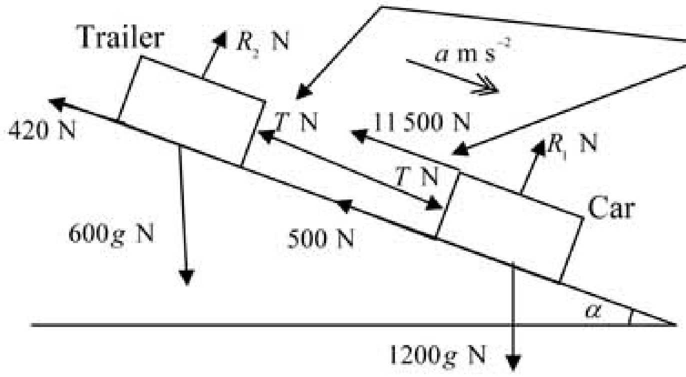
$\mathbf{F} = m\mathbf{a}$

$R(\rightarrow)\quad -B - 500 = 1200 \times (-10)$

$B = 12\,000 - 500 = 11\,500$

The magnitude of the force exerted by the brakes is $11\,500$ N.

b



Under braking, the force in the tow bar is a thrust and acts outwards in the directions shown in the diagram. When Newton's second law is applied to the whole system, the forces at the ends of the tow bar are of equal magnitude and in opposite directions and so they cancel one another out.

For the car and the trailer

$R(\square)$

$$\underbrace{1200g\sin\alpha + 600g\sin\alpha}_{\text{Components of weight}} - \underbrace{500 - 420}_{\text{Resistances}} - \underset{\substack{\uparrow \\ \text{Braking force}}}{11\,500} = \underset{\substack{\nwarrow \\ \text{Total Mass}}}{1800}a$$

$1800 \times 9.8 \times \dfrac{1}{14} - 12\,420 = 1800a$

$1260 - 12\,420 = 1800a$

$a = -\dfrac{11160}{1800} = -6.2$

You are given that $\sin\alpha = \dfrac{1}{14}$ and you should use that exact value here. It is not necessary to use your calculator to find a value for α. Any such value would only be approximate and would lead to inaccuracy.

The deceleration of the car and the trailer is $6.2\ \text{m s}^{-2}$.

c For the car alone

$R(\square\ )$

$T + 1200g \sin\alpha - 500 - 11\,500 = 1200a$

$T = 1200 \times (-6.2) - 1200 \times 9.8 \times \frac{1}{14} + 500 + 11\,500$

$= 3720$

The magnitude of the force exerted on the car by the trailer is 3700 N (2 s.f.).

The trailer exerts a force on the car through the thrust in the tow bar. That thrust acts down the plane in the same direction as the component of the weight. The braking force and the resistance act up the plane

d $a = -6.2,\ v = 0,\ s = 80,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-6.2) \times 80$

$u^2 = 2 \times 6.2 \times 80 = 992$

$u = \sqrt{992} = 31.496\ \ldots$

The maximum speed at which the car and trailer should travel down the hill to ensure that, when the brakes are fully applied, they can stop within 80 m is $31\ \mathrm{m\ s^{-1}}$ (2 s.f.).

If the car and trailer were travelling at a slower speed, they could stop in less than 80 m.

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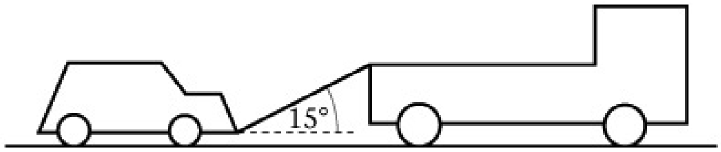
Review Exercise

Exercise A, Question 59

Question:



The figure above shows a lorry of mass 1600 kg towing a car of mass 900 kg along a straight horizontal road. The two vehicles are joined by a light tow-bar which is at an angle of 15° to the road. The lorry and the car experience constant resistances to motion of magnitude 600 N and 300 N respectively. The lorry's engine produces a constant horizontal force on the lorry of magnitude 1500 N. Find

a the acceleration of the lorry and the car,

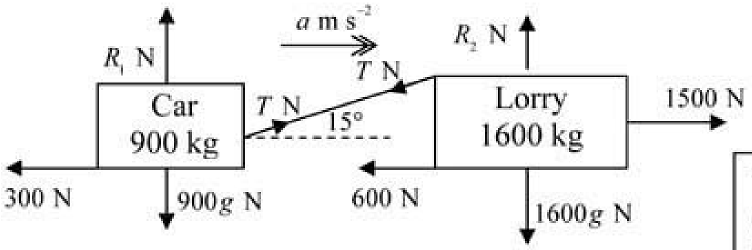
b the tension in the tow-bar.

When the speed of the vehicles is 6 m s^{-1}, the tow-bar breaks. Assuming that the resistance to the motion of the car remains of constant magnitude 300 N,

c find the distance moved by the car from the moment the tow-bar breaks to the moment when the car comes to rest.

d State whether, when the tow-bar breaks, the normal reaction of the road on the car is increased, decreased or remains constant. Give a reason for your answer.

Solution:



When you consider the whole system, the tension in the tow bar acting on the car and tension in the tow bar acting on the lorry are of equal magnitude and in opposite directions. When you resolve in any direction for the whole system, the tensions cancel each other out .

a For the lorry and the car

$R(\rightarrow)\quad 1500-600-300=2500a$

$$a=\frac{600}{2500}=0.24$$

The acceleration of the lorry and the car is 0.24 m s^{-2}.

b For the car alone

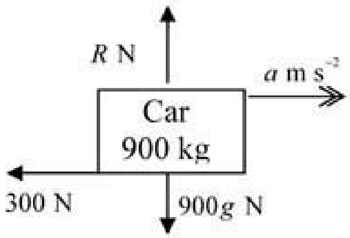
$$\mathbf{F}=m\mathbf{a}$$

$R(\rightarrow)\quad T\cos 15°-300=900\times 0.24$

$$T=\frac{900\times 0.24+300}{\cos 15°}=534.20\ \ldots$$

The tension in the tow bar is 530 N (2 s.f.).

c



After the tow bar breaks, the only force acting on the car in the horizontal direction is the constant resistance to motion.

After the tow bar breakes, for the car

$R(\rightarrow)\quad -300=900a \Rightarrow a=-\frac{1}{3}$

$u=6,\ v=0,\ a=-\frac{1}{3},\ s=?$

$v^2=u^2+2as$

$0^2=6^2-2\times\frac{1}{3}\times s$

$s = \frac{3}{2} \times 36 = 54$

The distance moved by the car from the moment the tow bar breaks to the moment when the car comes to rest is 54 m.

d After the tow bar has broken, in part **c**, R$(\uparrow)$

$$R = 900g.$$

Before the tow bar has broken, in part **a**,

R$(\uparrow)$ for car

$R_1 + T\sin 15° - 900g = 0 \Rightarrow R_1 = 900g - T\sin 15° < 900g$

So the normal reaction of the road on the car is increased when the tow bar breaks.

It is a common error to omit the vertical component of the tension here. That would lead to the incorrect conclusion that the normal reaction is unchanged.

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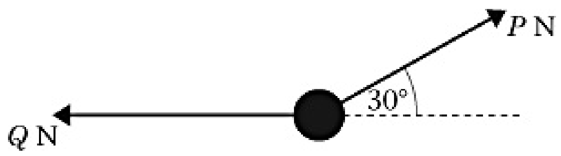
2 Review Exercise
Exercise A, Question 1

Question:



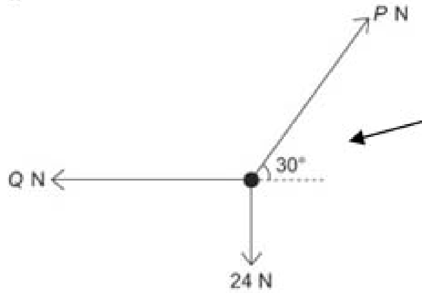
A particle of weight 24 N is held in equilibrium by two light inextensible strings. One string is horizontal. The other string is inclined at an angle of 30° to the horizontal, as shown. The tension in the horizontal string is Q newtons and the tension in the other string is P newtons. Find

a the value of P,

b the value of Q.

Solution:

a



Draw a diagram, showing the weight as well as the tensions in the strings.

R(↑)

$P\sin 30^\circ - 24 = 0$

Resolve vertically first, as there is only one unknown in the resulting equation.

$$\therefore P = \frac{24}{\sin 30^\circ} = 48$$

b R(→)

$$P\cos 30^\circ - Q = 0$$
$$\therefore 48\cos 30^\circ - Q = 0$$

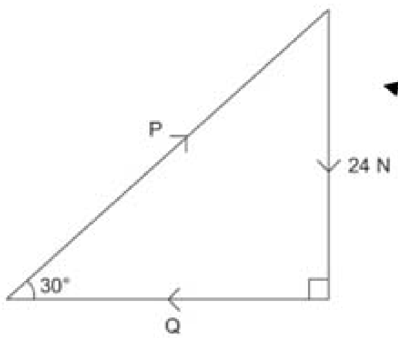
Then resolve horizontally and substitute the value of P, found in **a.**

$$\therefore Q = 48\cos 30^\circ = 41.6 \ (3\text{s.f.})$$

Give your answer to 3 s.f.

Alternative solution using triangle of forces

This is another method of solving problems involving equilibrium under 3 forces.



You can draw a triangle with sides representing each of the three forces.

From the triangle

a

Use trigonometry of the triangle to find P and to find Q.

$$\sin 30^\circ = \frac{24}{P}$$
$$\therefore P = \frac{24}{\sin 30^\circ} = 48$$

b

$$\tan 30^\circ = \frac{24}{Q}$$
$$\therefore Q = \frac{24}{\tan 30^\circ} = 41.6 \ (3\text{s.f.})$$

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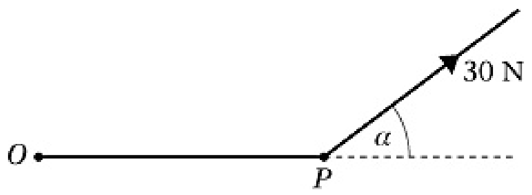
2 Review Exercise
Exercise A, Question 2

Question:



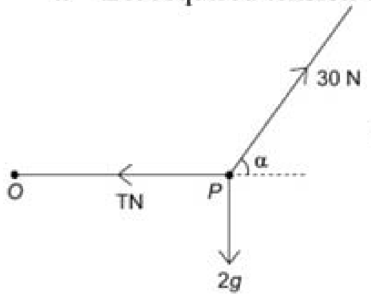
A particle P, of mass 2 kg, is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP horizontal, by a force of magnitude 30 N applied at an angle α to the horizontal, as shown.

a Find, to the nearest degree, the value of α.

b Find, in N to 3 significant figures, the magnitude of the tension in the string.

Solution:

a Let required tension be T N.



Draw a diagram, showing the weight $2g$ N as well as the force 30 N and the tension T N.

R(↑)

$30\sin\alpha - 2g = 0$

Resolve vertically first, as there is only one unknown in the resulting equation.

$$\therefore \sin\alpha = \frac{2g}{30} = 0.65\dot{3}$$

$\therefore \alpha = 40.8^\circ = 41^\circ$

Find arcsin $0.65\dot{3}°$.

b R(→)

$30\cos\alpha - T = 0$

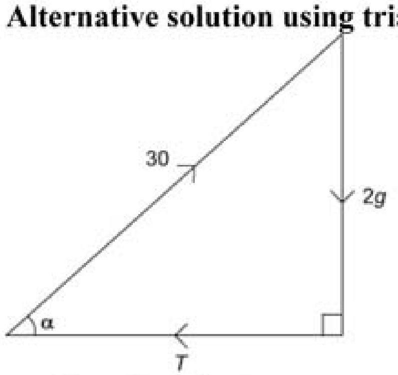
Resolve horizontally and substitute the value of α found in part **a**.

$$\therefore T = 30\cos\alpha = 22.7$$

Tension is 22.7N (3 s.f.).

Give your answer to 3 s.f.

Alternative solution using triangle of forces



You can draw a triangle with sides representing each of the three forces.

a From the triangle

$$\sin\alpha = \frac{2g}{30}$$
$$= 0.65\dot{3}$$
$$\therefore \alpha = 40.8^\circ = 41^\circ$$

Use trigonometry of the triangle to find α.

b

$$30^2 = (2g)^2 + T^2$$
$$\therefore T^2 = 900 - 384.16$$
$$= 515.84$$
$$\therefore T = 22.7\,\text{N}$$

Use Pythagoras' Theorem to find T.

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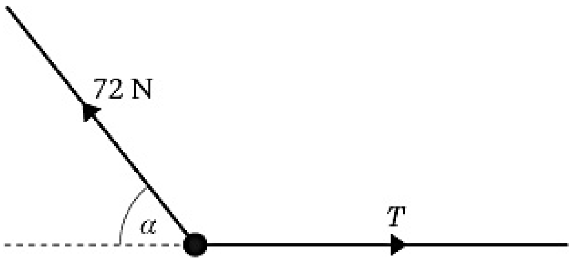
2 Review Exercise
Exercise A, Question 3

Question:



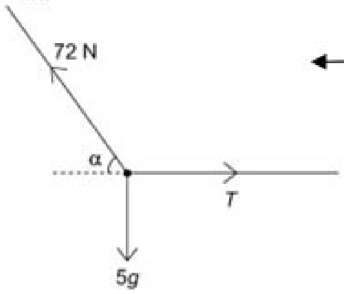
A body of mass 5 kg is held in equilibrium under gravity by two inextensible light ropes. One rope is horizontal, the other is at an angle α to the horizontal, as shown. The tension in the rope inclined at α to the horizontal is 72 N. Find

a the angle α, giving your answer to the nearest degree,

b the tension T in the horizontal rope, giving your answer to the nearest N.

Solution:

a



Draw a diagram, showing the weight 5th g N as well as the forces 72 N and T.

R(↑)

Resolve vertically first as there is only one unknown in the resulting equation.

$$
\begin{aligned}
72\sin\alpha - 5g &= 0 \\
\therefore \sin\alpha &= \frac{5g}{72} \\
&= 0.680\dot{5} \\
\therefore \alpha &= 42.9^\circ \\
&= 43^\circ
\end{aligned}
$$

Find $\arcsin 0.680\dot{5}$.

b R(→)

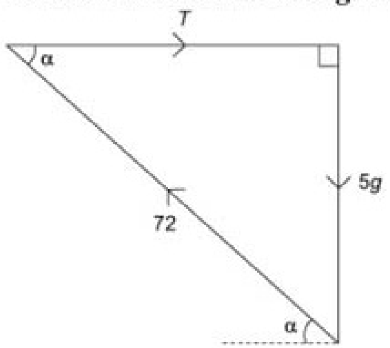
Resolve horizontally and substitute the value of α found in **a**.

$$T - 72\cos\alpha = 0$$

$$
\begin{aligned}
\therefore T &= 72\cos\alpha \\
&= 52.8 \\
\text{tension} &= 53\text{ N (2 s.f.)}
\end{aligned}
$$

Give your answer to the nearest Newton.

Alternative solution using triangle of forces



You can draw a triangle with sides representing each of the three forces.

From the triangle

a

$$\sin\alpha = \frac{5g}{72}$$
$$= 0.680\dot{5}$$
$$\therefore \alpha = 42.9^\circ \text{ (3 s.f.)}$$

Use trigonometry of the triangle to find α and to find T.

b

$$\cos\alpha = \frac{T}{72}$$
$$\therefore T = 72\cos\alpha$$
$$= 53\text{ N (2 s.f.)}$$

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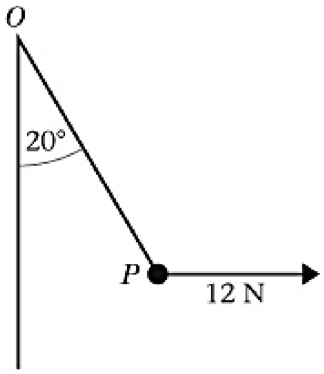
2 Review Exercise
Exercise A, Question 4

Question:



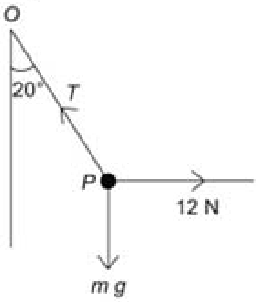
A particle P is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 12 N is applied to P. The particle P is in equilibrium with the string taut and OP making an angle of 20° with the downward vertical, as shown. Find

a the tension in the string,

b the weight of P.

Solution:

a



Draw a diagram, showing the tension T in the string and the weight mg of the particle.

Let T be the tension in the string, and let mg be the weight of P.

R(→)

Resolve horizontally first as there is only one unknown in the resulting equation.

$T \sin 20^\circ - 12 = 0$

$$\therefore T = \frac{12}{\sin 20} = 35.1$$

Tension in the string = 35.1 N (3 s.f.)

Give your answer to 3 s.f.

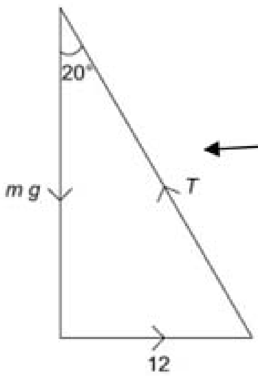
b R(↑)

Then resolve vertically and substitute the value of T found in **a.**

$$T \cos 20^\circ - mg = 0$$
$$\therefore 35.1 \cos 20^\circ = mg$$
$$\therefore \text{weight} = 33.0\text{ N} (3\text{s.f.})$$

Give your answer to 3 s.f.

Alternative solution using triangle of forces



You can draw a triangle with sides representing each of the three forces.

a From the triangle: $\sin 20^\circ = \dfrac{12}{T}$

$\therefore T = \dfrac{12}{\sin 20^\circ} = 35.1 (3\text{s.f.})$

Use trigonometry of the triangle to find T and to find mg.

b

$$\cos 20^\circ = \frac{mg}{T}$$
$$\therefore mg = 35.1 \cos 20^\circ = 33.0 \ (3\text{s.f.})$$

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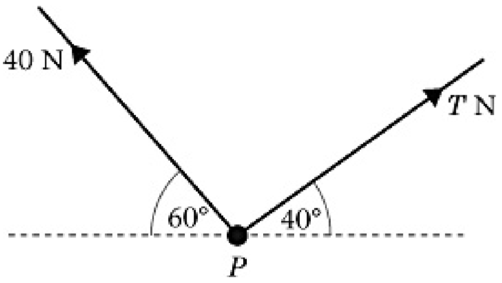
2 Review Exercise
Exercise A, Question 5

Question:



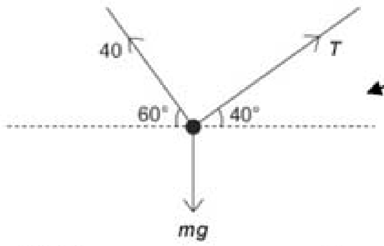
A particle P is held in equilibrium under gravity by two light, inextensible strings. One string is inclined at an angle of 60° to the horizontal and has a tension of 40 N. The other string is inclined at an angle of 40° to the horizontal and has a tension of T newtons, as shown. Find, to three significant figures,

a the value of T,

b the weight of P.

Solution:

a Let the weight of P be mg N.



Draw a diagram showing the tension T in the string and the weight mg of the particle.

R(→)

Resolve horizontally first as there is only one unknown in the resulting equation.

$T\cos 40^\circ - 40\cos 60^\circ = 0$

$$\therefore T = \frac{40\cos 60^\circ}{\cos 40^\circ} = 26.1 \text{ (3 s.f.)}$$

Give your answer to 3 s.f.

b R(↑)

$$40\sin 60^\circ + T\sin 40^\circ - mg = 0$$
$$\therefore 40\sin 60^\circ + 26.1\sin 40^\circ - mg = 0$$

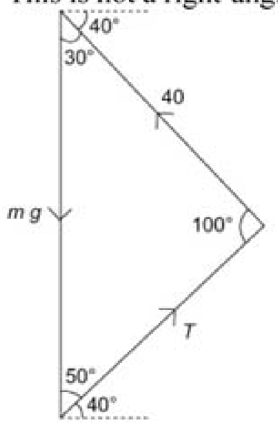
Then resolve vertically and substitute the value of T found in **a**.

$$\therefore mg = 40\sin 60^\circ + 26.1\sin 40^\circ = 51.4 \text{ (3 s.f.)}$$

Give your answer to 3 s.f.

Alternative solution using triangle of forces

This is not a right-angled triangle so use the sine rule.



You can draw a triangle with sides representing each of the three forces.

Calculate the angles in the triangle using $90^\circ - 60^\circ = 30; 90^\circ - 40^\circ = 50^\circ$

and angles of a triangle add up to 180° to give 100 as the third angle.

$$\frac{T}{\sin 30^\circ} = \frac{mg}{\sin 100^\circ} = \frac{40}{\sin 50^\circ}$$

$$\therefore T = \frac{40\sin 30^\circ}{\sin 50^\circ} = 26.1 \text{ (3 s.f.)}$$

$$mg = \frac{40\sin 100^\circ}{\sin 50^\circ} = 51.4 \text{ (3 s.f.)}$$

Use the sine rule to find T and mg.

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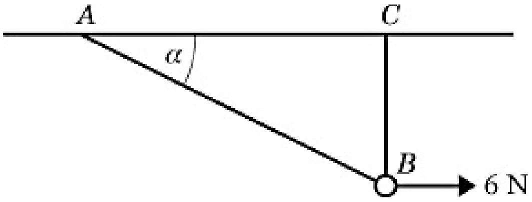
2 Review Exercise
Exercise A, Question 6

Question:



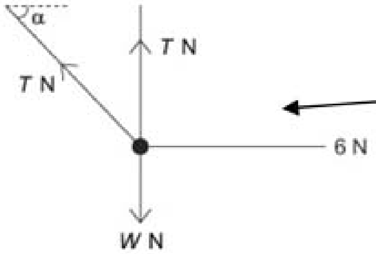
A smooth bead *B* is threaded on a light inextensible string. The ends of the string are attached to two fixed points *A* and *C* on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 6 N acting parallel to *AC*. The bead *B* is vertically below *C* and $\angle BAC = \alpha$, as shown in the diagram. Given that $\tan \alpha = \frac{3}{4}$, find

a the tension in the string,

b the weight of the bead.

Solution:

a



Draw a diagram showing the forces on the bead.
The tension in both parts of the string is the same.

Let the tension in the string be T N and let the weight of the bead be W N.

R(→)

Resolve horizontally first as there is only one unknown in the resulting equation.

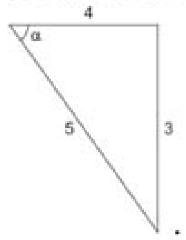
$6 - T\cos\alpha = 0$

$\therefore T \quad = \quad \dfrac{6}{\cos\alpha}$

As $\tan\alpha \; = \; \dfrac{3}{4}, \cos\alpha = \dfrac{4}{5}$

$\therefore T = 6 \div \dfrac{4}{5} = 7.5$

Use this triangle to find $\cos\alpha$ and $\sin\alpha$



b R(↑)

$T + T\sin\alpha - W = 0$

As $\tan\alpha = \dfrac{3}{4}, \sin\alpha = \dfrac{3}{5}$

$\therefore 7.5 + 7.5 \times \dfrac{3}{5} - W = 0$

$\therefore W = 12$

Resolve vertically and substitute the value of T found in part **a**.

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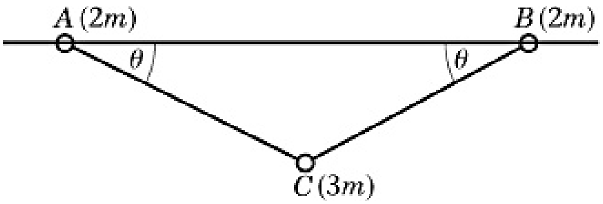
2 Review Exercise
Exercise A, Question 7

Question:



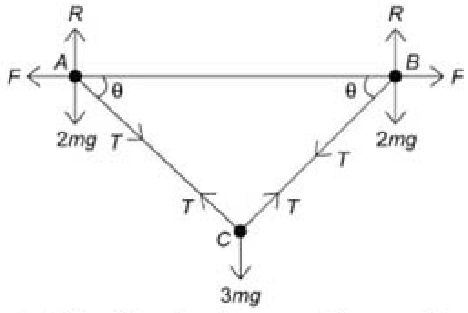
Two small rings, A and B, each of mass $2m$, are threaded on a rough horizontal pole. The coefficient of friction between each ring and the pole is μ. The rings are attached to the ends of a light inextensible string. A smooth ring C, of mass $3m$, is threaded on the string and hangs in equilibrium below the pole. The rings A and B are in limiting equilibrium on the pole, with

$\angle BAC = \angle ABC = \theta$, where $\tan\theta = \frac{3}{4}$, as shown in the diagram.

a Show that the tension in the string is $\frac{5}{2}mg$.

b Find the value of μ.

Solution:



Show all the forces acting on A, B and C.
The normal reactions R and the friction forces F will be the same at A as at B by symmetry.

a Consider the forces acting on the ring C.

Consider ring C first as there is only one unknown force at C.

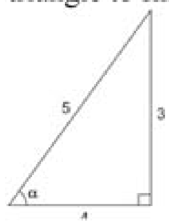
R(↑)

$2T\sin\theta - 3\,mg = 0$

$\therefore T = \dfrac{3\,mg}{2\sin\theta}$

As $\tan\theta = \dfrac{3}{4}, \sin\theta = \dfrac{3}{5}$

As $\tan\theta = \dfrac{3}{4}$, use 3, 4, 5 triangle to find $\sin\theta$ and $\cos\theta$.



$$\therefore T = \frac{3\,mg}{2\times\frac{3}{5}} = \frac{5\,mg}{2}$$

b Consider the forces acting on the ring A.

You could consider ring A or ring B.

R(→)

$T\cos\theta - F = 0$

$\therefore F = \dfrac{5\,mg}{2}\times\dfrac{4}{5} = 2\,mg$

Use the value of T given from part **a** to find F.

R(↑)

$$R - 2\,mg - T\sin\theta = 0$$

$$\therefore R = 2\,mg + \frac{5\,mg}{2}\times\frac{3}{5} = \frac{7\,mg}{2}$$

Use the value of T to find R.

The ring A is in limiting equilibrium

You were told that friction was limiting at A and B.

$$\therefore F = \mu R$$

$$\therefore 2\,mg = \mu\times\frac{7\,mg}{2}$$

$$\therefore \mu = \frac{4}{7}$$

Give μ as a fraction as this is an exact answer.

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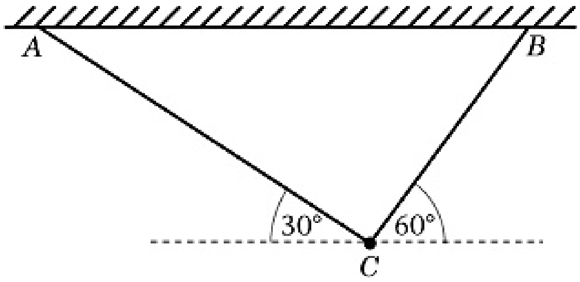
2 Review Exercise
Exercise A, Question 8

Question:

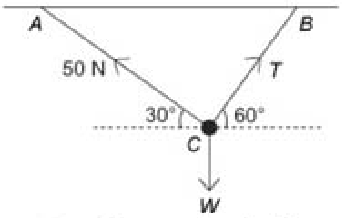


A particle of weight W newtons is attached at C to the ends of two light inextensible strings AC and BC. The other ends of the strings are attached to two fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at 30° and 60° respectively, as shown. Given the tension in AC is 50 N, calculate

a the tension in BC, to three significant figures,

b the value of W.

Solution:



Draw a diagram showing the three forces acting on the particle.

a Let the tension in the string BC be T N.

R(→)

$T\cos 60^\circ - 50\cos 30^\circ = 0$

Resolve horizontally first, as there is only one unknown in the resulting equation.

$$\therefore T = \frac{50\cos 30^\circ}{\cos 60^\circ}$$
$$= 86.6 \text{ (3 s.f.)}$$

Give your answer to 3 s.f.

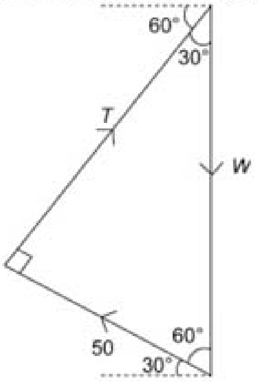
b R(↑)

$50\sin 30^\circ + T\sin 60^\circ - W = 0$

$$\therefore W = 50\sin 30^\circ + T\sin 60^\circ$$
$$= 50\sin 30^\circ + 86.6\sin 60^\circ$$
$$= 100$$

Resolve vertically and substitute the value of T found in part **a**.

Alternative solution using triangle of forces



Calculate the angles of the triangle using $90^\circ - 60^\circ = 30^\circ, 90^\circ - 30^\circ = 60^\circ$ and angles of a triangle add up to 180° to give 90° as the third angle.

From the triangle

$$\tan 60^\circ = \frac{T}{50}$$
$$\therefore T = 50\tan 60^\circ = 86.6\,(3\,\text{s.f.})$$
$$\cos 60^\circ = \frac{50}{W}$$
$$\therefore W = \frac{50}{\cos 60^\circ} = 100$$

Use trigonometry of the right angled triangle to find T and W.

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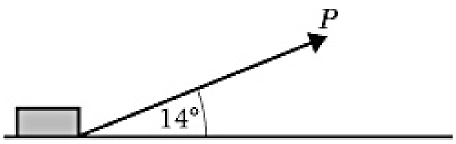
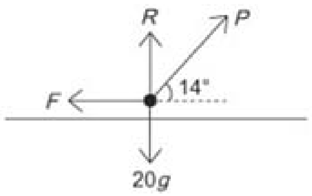
2 Review Exercise
Exercise A, Question 9

Question:



A small box of mass 20 kg rests on a rough horizontal floor. The coefficient of friction between the box and the floor is 0.25. A light inextensible rope is tied to the box and pulled with a force of magnitude P newtons at 14° to the horizontal as shown in the diagram. Given that the box is on the point of sliding, find the value of P, giving your answer to 1 decimal place.

Solution:



Draw a diagram showing the normal reaction R, the friction force F and the weight $20g$.

R(↑)

$R + P\sin 14^\circ - 20g = 0$ (1)

R(→)

$P\cos 14^\circ - F = 0$ (2)

As friction is limiting

$F = 0.25R$ (3)

The box is on the point of sliding and so the friction is limiting.

Substituting $F = 0.25R$ into (2)

$0.25R = P\cos 14^\circ$

Eliminate F and then eliminate R from the simultaneous equations to find P.

From equation (1)

$R = 20g - P\sin 14^\circ$

$\therefore 0.25(20g - P\sin 14^\circ) = P\cos 14^\circ$

Collect the terms in P together and factorise.

$$\therefore 5g = P(\cos 14^\circ + 0.25\sin 14^\circ)$$

$$\therefore P = \frac{5g}{\cos 14^\circ + 0.25\sin 14^\circ} = 47.5 \text{ (1 d.p.)}$$

Make P the subject of the formula.

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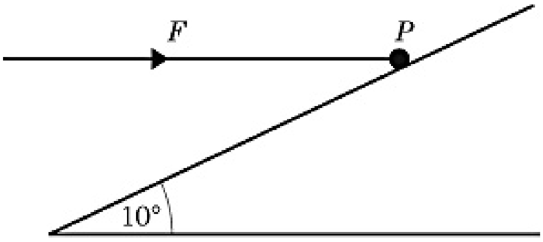
2 Review Exercise
Exercise A, Question 10

Question:



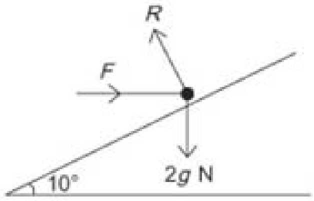
A smooth plane is inclined at an angle 10° to the horizontal. A particle *P* of mass 2 kg is held in equilibrium on the plane by a horizontal force of magnitude *F* newtons, as shown.

Find, to three significant figures,

a the normal reaction exerted by the plane on *P*.

b the value of *F*.

Solution:



Show the normal reaction and the weight $2g$. There is no friction as the plane is smooth.

a R(↑)

$R\cos 10^\circ - 2g = 0$

Resolving vertically eliminates force F and the equation has one unknown R.

$$\therefore R = \frac{2g}{\cos 10^\circ}$$
$$= 19.9 \text{ N (3 s.f.)}$$

b R(↑)

$$F - R\sin 10^\circ = 0$$
$$\therefore F = R\sin 10^\circ$$
$$= 3.46 \text{ (3 s.f.)}$$

Substitute the value of R found in **a**.

Alternative method

b Resolve along the plane

$F\cos 10^\circ - 2g\sin 10^\circ = 0$

Resolving along the plane gives an equation in one unknown F.

$$\therefore F = \frac{2g\sin 10^\circ}{\cos 10^\circ}$$
$$= 3.46 \text{ (3 s.f.)}$$

This is answer to **b**.

a Resolve perpendicular to the plane

$R - F\sin 10^\circ - 2g\cos 10^\circ = 0$

Substitute the value of F to find R.

$$\therefore R = F\sin 10^\circ + 2g\cos 10^\circ$$
$$= 19.9 \text{ (3 s.f.)}$$

This is answer to **a**.

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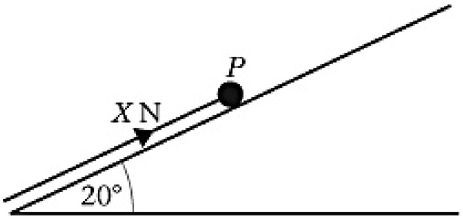
2 Review Exercise
Exercise A, Question 11

Question:



A particle *P* of mass 2.5 kg rests in equilibrium on a rough plane under the action of a force of magnitude *X* newtons acting up a line of greatest slope of the plane, as shown in the diagram. The plane is inclined at 20° to the horizontal. The coefficient of friction between *P* and the plane is 0.4. The particle is in limiting equilibrium and is on the point of moving up the plane. Calculate

a the normal reaction of the plane on *P*,

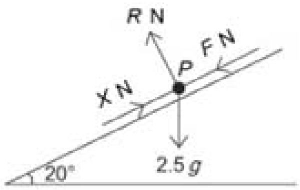
b the value of *X*.

The force of magnitude *X* newtons is now removed.

c Show that *P* remains in equilibrium on the plane.

Solution:

a



Draw a diagram showing the normal reaction R N, the friction force down the plane F N and the weight 2 mg.

R(↖)

$$R - 2.5\,g\cos 20^\circ = 0$$
$$\therefore R = 2.5\,g\cos 20^\circ$$
$$= 23\,(2\,\text{s.f.})$$

Resolve perpendicular to the plane first, as R is the only unknown in the resulting equation.

The normal reaction is 23 N.

Give your answer to 2 s.f. as you used $g = 9.8$ in your calculation.

b R(↗)

$$X - F - 2.5\,g\sin 20^\circ = 0$$
$$\therefore X = F + 2.5\,g\sin 20^\circ \quad *$$

Resolve parallel to the plane to find X in terms of F.

As friction is limiting,

$$F = \mu R$$
$$\text{i.e. } F = 0.4\,R$$

Use limiting friction and the value of R from part **a** to find the force F N.

and as $R = 23.0,\ F = 9.21$

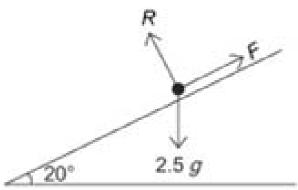
substitute into equation *

$\therefore X = 17.6$

$= 18\,(2\,\text{s.f.})$

Then X may be calculated. Again give X to 2 s.f.

c The force X is removed and the friction force will now act up the plane.



Draw a new diagram with F acting up the plane.

R(↗)

For equilibrium

$F - 2.5\,g\sin 20^\circ = 0$

$\therefore F = 8.38$ (3 s.f.)

Find the value of F which would maintain equilibrium.

R(↖)

$R - 2.5\,g\cos 20^\circ = 0$, as before

Check to see whether $F \le \mu R$.
Use 3 s.f. in your calculation.

$$\therefore R = 23.0$$
$$\mu R = 0.4 \times 23.0 = 9.21$$

This shows that $F < \mu R$, so the friction is not limiting, and equilibrium is maintained.

It is not possible for $F > \mu R$.
If $F \le \mu R$ equilibrium is maintained.

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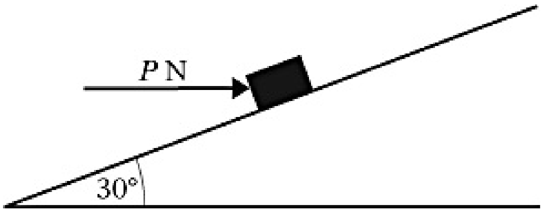
2 Review Exercise
Exercise A, Question 12

Question:



A parcel of weight 10 N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P newtons acts on the parcel, as shown. The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is 18 N. The coefficient of friction between the parcel and the plane is μ. Find

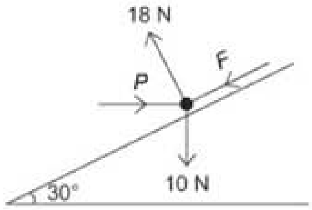
a the value of P,

b the value of μ.

The horizontal force is removed.

c Determine whether or not the parcel moves.

Solution:



Draw a diagram showing the weight, the normal reaction and the friction force F.

F acts down the plane as the parcel is on the point of slipping up the plane.

a R(↖)

$18 - 10\cos 30^\circ - P\sin 30^\circ = 0$
$\therefore P\sin 30^\circ = 18 - 10\cos 30^\circ$

Resolve perpendicular to the plane and make P the subject of the equation.

$$\therefore P = \frac{18 - 10\cos 30^\circ}{\sin 30^\circ}$$
$$= 18.68\ (4\text{s.f.})$$
$$= 18.7\ (3\text{s.f.})$$

Answer is given to 3 s.f. as g is not used in the calculation.

b R(↗)
$P\cos 30^\circ - F - 10\sin 30^\circ = 0$

Resolve along the plane to find F.

$\therefore F = P\cos 30^\circ - 10\sin 30^\circ$
$= 11.2$

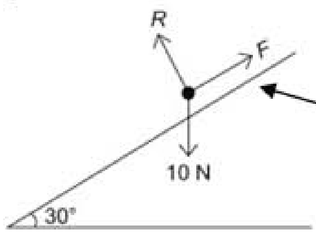
As the friction is limiting,

$$F = \mu R$$
$$\therefore \mu = \frac{11.2}{18}$$
$$= 0.62\ (2\text{ s.f.})$$

Use $\mu = \dfrac{F}{R}$ and give your answer to 2 s.f.

P is now removed.

c



Draw a new diagram.

F now acts up the plane as any motion would be down the plane.

The friction now acts up the plane and the normal reaction will be different. Let the normal reaction be R.
R(↗)

If equilibrium is maintained

$$F - 10\sin 30^\circ = 0$$
$$\therefore F = 10\sin 30^\circ$$
$$= 5$$

Find the force F which would maintain equilibrium.

R(↖)

$$R - 10\cos 30^\circ = 0$$
$$\therefore R = 10\cos 30^\circ$$
$$= 8.66$$

Find the normal reaction force R and use the value of μ found in part **b** to calculate μR.

The value of μ is unchanged

$$\therefore \mu R = 0.599 \times 8.66$$
$$= 5.19 \text{ (3 s.f.)}$$

as $F < \mu R$, equilibrium is maintained, and the parcel does not move

Check that $F \leq \mu R$. As $F < \mu R$ the equilibrium is not limiting.

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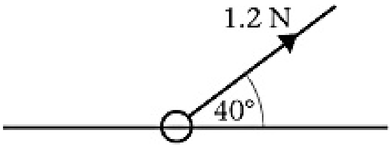
2 Review Exercise
Exercise A, Question 13

Question:

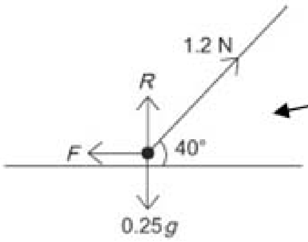


A small ring of mass 0.25 kg is threaded on a fixed rough horizontal rod. The ring is pulled upwards by a light string which makes an angle 40° with the horizontal, as shown. The string and the rod are in the same vertical plane. The tension in the string is 1.2 N and the coefficient of friction between the ring and the rod is μ. Given that the ring is in limiting equilibrium, find

a the normal reaction between the ring and the rod,

b the value of μ.

Solution:



Draw a diagram showing the forces acting on the ring. Include the weight $0.25g$, the normal reaction R and the friction force F.

a R($\uparrow$)

$R + 1.2\sin 40^\circ - 0.25g = 0$

$\therefore R = 0.25g - 1.2\sin 40^\circ$
$= 1.68$

Resolve vertically first as R is the only unknown force in the resulting equation.

$\therefore$ Normal reaction is 1.7 N (2 s.f.)

Give answer to 2 s.f. as $g = 9.8$ has been used in the calculation.

b R($\rightarrow$)

$1.2\cos 40^\circ - F = 0$

$\therefore F = 0.919$

Then resolve horizontally to find F.

as the ring is on limiting equilibrium $F = \mu R$

$\therefore \mu = 0.548$
$= 0.55$ (2 s.f.)

Use $F = \mu R$ to find $\mu = \dfrac{F}{R}$. Give final answer to 2 s.f. but use values of F and K to 3 s.f. in calculation of μ.

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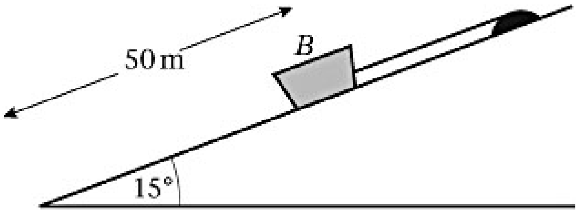
2 Review Exercise
Exercise A, Question 14

Question:



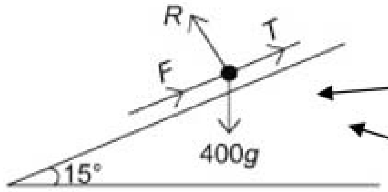
The diagram shows a boat *B* of mass 400 kg held at rest on a slipway by a rope. The boat is modelled as a particle and the slipway as a rough plane inclined at 15° to the horizontal. The coefficient of friction between *B* and the slipway is 0.2. The rope is modelled as a light, inextensible string, parallel to a line of greatest slope of the plane. The boat is in equilibrium and on the point of sliding down the slipway.

a Calculate the tension in the rope.

The boat is 50 m from the bottom of the slipway. The rope is detached from the boat and the boat slides down the slipway.

b Calculate the time taken for the boat to slide to the bottom of the slipway.

Solution:



Draw a diagram showing the forces acting.

The friction acts up the plane, as the motion would be down the plane.

Let T be the tension, F the friction and R the normal reaction.

a R(↖)

$$R - 400\,g\cos 15^\circ = 0$$
$$\therefore R = 400\,g\cos 15^\circ$$
$$= 3786$$

First find force R.

As friction is limiting, $F = \mu R$

$$\therefore \text{F} = 0.2 \times 3786$$
$$= 757$$

Use $F = \mu R$ to find F.

R(↗)

$$T + F - 400\,g\sin 15^\circ = 0$$

Then resolve along the plane to find T.

$$\therefore T = 400\,g\sin 15^\circ - F$$
$$= 257.3 = 257\ (3\text{ s.f.})\ \text{or}\ 260\,(2\text{ s.f.})$$

b When the rope is detached, the resultant force is 257 N acting down the plane.

The forces were in equilibrium, when T is removed the resultant force is equal and opposite to T.

Using Newton's Second Law $F = ma$,

$$257 = 400\,a$$
$$\therefore a = \frac{257}{400} = 0.643$$

Find the constant acceleration a, then use one of the constant acceleration formulae to find t.

Consider the motion down the plane.
$u = 0, a = 0.643, s = 50, t = ?$
Use

Choose this equation as it relates u, a, s and t.

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
\therefore 50 &= 0 + \frac{1}{2} \times 0.643t^2 \\
\therefore t^2 &= \frac{2 \times 50}{0.643} \\
&= 155.5\ldots \\
\therefore t &= 12.5 \text{ (3 s.f.) or } t = 12 \text{ (1 s.f.)}
\end{aligned}
$$

Make t^2 the subject of the equation, then square root to obtain t.

Time to slide down is 12.5 s.

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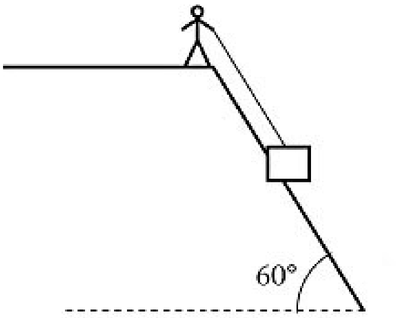
2 Review Exercise
Exercise A, Question 15

Question:



A heavy package is held in equilibrium on a slope by a rope. The package is attached to one end of the rope, the other end being held by a man standing at the top of the slope. The package is modelled as a particle of mass 20 kg. The slope is modelled as a rough plane inclined at 60° to the horizontal and the rope as a light inextensible string. The string is assumed to be parallel to a line of greatest slope of the plane, as shown in the diagram. At the contact between the package and the slope, the coefficient of friction is 0.4.

a Find the minimum tension in the rope for the package to stay in equilibrium on the slope.

The man now pulls the package up the slope. Given that the package moves at constant speed,

b find the tension in the rope.

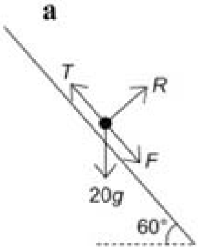
c State how you have used, in your answer to part **b**, the fact that the package moves

i up the slope,

ii at constant speed.

Solution:

a



Draw a diagram, showing the forces acting.

The friction is directed up the plane to prevent slipping down the plane.

Let T be the minimum tension, F the force of friction and R the normal reaction.

R(↗)

$$R - 20g\cos 60^\circ = 0$$
$$\therefore R = 20g\cos 60^\circ$$
$$= 98$$

Resolve perpendicular to the plane first as the resulting equation has only one unknown.

As the friction is limiting, $F = \mu R$.

$$\therefore F = 0.4 \times 98$$
$$= 39.2$$

When the tension is a minimum the friction is limiting.

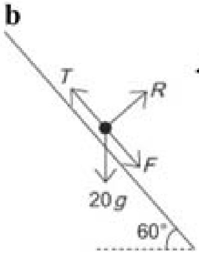
R(↖)

$$T + F - 20g\sin 60^\circ = 0$$
$$\therefore T = 20g\sin 60^\circ - F$$
$$= 130.5$$

Resolve along the plane and substitute the value for F, to give T.

Tension $= 131$ N (3 s.f.) or 130 N (2 s.f.)

b



Draw a new diagram with F directed down the plane.

R(↗)
$R = 98$ as before

As the 'particle' is moving, friction is limiting, so $F = 39.2$ as before.

Use $F = \mu R$ with μ and R the same as in part **a**.

R(↖)
$T - F - 20g\sin 60^\circ = 0$
$\therefore T = 39.2 + 20g\sin 60^\circ$

Tension $= 209\text{ N}\,(3\text{s.f.})$ or $210\text{ N}\,(2\text{s.f.})$

N.B. for $131 < T < 209$ the package is in equilibrium.

c
i friction acts down the slope and has magnitude $0.4R$

Friction opposes motion.

ii there is no acceleration so the net force on the package is zero

This is a result of Newton's First and Second Laws.

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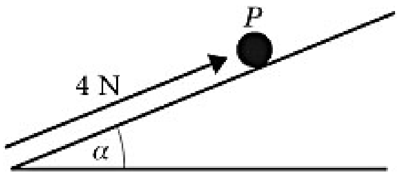
2 Review Exercise
Exercise A, Question 16

Question:



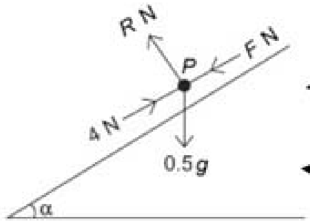
A particle P of mass 0.5 kg is on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The particle is held at rest on the plane by the action of a force of magnitude 4 N acting up the plane in a direction parallel to a line of greatest slope of the plane, as shown. The particle is on the point of slipping up the plane.

a Find the coefficient of friction between P and the plane.

The force of magnitude 4 N is removed.

b Find the acceleration of P down the plane.

Solution:



Draw a diagram showing the forces acting on P.

Friction acts down the plane to oppose slipping up the plane.

a Let the normal reaction be R N, and the friction be F N.

R(↖)

$$R - 0.5\,g\cos\alpha = 0$$
$$\therefore R = 0.5\,g \times \frac{4}{5}$$
$$= 3.92\,(3\text{s.f.})$$

Find R by resolving perpendicular to the plane.

R(↗)

$$4 - F - 0.5\,g\sin\alpha = 0$$
$$\therefore F = 4 - 0.5\,g\sin\alpha$$
$$= 1.06\ (3\text{s.f.})$$

Find F by resolving parallel to the plane.

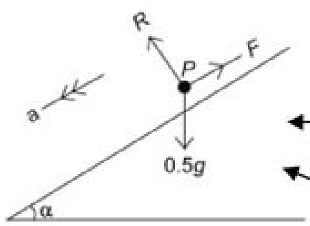
Use $F = \mu R$, as the friction is limiting.

$$\therefore \mu = \frac{F}{R} = \frac{1.06}{3.92}$$
$$\therefore \mu = 0.270\ (3\text{s.f.})$$

As the particle is on the point of slipping, $F = \mu R$.

b The force of 4 N is removed.

The friction will now act up the plane



Friction acts up the plane, in the opposite direction to the motion.

The forces perpendicular to the plane are unaltered and so R is unchanged.

As before: $R = 3.92$ and

$$F = \mu R$$
$$= 1.06$$

R(↗)

Also μ and therefore F are unchanged.

$0.5\,g\sin\alpha - F = 0.5\,a$

use Newton's Second Law along the plane $F = ma$.

∴

$$\therefore a = \left(0.5\,g \times \frac{3}{5} - 1.06\right) \div 0.5$$
$$= 3.76 \text{ (3s.f.)}$$

Rearrange the equation to find a.

∴ Acceleration is 3.76 m s^{-2} (3 s.f.) down the plane
or 3.8 m s^{-2} (2 s.f.).

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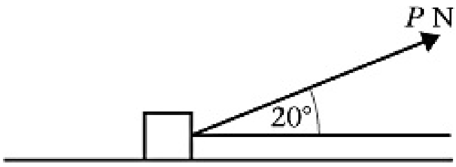
2 Review Exercise
Exercise A, Question 17

Question:



A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of 20° with the ground, as shown. The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is P newtons.

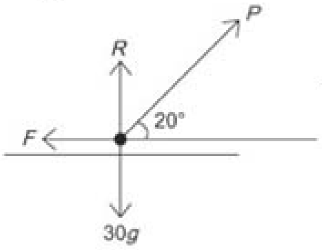
a Find the value of P.

The tension in the rope is now increased to 150 N.

b Find the acceleration of the box.

Solution:

a



Draw a diagram showing the forces acting on the box.

As the box is moving $F = \mu R$
i.e. $F = 0.4\,R \qquad (1)$

The friction is limiting.

R(↑)

Resolve vertically and horizontally to give two equations.

$$R + P\sin 20^\circ - 30\,g = 0$$

$$\therefore R = 30\,g - P\sin 20^\circ \qquad (2)$$

R(→)

$$P\cos 20^\circ - F = 0 \qquad (3)$$

$$\therefore F = P\cos 20^\circ \qquad (3)$$

As the speed is constant, the resultant force is zero

Substitute R from equation (2) and F from equation (3) into equation (1).

Then $P\cos 20^\circ = 0.4(30\,g - P\sin 20^\circ)$

$$\therefore P\cos 20^\circ + 0.4\,P\sin 20^\circ = 0.4\times 30\,g$$

$$\therefore P(\cos 20^\circ + 0.4\sin 20^\circ) = 0.4\times 30\,g$$

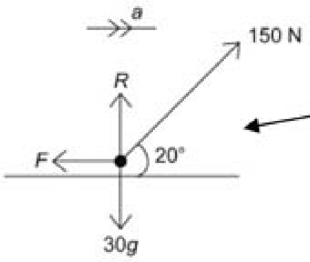
$$\therefore P = \frac{12\,g}{(\cos 20^\circ + 0.4\sin 20^\circ)}$$

$$= 125.3$$

$$= 125\,(3\text{s.f.}) \text{ or } 130\,(2\text{s.f.})$$

Solve simultaneous equations to find the value of P.

b



Draw a new diagram.

Again $F = 0.4\,R$

R(↑)

$R + 150\sin 20^\circ - 30\,g = 0$

$$\begin{aligned} \therefore R &= 30\,g - 150\sin 20^\circ \\ &= 242.7 \\ \text{As } F &= 0.4\,R \\ F &= 97.1 \end{aligned}$$

Resolve vertically to find R.

Use limiting equilibrium to find F.

R(→)

$150\cos 20^\circ - 97.1 = 30\,a$

$$\begin{aligned} \therefore 30\,a &= 43.88 \\ \text{i.e. } a &= 1.46 \end{aligned}$$

$\therefore$ acceleration of the box is 1.46 m s^{-2}

Use Newton's Second Law $F = ma$ along the plane.

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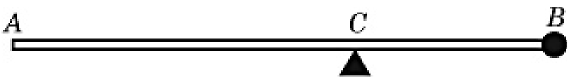
2 Review Exercise
Exercise A, Question 18

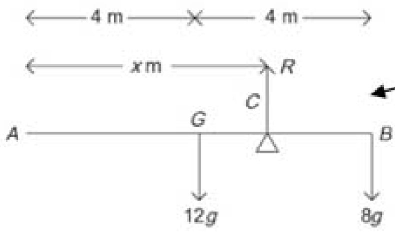
Question:



A uniform rod AB has length 8 m and mass 12 kg. A particle of mass 8 kg is attached to the rod at B. The rod is supported at a point C and is in equilibrium in a horizontal position, as shown.

Find the length of AC.

Solution:



Draw a diagram showing weights and distances.

Let the length $AC = x$ m.
Let G be the mid-point of AB.

The distance $GC = (x-4)$ m
The distance $BC = (8-x)$ m.

You will need these distances in the moments equation.

As the beam is uniform the weight $12g$ acts through G.

Take moments about C.

Anticlockwise moment $=$
$12\,g(x-4)$

Use moment $=$ force $\times$ distance.

Clockwise moment $=$
$8\,g(8-x)$

$$\therefore 12\,g(x-4) = 8\,g(8-x)$$

Use anticlockwise moment $=$ clockwise moment.

$$\therefore 12\,g\,x - 48\,g = 64\,g - 8\,g\,x$$

$$\therefore 20\,g\,x = 112\,g$$

$$\therefore x = \frac{112}{20} = 5.6$$

Make x the subject of the formula.

$\therefore$ The length AC is 5.6 m.

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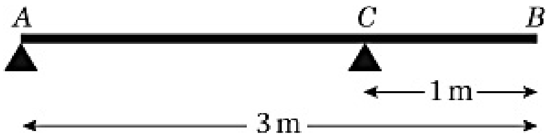
2 Review Exercise
Exercise A, Question 19

Question:



A uniform beam *AB* has mass 12 kg and length 3 m. The beam rests in equilibrium in a horizontal position, resting on two smooth supports. One support is at end *A*, the other at a point *C* on the beam, where *BC* = 1 m, as shown in the diagram. The beam is modelled as a uniform rod.

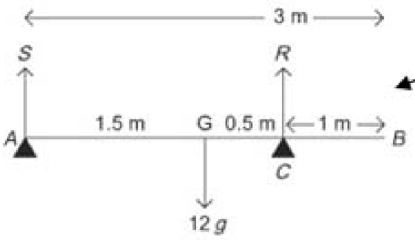
a Find the reaction on the beam at *C*.

A woman of mass 48 kg stands on the beam at the point *D*. The beam remains in equilibrium. The reactions on the beam at *A* and *C* are now equal.

b Find the distance *AD*.

Solution:

a



Draw a diagram showing the reactions at the supports and the weight of the beam.

As the beam is uniform the weight acts through G, the mid-point of AB.
Let the reaction at A be S N and the reaction at C be R N.

Take moments about A.

Distance $AC = 3\text{ m} - 1\text{ m} = 2\text{ m}$

The weight of the beam acts at its mid-point G.
$AG = \frac{1}{2} \times 3\text{ m} = 1\frac{1}{2}\text{ m}$

$$R \times 2 = 12\,g \times 1.5$$

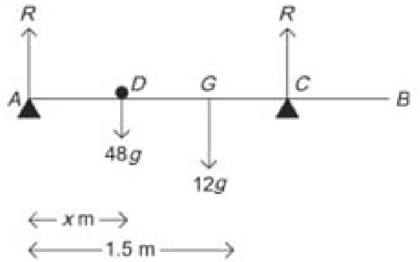
$$R = \frac{12\,g \times 1.5}{2}$$

$$\therefore R = 88.2\text{ N (3 s.f.)}$$

Use anticlockwise moment = clockwise moment.

Make R the subject of the equation.

b



Draw a new diagram showing the forces acting and the distances required.

Let the distance AD be x m and let the reactions at A and C be R N.

R(↑)

$$R + R - 12\,g - 48\,g = 0$$

$$\therefore R = 30\,g$$

Resolve to find the new value of R.

Take moments about A.
$12\,g \times 1.5 + 48\,g \times x - 30\,g \times 2 = 0$

Use clockwise moment = anticlockwise moment.

$$\therefore 48\,gx = 60\,g - 18\,g$$

$$= 42\,g$$

$$\therefore x = \frac{42}{48} = 0.875\text{ m}$$

Make x the subject of the formula.

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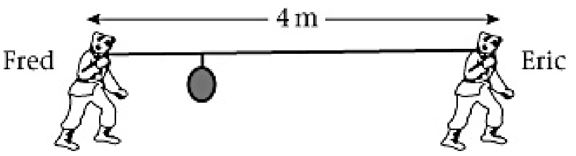
2 Review Exercise
Exercise A, Question 20

Question:

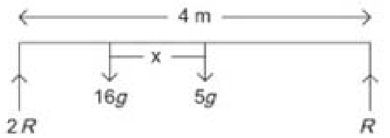


Two men, Eric and Fred, set out to carry a water container across a desert, using a long uniform pole. The length of the pole is 4 m and its mass is 5 kg. The ends of the pole rest in equilibrium on the shoulders of the two men, with the pole horizontal. The water container has mass 16 kg and is suspended from the pole by means of a light rope, which is short enough to prevent the container reaching the ground, as shown. Eric has a sprained ankle, so Fred fixes the rope in such a way that the vertical force on his shoulder is twice as great as the vertical force on Eric's shoulder.

a Find the vertical force on Eric's shoulder.

b Find the distance from the centre of the pole to the point at which the rope is fixed.

Solution:



Draw a diagram showing the forces acting.

Let the force on Eric's shoulder be R and the force on Fred's shoulder be $2R$.

R(↑)

$3R - 16g - 5g = 0$

$\therefore 3R = 21g$

$R = 7g$

Resolve vertically to find R.

Take moments about Fred's shoulder.
Let distance from centre of pole to point at which rope is fixed $= x$ m.
Then

$5g \times 2 + 16g \times (2 - x) = R \times 4$

$\therefore 10g + 32g - 16gx = 28g$

$\therefore 16gx = 14g$

$\therefore x = \frac{7}{8}$ m

As the pole is uniform the weight acts at the middle i.e. 2 m from left hand end (Fred's shoulder).

Use clockwise moment = anticlockwise moment

$(2 - x)$ is distance from Fred's shoulder to weight $16g$.

Make x the subject of the formula.

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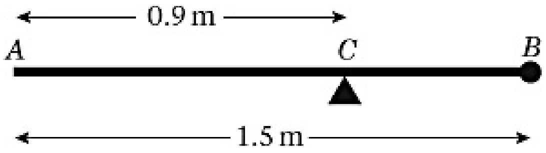
2 Review Exercise
Exercise A, Question 21

Question:



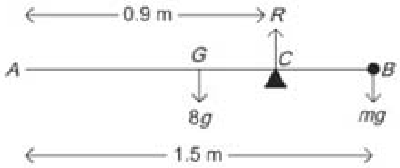
A uniform rod *AB* has length 1.5 m and mass 8 kg. A particle of mass *m* kg is attached to the rod at *B*. The rod is supported at the point *C*, where $AC = 0.9$ m, and the system is in equilibrium with *AB* horizontal, as shown.

a Show that $m = 2$.

A particle of mass 5 kg is now attached to the rod at *A* and the support is moved from *C* to a point *D* of the rod. The system, including both particles, is again in equilibrium with *AB* horizontal.

b Find the distance *AD*.

Solution:



Draw a diagram, showing the forces and distances given.

a The weight $8g$ N acts at the mid-point of AB.

$$\begin{aligned} \text{So } AG &= 0.75\text{ m} \\ \therefore GC &= 0.9\text{ m} - 0.75\text{ m} = 0.15\text{ m} \\ \text{Also } CB &= 0.75\text{ m} - 0.15\text{ m} = 0.6\text{ m} \\ \text{or } CB &= 1.5\text{ m} - 0.9\text{ m} = 0.6\text{ m} \end{aligned}$$

Calculate the other lengths needed in the moments equation.

Take moments about C.

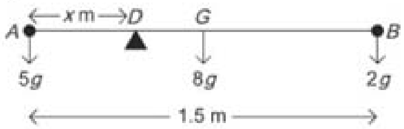
$mg \times 0.6 = 8\,g \times 0.15$

$$\therefore m = \frac{8 \times 0.15}{0.6}$$

$$\therefore m = 2$$

Use clockwise moment = anticlockwise moment.

b



Draw a new diagram to illustrate the new situation.

Let the distance $AD = x$ m.

$$\begin{aligned} \text{Then } DG &= (0.75 - x)\text{ m} \\ \text{and } BD &= (1.5 - x)\text{ m} \end{aligned}$$

Express distances from D in terms of x.

Take moments about D.

Use clockwise moment = anticlockwise moment.

$$2\,g \times (1.5 - x) + 8\,g \times (0.75 - x) = 5\,gx$$

$$\therefore 3\,g - 2\,gx + 6\,g - 8\,gx - 5\,gx = 0$$

$$\therefore 9\,g = 15\,gx$$

$$\therefore x = \frac{9}{15} = 0.6\text{ m}$$

Make x the subject of the formula.

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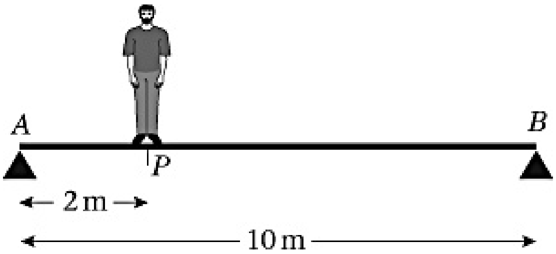
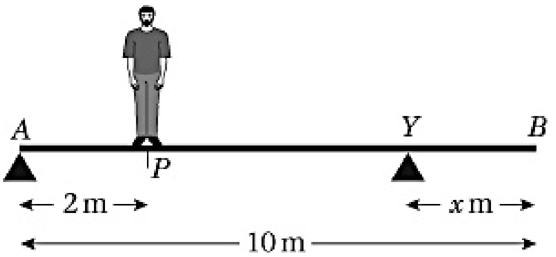
2 Review Exercise
Exercise A, Question 22

Question:



A uniform steel girder *AB*, of mass 150 kg and length 10 m, rests horizontally on two supports at *A* and *B*. A man of mass 90 kg stands on the girder at the point *P*, where $AP = 2$ m, as shown. By modelling the girder as a uniform rod and the man as a particle.

a find the magnitude of the reaction at *B*.



The support *B* is moved to a point *Y* on the girder, where $BY = x$ metres, as shown. The man remains on the girder at *P*. The magnitudes of the reactions at the two supports are now equal.

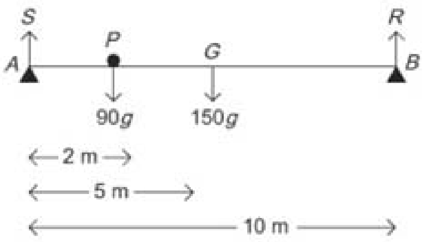
Find

b the magnitude of the reaction at each support,

c the value of *x*.

Solution:

a



Draw a diagram showing the weights acting, the reaction at R and distances.

Let G be the mid-point of AB. As the girder is uniform the weight acts through G.
Let the reaction at B be R N.

The reaction at A is not needed in this part of the question.

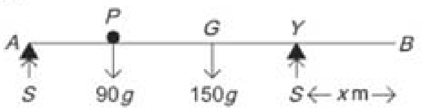
Take moments about A.

$$R\times 10-150\,g\times 5-90\,g\times 2=0$$

$$\begin{aligned}\therefore 10\,R &= 750\,g+180\,g\\ \therefore R &= 930\,g\div 10\\ &= 93\,g\\ &= 911\text{ N}\,(3\text{ s.f.})\\ &\text{or } 910\text{ N }(2\text{ s.f.})\end{aligned}$$

Make R the subject of the equation.

b



Draw a new diagram showing the weights acting and the two equal reactions.

Let the reactions at A and Y be S N.

$$\text{R}(\uparrow)$$

$$\begin{aligned}2\,S-90\,g-150\,g &= 0\\ \therefore 2\,S=240\,g\Rightarrow S &= 120\,g=1176\text{ N}\\ \therefore\quad S &= 1200\text{ N }(2\text{ s.f.})\end{aligned}$$

Resolve to find S.

c Take moments about A.

Take moments about either point A or point Y.

$$\begin{aligned}S\times(10-x) &= 150\,g\times 5-90\,g\times 2\\ \therefore 10-x &= \frac{750\,g+180\,g}{S}=\frac{930\,g}{120\,g}\\ \therefore x &= 10-\frac{93}{12}\\ &= 2.25\text{ m}\end{aligned}$$

Make x the subject of the formula.

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2 Review Exercise
Exercise A, Question 23

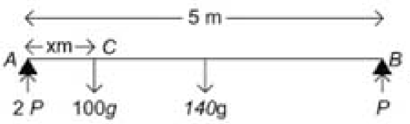
Question:

A footbridge across a stream consists of a uniform horizontal plank *AB* of length 5 m and mass 140 kg, supported at the ends *A* and *B*.
A man of mass 100 kg is standing at a point *C* on the footbridge. Given that the magnitude of the force exerted by the support at *A* is twice the magnitude of the force exerted by the support at *B*, calculate

a the magnitude, in N, of the force exerted by the support at *B*,

b the distance *AC*.

Solution:



Draw a diagram showing forces and distances.

a R(↑)

$2P + P - 140g - 100g = 0$

Let the reaction at B be P and at A be $2P$.
Let the distance $AC = x$ m.

$$\begin{aligned} \therefore 3P &= 240g \\ \text{i.e. } P &= 80g \\ &= 784 \text{ N} \\ &= 780 \text{ N (2 s.f.)} \end{aligned}$$

Resolve vertically to find P.

b Let distance AC be x m.

Take moments about A (or about B) to find the distance x.

Take moments about A.
$P \times 5 = 140g \times 2.5 + 100gx$

Use total anticlockwise moment = clockwise moment.

Substituting $P = 784$
$784 \times 5 - 140 \times 9.8 \times 2.5 = 100 \times 9.8x$
$\therefore x = 0.5$
The distance AC is 0.5 m.

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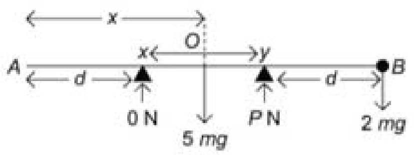
2 Review Exercise
Exercise A, Question 24

Question:

A non-uniform thin straight rod AB has length $3d$ and mass $5m$. It is in equilibrium resting horizontally on supports at the points X and Y, where $AX = XY = YB = d$. A particle of mass $2m$ is attached to the rod at B. Given that the rod is on the point of tilting about Y, find the distance of the centre of mass of the rod from A.

Solution:



Let the required distance be x m.

When the rod begins to tilt about Y the force exerted by the support at $X = 0$ N.

This is an important fact which you should know.

Take moments about Y.

$$5\,mg(2d - x) = 2\,mg\,d$$
$$\therefore 10\,mgd - 5\,mg\,x = 2\,mg\,d$$
$$\therefore 10\,mgd - 2\,mg\,d = 5\,mg\,x$$

Total of the moments anti-clockwise = total of the moments clockwise.

$$\text{i.e. } 8\,mgd = 5\,mgx$$
$$\therefore x = \frac{8}{5}d = 1.6\,d$$

Make x the subject of the formula.

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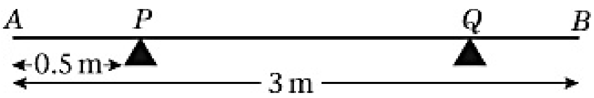
2 Review Exercise
Exercise A, Question 25

Question:

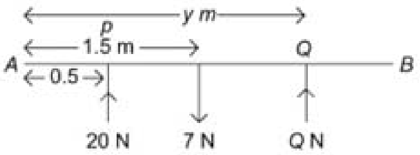


A uniform rod *AB* has weight 70 N and length 3 m. It rests in a horizontal position on two smooth supports placed at *P* and *Q*, where $AP = 0.5$ m as shown in the diagram. The reaction on the rod at *P* has magnitude 20 N. Find

a the magnitude of the reaction on the rod at *Q*,

b the distance *AQ*.

Solution:



Let the reaction at Q be Q N and let the distance $AQ = y$ m.

a R$(\uparrow)$

$20 + Q - 70 = 0$

$$\therefore Q = 70 - 20 = 50$$

Resolve to find Q.

$\therefore$ The reaction at Q is 50 N.

b Take moments about P.

$70 \times 1 = Q(y - 0.5)$

Total clockwise moment = total anticlockwise moment.

But $Q = 50$

$$\therefore 50(y - 0.5) = 70$$
$$\text{i.e. } 50y = 95$$
$$\therefore y = 1.9$$

Substitute Q and make y the subject of the formula.

The distance AQ is 1.9 m.

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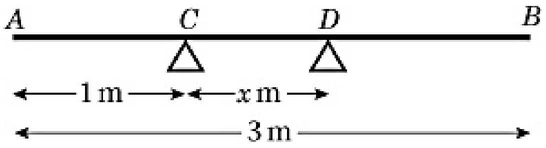
2 Review Exercise
Exercise A, Question 26

Question:



A uniform plank AB has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where AC = 1 m and CD = x m, as shown. The reaction of the support on the plank at D has magnitude 80 N. Modelling the plank as a rod.

a show that x = 0.75.

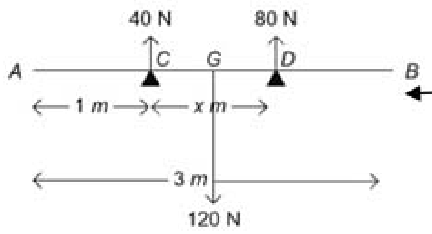
A rock is now placed at B and the plank is on the point of tilting about D. Modelling the rock as a particle, find

b the weight of the rock,

c the magnitude of the reaction of the support on the plank at D.

d State how you have used the model of the rock as a particle.

Solution:



Draw a diagram showing weights, distances and reactions relevant to the problem.

The reaction at C is not relevant to the problem because we are taking moments about C.

a Let G be the mid-point of AB.
The weight of the plank acts through G as the plank is modelled as a uniform rod.

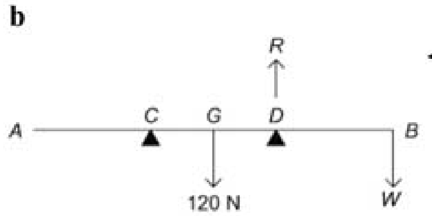
Take moments about C.

The weight of the plane acts at the mid-point.

$$80\,x = 120 \times 0.5$$
$$\therefore x = \frac{120 \times 0.5}{80}$$
$$\therefore x = 0.75$$

Make x the subject of the equation.

b



Draw a new diagram, showing the rock of weight W at point B.

When the plank is on the point of tilting about D, the reaction at C is zero.

Take moments about D.

Take moments about D as the resulting equation has one unknown W.

$$120 \times 0.25 = W \times 1.25$$
$$\therefore W = \frac{120 \times 0.25}{1.25} = 24\text{ N}$$

Distance $DB = CB - CB = 2\text{m} - 0.75\text{ m}$

c $R(\uparrow)$

Resolve vertically or take moments about point C to find R.

$$R - 120 - W = 0$$
$$\therefore R = 120 + 24 = 144\text{ N}$$

d The weight of the rock acts precisely at B.

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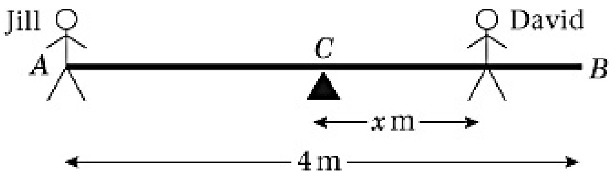
2 Review Exercise
Exercise A, Question 27

Question:



A seesaw in a playground consists of a beam *AB* of length 4 m which is supported by a smooth pivot at its centre *C*. Jill has mass 25 kg and sits on the end *A*. David has mass 40 kg and sits at a distance *x* metres from *C*, as shown. The beam is initially modelled as a uniform rod. Using this model,

a find the value of *x* for which the seesaw can rest in equilibrium in a horizontal position.

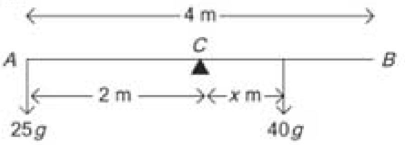
b State what is implied by the modelling assumptions that the beam is uniform.

David realises that the beam is not uniform as he finds he must sit at a distance 1.4 m from *C* for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 15 kg. Using this model,

c find the distance of the centre of mass of the beam from *C*.

Solution:

a



Draw a diagram, showing weights and distances.

Take moments about C.

As C is the mid-point the distance $AC = 2$ m.

$$25\,g \times 2 = 40\,g \times x$$

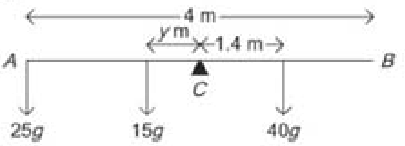
$$\therefore x = \frac{50\,g}{40\,g}$$

$$x = 1.25$$

b The weight of the beam acts through its mid-point at C.

The weight has zero moment about C.

c



Draw a new diagram showing weights and distances.

Let the distance of the centre of mass from C be y m.
Take moments about C.
$25\,g \times 2 + 15\,g \times y = 40\,g \times 1.4$

Use total anticlockwise moment = total clockwise moment.

$$\therefore 15\,gy = 56\,g - 50\,g$$

$$\therefore y = \frac{6\,g}{15\,g} = 0.4$$

Make y the subject of the formula.

$\therefore$ Distance of centre of mass from C is 0.4 m.

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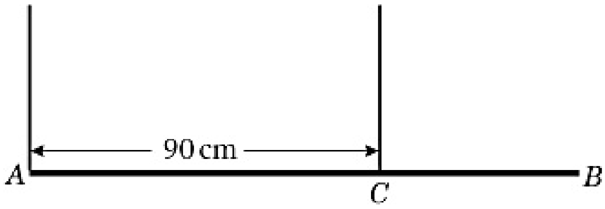
2 Review Exercise
Exercise A, Question 28

Question:



A steel girder AB has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end A. The other cable is attached to the point C on the girder, where $AC = 90$ cm, as shown. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

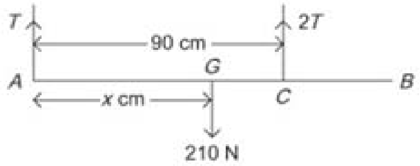
a the tension in the cable at A,

b show that $AB = 120$ cm.

A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.

c Find the value of W.

Solution:



Draw a diagram showing forces and distances.

Let the weight act through G at x cm from A.

a Let the tension in the cable at A be T N and at C be $2T$ N.

$R(\uparrow)$

$$2T + T - 210 = 0 \Rightarrow 3T - 210 = 0$$
$$\therefore T = 70 \text{ N}$$

Resolve vertically as T is the only unknown in this equation.

b Take moments about A.

It is also valid to take moments about C.

$$2T \times 90 = 210 \times x$$
$$\therefore x = \frac{180T}{210}$$

Express x in terms of T and use the value of T found in part **a** to give x.

Substitute $T = 70$

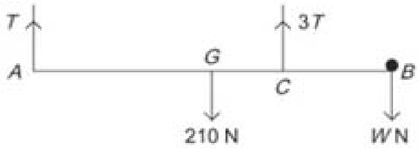
$$\therefore x = \frac{180 \times 70}{210} = 60 \text{ cm}$$

As G is the mid-point of AB

$\therefore AB = 120$ cm

You were told that the girder is modelled as a uniform rod, so $AB = 2x$ cm.

c



Draw a new diagram, showing the load at B.

$R(\uparrow)$

$$T + 3T - 210 - W = 0$$
$$\therefore 4T - W = 210 \qquad (1)$$

Resolve and take moments to give two equations in T and W.

Take moments about A.

$$3T \times 90 = W \times 120 - 210 \times 60$$

You could take moments about C to give a second equation.

Divide equation by 30.

$$\therefore 9T - 4W = 420 \qquad (2)$$

Solve the simultaneous equations.

$9 \times (1) - 4 \times (2)$ gives

$$-9W + 16W = 1890 - 1680$$
$$\text{i.e. } 7W = 210$$
$$\therefore W = 30$$

Solve the equation to find W.

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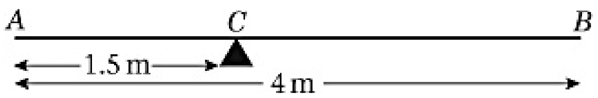
2 Review Exercise
Exercise A, Question 29

Question:

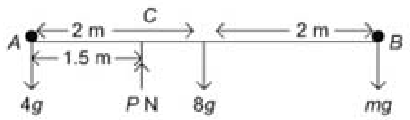


A uniform rod *AB* has mass 8 kg and length 4 m. A particle of mass 4 kg is attached to the rod at *A* and a particle of mass *M* kg is attached to the rod at *B*. The rod is supported at the point *C*, where *AC* = 1.5 *m*, and rests in equilibrium in a horizontal position, as shown in the diagram.

a Find the value of *M*.

b State how you used the information that the rod is uniform.

Solution:



Using total of anti-clockwise moments = total of clockwise moments.

a Take moments about *C*.

$$\begin{aligned} 4g\times1.5 &= 8g\times0.5+Mg\times2.5 \\ \therefore 6g-4g &= 2.5Mg \\ \therefore \frac{2g}{2.5g} &= M \\ \therefore M &= 0.8 \end{aligned}$$

Make *M* the subject of the formula.

b The weight acts through the mid-point of the rod.

For a uniform rod its weight acts through the mid-point.

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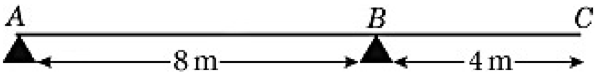
2 Review Exercise
Exercise A, Question 30

Question:



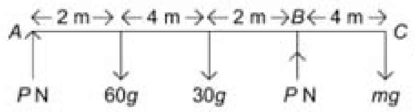
A uniform plank *ABC*, of length 12 m and mass 30 kg, is supported in a horizontal position at the points *A* and *B*, where *AB* = 8 m and *BC* = 4 m, as shown in the diagram. A woman of mass 60 kg stands on the plank at a distance of 2 m from *A*, and a rock of mass *M* kg is placed on the plank at the end *C*. The plank remains in equilibrium. The plank is modelled as a uniform rod, and the woman and the rock as particles.

Given that the forces exerted by the supports on the plank at *A* and *B* are equal in magnitude,

a find **i** the value of *M*, **ii** the magnitude of the force exerted by the support at *A* on the plank.

b State how you used the modelling assumption that the rock is a particle.

Solution:



Let the reactions at A and B be P N.

a i $R(\uparrow)$

$2P - 60g - 30g - Mg = 0$

$\therefore 2P - Mg = 90g \qquad (1)$

Take moments about A.

$P \times 8 = 60g \times 2 + 30g \times 6 + Mg \times 12$

$\therefore 8P - 12Mg = 300g$

$\div$ through by 4

Then $2P - 3Mg = 75g \qquad (2)$

Solve by subtracting equation (2) from equation (1).

As there are two unknowns, P and M, you need two equations.

You can resolve and you can take moments about A or B.

Using anticlockwise moment = clockwise moment.

Then $2Mg = 15g$

$\therefore M = \dfrac{15g}{2g}$

$= 7.5$ kg

Then solve simultaneous equations to find M.

ii Substitute into equation (1)

$\therefore 2P = 90g + 7.5g$

$\therefore P = \dfrac{97.5g}{2}$

$= 48.75g$

$\therefore$ Force has magnitude 477.75 N = 480 N (2 s.f.).

Substitute into one of the equations to find P.

A particle is a point mass and its weight acts at C.

b Assumed that the centre of mass acts at the point C.

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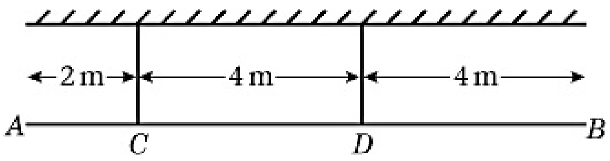
2 Review Exercise
Exercise A, Question 31

Question:



A light rod AB has length 10 m. It is suspended by two light vertical cables attached to the rod at the points C and D, where $AC = 2$ m, $CD = 4$ m, and $DB = 4$ m, as shown in the diagram. A load of weight 60 N is attached to the rod at A and a load of weight XN is attached to the rod at B. The rod is hanging in equilibrium in a horizontal position. Find, in terms of X,

a the tension in the cable at C,

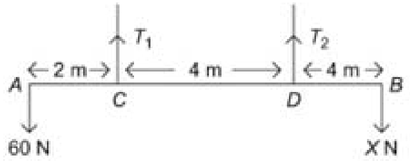
b the tension in the cable at D,

c Hence show that $15 \leq X \leq 90$.

If the tension in either cable exceeds 120 N that cable breaks.

d Find the maximum possible value of X.

Solution:



Let the tensions in the tables be T_1 and T_2 respectively.

a Take moments about *D*.

$$60 \times 6 = T_1 \times 4 + X \times 4$$
$$\therefore T_1 \times 4 = 360 - 4X$$
$$\therefore T_1 = 90 - X$$

Using anticlockwise moment = clockwise moment.

b Take moments about *C*.

$$T_2 \times 4 + 60 \times 2 = X \times 8$$
$$\therefore T_2 \times 4 = 8X - 120$$
$$\therefore T_2 = 2X - 30$$

Again using anticlockwise moment = clockwise moment.

c As T_1, T_2 are both positive

As the cables are under tension, T_1 and T_2 are both positive.

$90 - X \geq 0 \Rightarrow X \leq 90$
and $2X - 30 \geq 0 \Rightarrow X \geq 15$
so $15 \leq X \leq 90$

d T_1 cannot reach 120.

$T_1 = 90 - X$ which cannot exceed 90 (from part **a**).

If $T_2 > 120$, then

$$2X - 30 > 120$$
$$\therefore 2X > 150$$
$$\text{i.e. } X > 75$$

$T_2 = 2x - 30$, was found in part **b**.

The maximum possible value of *X* is 75.

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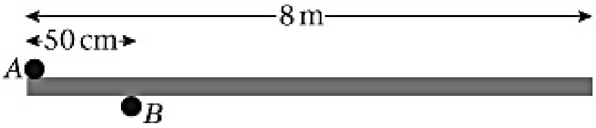
2 Review Exercise
Exercise A, Question 32

Question:



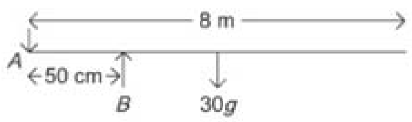
A large uniform plank of wood of length 8 m and mass 30 kg is held in equilibrium by two small steel rollers *A* and *B*, ready to be pushed into a saw-mill. The centres of the rollers are 50 cm apart. One end of the plank presses against roller *A* from underneath, and the plank rests on top of roller *B*, as shown in the diagram. The rollers are adjusted so that the plank remains horizontal and the force exerted on the plank by each roller is vertical.

a Suggest a suitable model for the plank to determine the forces exerted by the rollers.

b Find the magnitude of the force exerted on the plank by the roller at *B*.

c Find the magnitude of the force exerted on the plank by the roller at *A*.

Solution:



Let the forces at *A* and *B* be *A* N and *B* N.

a Model the plank as a uniform rod.

This implies the plank has negligible thickness.

b Take moments about *A*.

$$B \times 0.5 = 30g \times 4$$
$$\therefore B = \frac{120g}{0.5}$$
$$= 240g = 2400\,(2\text{ s.f.})$$

Using total moment anticlockwise = total moment clockwise.

c $R(\uparrow)$

$$B - A - 30g = 0$$
$$\therefore A = B - 30g$$
$$= 240g - 30g$$
$$= 210g = 2100\,(2\text{ s.f.})$$

Resolve or take moments about *B*, to find the force exerted at *A*.

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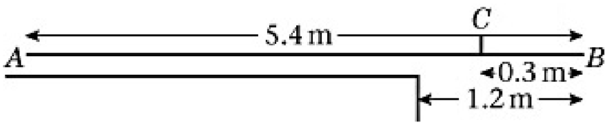
2 Review Exercise
Exercise A, Question 33

Question:



A plank of wood AB has length 5.4 m. It lies on a horizontal platform, with 1.2 m projecting over the edge, as shown in the diagram. When a girl of mass 50 kg stands at the point C on the plank, where $BC = 0.3$ m, the plank is on the point of tilting. By modelling the plank as a uniform rod and the girl as a particle,

a find the mass of the plank.

The girl places a rock on the end of the plank at A. By modelling the rock also as a particle,

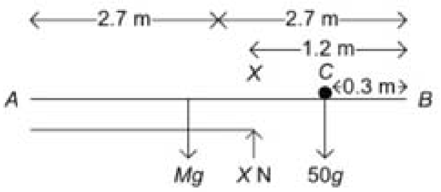
b find, to two significant figures, the smallest mass of the rock which will enable the girl to stand on the plank at B without tilting.

c State briefly how you have used the modelling assumption that

i the plank is uniform,

ii the rock is a particle.

Solution:



Let the edge of the platform be point X. Let the mass of the plank be M kg.

a When it is on the point of tilting the force of the platform on the rod acts at X (the edge of the platform).

The weight of the plank acts at its mid-point.

Take moments about X.

$Mg \times (2.7 - 1.2) = 50\,g(1.2 - 0.3)$

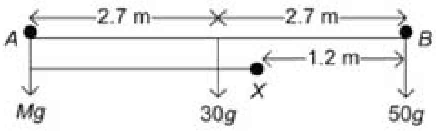
Using anti-clockwise moment = clockwise moment.

$$\begin{aligned}
\therefore Mg \times 1.5 &= 50\,g \times 0.9 \\
\therefore 1.5\,Mg &= 45\,g \\
\therefore Mg &= \frac{45\,g}{1.5} \\
\therefore M &= 30
\end{aligned}$$

Make M the subject of the formula.

$\therefore$ The mass of the plank is 30 kg.

b



This is a new situation so draw a new diagram. Let the mass of the rock be m kg.

Take moments about X again.

Using anticlockwise moment = clockwise moment.

$30\,g \times (2.7 - 1.2) + mg \times (5.4 - 1.2) = 50\,g \times 1.2$

$$\begin{aligned}
\therefore mg \times 4.2 &= 50\,g \times 1.2 - 30\,g \times 1.5 \\
\text{i.e. } 4.2\,mg &= 60\,g - 45\,g \\
&= 15\,g
\end{aligned}$$

Make m the subject and evaluate to 2 s.f.

$$\begin{aligned}
\therefore m &= \frac{15\,g}{4.2\,g} \\
&= 3.6 \text{ (2 s.f.)} \\
\therefore \text{mass} &= 3.6 \text{ kg (2 s.f.)}
\end{aligned}$$

c i Plank is uniform $\rightarrow$ weight acts through the mid-point.

ii Rock is a particle $\rightarrow$ mass of rock acts through the end-point A.

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2 Review Exercise
Exercise A, Question 34

Question:

Three forces F_1, F_2 and F_3 act on a particle.

$F_1 = (-3i + 7j)$ N , $F_2 = (i - j)$ N , $F_3 = (pi + qj)$ N

a Given that the particle is in equilibrium, determine the value of p and the value of q.

The resultant of the forces F_1 and F_2 is **R.**

b Calculate, in N, the magnitude of **R.**

c Calculate to the nearest degree, the angle between the line of action of **R** and the vector **j**.

Solution:

a If the particle is in equilibrium then $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

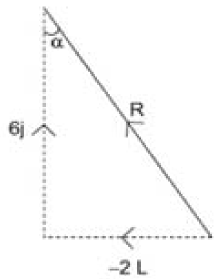
Use the condition for equilibrium.

$$\therefore -3\mathbf{i} + 7\mathbf{j} + \mathbf{i} - \mathbf{j} + p\mathbf{i} + q\mathbf{j} = 0$$
$$\therefore (p-2)\mathbf{i} + (q+6)\mathbf{j} = 0$$

Collect **i** terms and **j** terms.

$\therefore p = 2$ and $q = -6$

b The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is



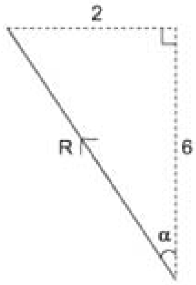
$$\begin{aligned} \mathbf{R} &= \mathbf{F}_1 + \mathbf{F}_2 \\ \therefore \mathbf{R} &= -3\mathbf{i} + 7\mathbf{j} + \mathbf{i} - \mathbf{j} \\ &= -2\mathbf{i} + 6\mathbf{j} \end{aligned}$$

$\therefore$ The magnitude of **R** is $\sqrt{(-2)^2 + 6^2}$

$$\begin{aligned} &= \sqrt{4+36} \\ &= \sqrt{40} \\ &= 2\sqrt{10} \text{ or } 6.32 \text{ (3 s.f.)} \end{aligned}$$

Use Pythagoras' Theorem to find the magnitude of **R**.

c



Let the angle between **R** and **j** be α.
Then

$$\begin{aligned} \tan\alpha &= \frac{2}{6} \\ \therefore \alpha &= \arctan\left(\frac{1}{3}\right) \\ &= 18^\circ \text{ (nearest degree)} \end{aligned}$$

Use trigometry to find the required angle.

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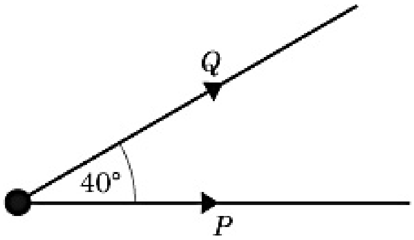
2 Review Exercise
Exercise A, Question 35

Question:



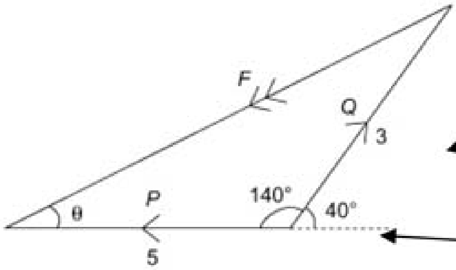
Two forces **P** and **Q**, act on a particle. The force **P** has magnitude 5 N and the force **Q** has magnitude 3 N. The angle between the directions of **P** and **Q** is 40°, as shown in the diagram. The resultant of **P** and **Q** is **F**.

a Find, to three significant figures, the magnitude of **F**.

b Find, in degrees to one decimal place, the angle between the directions of **F** and **P.**

Solution:

a Draw a vector triangle:-



Draw a triangle to illustrate $\mathbf{P}+\mathbf{Q}=\mathbf{F}$.

The angle in the triangle $=180^\circ-40^\circ$
$=140^\circ$

Use the cosine rule:

$$|\mathbf{F}|^2 = 5^2+3^2-2\times5\times3\times\cos140^\circ$$
$$\therefore|\mathbf{F}| = 7.55 \text{ (3 s.f.)}$$

b Use the sine rule:

$$\frac{\sin\theta}{3} = \frac{\sin140^\circ}{|\mathbf{F}|}$$
$$\therefore\sin\theta = 3\times\frac{\sin140^\circ}{7.55}$$
$$= 0.255$$
$$\therefore\theta = 14.8^\circ \text{ (1 d.p.)}$$

Rearrange to make $\sin\theta$ the subject of the formula.

$\theta=\arcsin0.255$

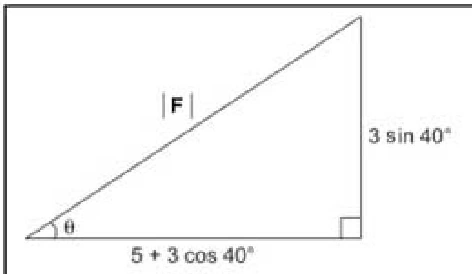
Alternative method

a $\mathbf{P}=\begin{pmatrix}5\\0\end{pmatrix}$ and $\theta=\begin{pmatrix}3\cos40^\circ\\3\sin40^\circ\end{pmatrix}$

Express the forces in component form.

$$\therefore\mathbf{P}+\mathbf{Q} = \begin{pmatrix}5+3\cos40^\circ\\3\sin40^\circ\end{pmatrix}$$
$$\therefore|\mathbf{F}|^2 = (5+3\cos40^\circ)^2+(3\sin40^\circ)^2$$
$$\therefore|\mathbf{F}| = 7.55 \text{ (3 s.f.)}$$

Use $\mathbf{F}=\mathbf{P}+\mathbf{Q}$ to give the components of $\mathbf{F}$.



Use Pythagoras' Theorem to find $|\mathbf{F}|$

b

$$\tan\theta = \frac{3\sin40^\circ}{5+3\cos40^\circ}$$
$$\therefore\theta = 14.8^\circ \text{ (1 d.p.)}$$

Use trigonometry of the triangle to find θ.

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2 Review Exercise
Exercise A, Question 36

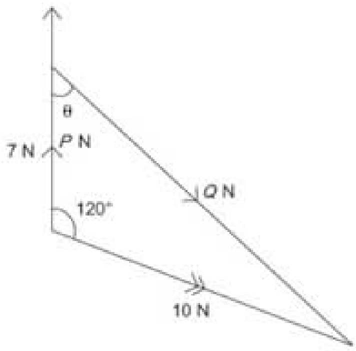
Question:

Two forces **P** and **Q** act on a particle. The force **P** has magnitude 7 N and acts due north. The resultant of **P** and **Q** is a force of magnitude 10 N acting in a direction with bearing 120°. Find

a the magnitude of **Q**,

b the direction of **Q**, giving your answer as a bearing.

Solution:



Draw a triangle to illustrate **P** and **Q** and their resultant.

a Use the cosine rule to find **Q**

$$\begin{aligned}\mathbf{Q}^2 &= 7^2 + 10^2 - 2\times 7\times 10\times \cos 120^\circ \\ &= 219 \\ \therefore \mathbf{Q} &= 14.8\end{aligned}$$

Find **Q** by using the cosine rule.

b

$$\begin{aligned}\frac{\sin\theta}{10} &= \frac{\sin 120^\circ}{\mathrm{Q}} \\ \therefore \sin\theta &= 10\times\frac{\sin 120^\circ}{14.8} \\ &= 0.585 \\ \therefore \theta &= 35.8^\circ\end{aligned}$$

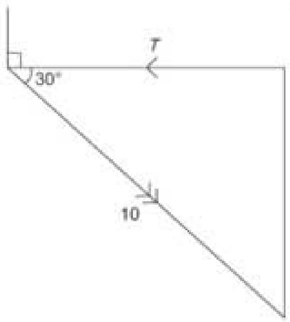
Find θ by using the sine rule, then calculate $180-\theta$, the

$\therefore$ The direction of **Q** is $180^\circ - 35.8^\circ = 144.2^\circ$.

Alternative method

a Resultant $\mathbf{P}+\mathbf{Q} = \begin{pmatrix}10\cos 30^\circ \\ -10\sin 30^\circ\end{pmatrix}$

Express the forces in component form.



Express the resultant force and the force **P** in component form.

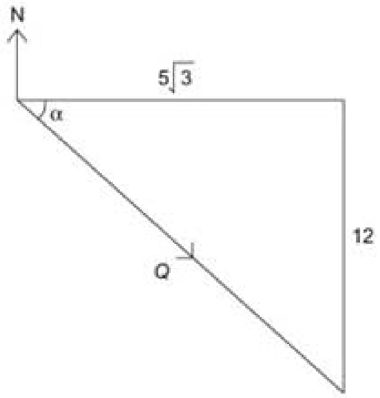
Also $\mathbf{P} = \begin{pmatrix}0\\7\end{pmatrix}$

$\therefore \mathbf{Q} = \begin{pmatrix}10\cos 30^\circ \\ -10\sin 30^\circ\end{pmatrix} - \begin{pmatrix}0\\7\end{pmatrix} = \begin{pmatrix}5\sqrt{3}\\-12\end{pmatrix}$

Find **Q** in resultant form.

Magnitude $= \sqrt{(5\sqrt{3})^2 + 12^2} = 14.8$ (3 s.f.)

b



Use a sketch to find the required bearing.

In the diagram $\tan\alpha = \dfrac{12}{5\sqrt{3}}$

$\therefore \alpha = 54.2^\circ$

$$\begin{aligned}\therefore \text{bearing} &= 90^\circ + 54.2^\circ \\ &= 144^\circ (3\text{ s.f.})\end{aligned}$$

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2 Review Exercise
Exercise A, Question 37

Question:

Two forces $F_1 = (2i + 3j)$ N and $F_2 = (\lambda i + \mu j)$ N , where λ and μ are scalars, act on a particle. The resultant of the two forces is **R**, where **R** is parallel to the vector $i + 2j$.

a Find, to the nearest degree, the acute angle between the line of action of **R** and the vector **i**.

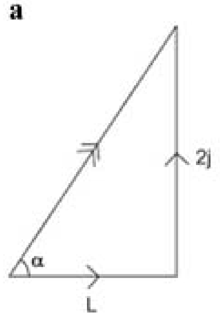
b Show that $2\lambda - \mu + 1 = 0$.

Given that the direction of F_2 is parallel to **j**,

c find, to three significant figures, the magnitude of **R**.

Solution:

a



Draw a diagram to show the vector $\mathbf{i}+2j$.

Let the required angle be α.
Then $\tan\alpha = 2$
$\therefore \alpha = 63.4^\circ$

As $\mathbf{R}$ is parallel to $\mathbf{i}+2\mathbf{j}$, $\mathbf{R}$ also makes an angle α with $\mathbf{i}$.

b As $\mathbf{F}_1+\mathbf{F}_2=\mathbf{R}$
$(2\mathbf{i}+3\mathbf{j})+(\lambda\mathbf{i}+\mu\mathbf{j}) = k(\mathbf{i}+2\mathbf{j})$

Let $\mathbf{R}=k(\mathbf{i}+2\mathbf{j})$ where k is constant.

where k is a constant.
$\therefore 2+\lambda = k$ and $3+\mu = 2k$ *

Put $\mathbf{i}$ components equal.

Put $\mathbf{j}$ components equal.

Eliminate k from these two equations.
Then

$$2(2+\lambda) = 3+\mu$$
$$\therefore 2\lambda-\mu+1 = 0 \quad *$$

c If $\mathbf{F}_2$ is parallel to $\mathbf{j}$ then $\lambda=0$.
Substituting $\lambda=0$ into * gives

$\mu=1$ and $k=2$

Substitute $k=2$ into $\mathbf{R}=k(\mathbf{i}+2\mathbf{j})$

$$\therefore \mathbf{R} = 2\mathbf{i}+4\mathbf{j}$$
$$\therefore |\mathbf{R}| = \sqrt{2^2+4^2}$$
$$= \sqrt{20}$$
$$= 4.47 \text{ (3 s.f.)}$$

The magnitude of $x\mathbf{i}+y\mathbf{j}=\sqrt{x^2+y^2}$ from Pythagoras Theorem.

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2 Review Exercise
Exercise A, Question 38

Question:

A force **R** acts on a particle, where $R = (7i + 16j)$ N .

Calculate

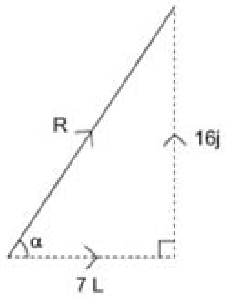
a the magnitude of **R**, giving your answers to one decimal place,

b the angle between the line of action of **R** and **i**, giving your answer to the nearest degree.

The force **R** is the resultant of two forces **P** and **Q**. The line of action of P is parallel to the vector $(i + 4j)$ and the line of action of **Q** is parallel to the vector $(i + j)$.

c Determine the forces **P** and **Q** expressing each in terms of **i** and **j**.

Solution:



Use Pythagoras to find the magnitude.

a The magnitude of

$$\mathbf{R} = \sqrt{7^2 + 16^2} = 17.5 \text{ (1 d.p.)}$$

b

$$\tan\alpha = \frac{16}{7}$$

$$\therefore \alpha = \arctan\left(\frac{16}{7}\right) = 66^\circ \text{ (nearest degree)}$$

Use trigonometry to find the angle and give it to the nearest degree.

c Let

$$\mathbf{P} = \lambda(\mathbf{i} + 4\mathbf{j}) \text{ and}$$
$$\mathbf{Q} = \mu(\mathbf{i} + \mathbf{j})$$

Express **P** and **Q** as multiples of the vectors $\mathbf{i} + 4\mathbf{j}$ and $\mathbf{i} + \mathbf{j}$ respectively.

As $\mathbf{P} + \mathbf{Q} = \mathbf{R}$

$\therefore \lambda(\mathbf{i} + 4\mathbf{j}) + \mu(\mathbf{i} + \mathbf{j}) = (7\mathbf{i} + 16\mathbf{j})$

$\mathbf{P} + \mathbf{Q} = \mathbf{R}$ is the vector addition law.

Equating **i** components

$$\lambda + \mu = 7 \qquad (1)$$

Equating **j** components

$$4\lambda + \mu = 16 \qquad (2)$$

Subtract $(2) - (1)$

$$3\lambda = 9$$
$$\therefore \lambda = 3$$

Solve the simultaneous equations to find λ and μ.

Substitute into equation (1)

$$\therefore 3 + \mu = 7$$
$$\therefore \mu = 4$$

$$\therefore \mathbf{P} = 3(\mathbf{i} + 4\mathbf{j}) = 3\mathbf{i} + 12\mathbf{j} \text{ and}$$
$$\mathbf{Q} = 4(\mathbf{i} + \mathbf{j}) = 4\mathbf{i} + 4\mathbf{j}$$

Substitute λ and μ to find **P** and **Q** in terms of **i** and **j**.

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2 Review Exercise
Exercise A, Question 39

Question:

A particle P moves in a straight line with constant velocity. Initially P is at the point A with position vector $(2i - j)$ m relative to a fixed origin O, and 2s later it is at the point B with position vector $(6i + j)$ m.

a Find the velocity of P.

b Find, in degrees to one decimal place, the size of the angle between the direction of motion of P and the vector **i**.

Three second after it passes B the particle P reaches the point C.

c Find, in metres to one decimal place, the distance OC.

Solution:

a

$$\begin{aligned}\text{velocity} &= \frac{\text{difference in displacement}}{\text{time}}\\ &= \frac{(6\mathbf{i}+\mathbf{j})-(2\mathbf{i}-\mathbf{j})}{2}\\ &= \frac{4\mathbf{i}+2\mathbf{j}}{2}\\ &= 2\mathbf{i}+\mathbf{j}\end{aligned}$$

Use the definition for constant velocity.

b

Let the angle between v and $\mathbf{i}$ be α.

This angle gives the direction of motion of P which is asked for in the question.

Then $\tan\alpha = \frac{1}{2}$

$$\begin{aligned}\therefore \alpha &= \arctan\left(\frac{1}{2}\right)\\ &= 26.6^\circ \text{ (1 d.p.)}\end{aligned}$$

c

$$\begin{aligned}\text{Displacement} &= \text{velocity} \times \text{time}\\ \text{In 3 s displacement} &= 3(2\mathbf{i}+\mathbf{j})\\ \therefore \overrightarrow{BC} &= 6\mathbf{i}+3\mathbf{j}\end{aligned}$$

But

$$\begin{aligned}\overrightarrow{OB} &= 6\mathbf{i}+\mathbf{j}\\ \therefore \overrightarrow{OC} &= \overrightarrow{OB}+\overrightarrow{BC}\\ &= 12\mathbf{i}+4\mathbf{j}\end{aligned}$$

Use the vector law of addition to find $\overrightarrow{OC}$.

The distance

$$\begin{aligned}OC &= \sqrt{12^2+4^2}\\ &= \sqrt{144+16}\\ &= \sqrt{160}\\ &= 12.6 \text{ m (1 d.p.)}\end{aligned}$$

Use Pythagoras' Theorem to find the distance OC.

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2 Review Exercise
Exercise A, Question 40

Question:

A particle P is moving with constant velocity $(5\mathbf{i} - 3\mathbf{j})$ m s^{-1}. At time $t = 0$, its position vector, with respect to a fixed origin O, is $(-2\mathbf{i} + \mathbf{j})$ m. Find, to three significant figures.

a the speed of P,

b the distance of P from O when $t = 2s$.

Solution:

a The velocity $u = 5\mathbf{i} - 3\mathbf{j}$
The speed

$$|u| = \sqrt{5^2 + (-3)^2}$$
$$|u| = \sqrt{34}$$
$$= 5.83 \text{ M s}^{-1}$$

Use the formula for magnitude of a vector.

b The displacement from $t = 0$ to $t = 2$ is $(5\mathbf{i} - 3\mathbf{j}) \times 2$

Find the displacement between $t = 0$ and $t = 2$, using velocity $\times$ time.

The position vector of P at $t = 2$ is $(-2\mathbf{i} + \mathbf{j}) + (10\mathbf{i} - 6\mathbf{j})$

Use the vector law of addition.

$= 8\mathbf{i} - 5\mathbf{j}$
$\therefore$ The distance of P from O is

$$\sqrt{8^2 + (-5)^2}$$
$$= \sqrt{64 + 25}$$
$$= \sqrt{89}$$
$$= 9.43$$

Use the formula for the magnitude of a vector.

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2 Review Exercise
Exercise A, Question 41

Question:

A boat *B* is moving with constant velocity. At noon, *B* is at the point with position vector $(3i - 4j)$ km with respect to a fixed origin *O*. At 1430 on the same day, *B* is at the point with position vector $(8i + 11j)$ km.

a Find the velocity of *B*, giving your answer in the form $pi + qj$.

At time *t* hours after noon, the position vector of *B* is **b** km.

b Find, in terms of *t*, an expression for **b**.

Another boat *C* is also moving with constant velocity. The position vector of *C*, **c** km, at time *t* hours after noon, is given by

$c = (-9i + 20j) + t(6i + \ - j)$, where λ is a constant.

Given that *C* intercepts *B*,

c find the value of λ,

d show that, before *C* intercepts *B*, the boats are moving with the same speed.

Solution:

a Displacement between noon and 14.30 is

$$(8\mathbf{i}+11\mathbf{j}) \quad - \quad (3\mathbf{i}-4\mathbf{j}) = (5\mathbf{i}+15\mathbf{j})$$

$$\therefore \text{velocity} \quad = \quad (5\mathbf{i}+15\mathbf{j}) \div 2\frac{1}{2} \text{ km/h}$$

$$= \quad (2\mathbf{i}+6\mathbf{j}) \text{ km/h.}$$

Use velocity $= \dfrac{\text{displacement}}{\text{time}}$

b The position vector t hours after noon is

$\mathbf{b} = (3\mathbf{i}-4\mathbf{j})+(2\mathbf{i}+6\mathbf{j})t$ for boat B.

Use vector law of addition.

c For boat C

$\mathbf{c} = (-9\mathbf{i}+20\mathbf{j})+t(6\mathbf{i}+\lambda\mathbf{j})$

When C intercepts B

$$\begin{pmatrix} 3+2t \\ -4+6t \end{pmatrix} = \begin{pmatrix} -9+6t \\ 20+\lambda t \end{pmatrix}$$

Equate the position vectors of B and of C.

$$\therefore 3+2t = -9+6t$$
$$\Rightarrow 12 = 4t$$
$$\therefore t = 3$$

Equate the **i** components and solve to give the value for t.

Also

$$-4+6t = 20+\lambda t \text{ at } t=3$$
$$\therefore -4+18 = 20+3\lambda$$
$$\therefore -6 = 3\lambda$$
$$\therefore \lambda = -2$$

Equate the **j** components and use your value for t to give λ.

d

The velocity of boat B $= (2\mathbf{i}+6\mathbf{j})$ kmh^{-1}

The velocity of boat C $= (6\mathbf{i}-2\mathbf{j})$ kmh^{-1}

The magnitude of the velocities are $\sqrt{2^2+6^2}$ and $\sqrt{6^2+(-2)^2}$ which are equal.

speed of boat B = speed of boat C

$$= \sqrt{36+4} = \sqrt{40} \text{ kmh}^{-1}$$

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2 Review Exercise
Exercise A, Question 42

Question:

A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. When $t = 0$, P has velocity $(3i + 2j)$ m s^{-1} and at time $t = 4$s, P has velocity $(15i - 4j)$ m s^{-1}. Find

a the acceleration of P in terms of **i** and **j**,

b the magnitude of **F**,

c the velocity of P at time $t = 6$ s.

Solution:

a As the force is constant, the acceleration is constant.
Given $u = 3\mathbf{i} + 2\mathbf{j}, v = 15\mathbf{i} - 4\mathbf{j}$ and $t = 4$, to find a use

You may use constant acceleration formulae.

$$v = u + at \text{ or } v - u = at$$
$$\therefore 15\mathbf{i} - 4\mathbf{j} - (3\mathbf{i} + 2\mathbf{j}) = 4a$$
$$\text{i.e. } 12\mathbf{i} - 6\mathbf{j} = 4a$$
$$\therefore a = 3\mathbf{i} - 1.5\mathbf{j}$$

Choose this as it connects u, v, a and t.

b Use

$$\mathbf{F} = ma$$
$$\mathbf{F} = 2(3\mathbf{i} - 1.5\mathbf{j})$$
$$= 6\mathbf{i} - 3\mathbf{j}$$

Use Newton's Second Law.

$$\therefore |\mathbf{F}| = \sqrt{6^2 + (-3)^2}$$
$$= \sqrt{45}$$
$$= 6.71$$

Use the formula for magnitude of a vector.

c Use

$$v = u + at \text{ with } t = 6$$
$$\therefore v = 3\mathbf{i} + 2\mathbf{j} + 6(3\mathbf{i} - 1.5\mathbf{j})$$
$$= 21\mathbf{i} - 7\mathbf{j}$$

Use the value for a found in part **a**.

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2 Review Exercise
Exercise A, Question 43

Question:

*The horizontal unit vectors **i** and **j** are due east and due north respectively.*

A ship S is moving with constant velocity $(-2.5\mathbf{i}+6\mathbf{j})\text{km h}^{-1}$. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i}+5\mathbf{j})\text{km}$. Find

a the speed of S,

b the bearing on which S is moving.

The ship is heading directly towards a submerged rock R. A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

c Find the position vector of R.

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of 5 km h^{-1}. Assuming that S continues to move with this new constant vector, find

d an expression for the position vector of the ship t hours after 1400.

e The time when S will be due east of R.

f The distance of S from R at the time 1500.

Solution:

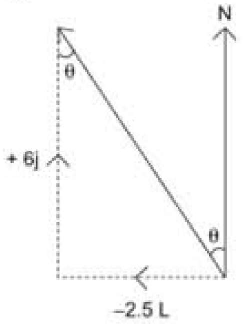
a $v = -2.5\mathbf{i} + 6\mathbf{j}$

$$\therefore \text{speed} = |v| = \sqrt{(-2.5)^2 + 6^2} = \sqrt{42.25} = 6.5 \text{ kmh}^{-1}$$

You know that speed is the magnitude of velocity.

Use the formula for magnitude of a vector.

b



The velocity vector indicates the direction of motion, so draw a diagram showing this vector.

$$\tan\theta = \frac{2.5}{6}$$
$$\therefore \theta = 22.6^\circ$$

Use trigonometry to find θ.

The bearing on which S moves is 337°.

Use geometry $360^\circ - 22.6^\circ$ to find the bearing (measured clockwise from North).

c At 15.00 the ship has moved for 3 hours since noon.

$$\text{Its displacement} = (-2.5\mathbf{i} + 6\mathbf{j}) \times 3 = -7.5\mathbf{i} + 18\mathbf{j}$$

Use $\text{velocity} \times \text{time} = \text{displacement}$.

It started at $16\mathbf{i} + 5\mathbf{j}$ so finishes at $8.5\mathbf{i} + 23\mathbf{j}$.

Use vector addition $16\mathbf{i} + 5\mathbf{j} - 7.5\mathbf{i} + 18\mathbf{j}$

This is position vector of R.

d At 14.00 the ship has moved for 2 hours since noon to $(-2.5\mathbf{i} + 6\mathbf{j}) \times 2$

$= -5\mathbf{i} + 12\mathbf{j}$ from noon position.

Use $\text{velocity} \times \text{time}$.

$\therefore$ At 14.00 ship is at $16\mathbf{i} + 5\mathbf{j} - 5\mathbf{i} + 12\mathbf{j}$

$= 11\mathbf{i} + 17\mathbf{j}$

Use vector addition.

It then moves North at 5 kmh^{-1} for t hours.

Its displacement is $5t\mathbf{j}$

$\therefore$ t hours after 14.00 ship is at

$11\mathbf{i}+(17+5t)\mathbf{j}$

Use vector addition.

e Ship is due east of R when

Equate **j** components and find t.

$$23\mathbf{j} = (17+5t)\mathbf{j}$$
$$\text{i.e. } t = \frac{6}{5}\text{ hrs}$$
$$= 1\text{ hr } 12\text{ min}$$

The time at which ship is due east of R is 15.12

Use $t=\frac{6}{5}$ hrs after 14.00 to give the time.

f At 16.00 the ship is at

$11\mathbf{i}+(17+10)\mathbf{j}=11\mathbf{i}+27\mathbf{j}$

Substitute t=2 into the position vector for S.

The distance of S from R is

$$\sqrt{\left((11-8.5)^2+(27-23)^2\right)}$$
$$= \sqrt{\left((2.5)^2+4^2\right)}$$
$$= \sqrt{22.25}$$
$$= 4.72\text{ km (3 s.f.)}$$

Use the formula, based on Pythagoras' Theorem, for the distance between two points.

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2 Review Exercise
Exercise A, Question 44

Question:

The horizontal unit vectors **i** *and* **j** *are due east and due north respectively.*

A model boat *A* moves on a lake with constant velocity $(-\mathbf{i}+6\mathbf{j})\,\mathrm{m\,s^{-1}}$. At time $t=0$, *A* is at the point with position vector $(2\mathbf{i}-10\mathbf{j})\mathrm{m}$. Find

a the speed of *A*,

b the direction in which *A* is moving, giving your answer as a bearing.

At time $t=0$, a second boat *B* is at the point with position vector $(-26\mathbf{i}+4\mathbf{j})\mathrm{m}$.

Given that the velocity of *B* is $(3\mathbf{i}+4\mathbf{j})\,\mathrm{m\,s^{-1}}$,

c show that *A* and *B* will collide at a point *P* and find the position vector of *P*.

Given instead that *B* has speed $8\,\mathrm{m\,s^{-1}}$ and moves in the direction of the vector $(3\mathbf{i}+4\mathbf{j})$.

d find the distance of *B* from *P* when $t=7s$.

Solution:

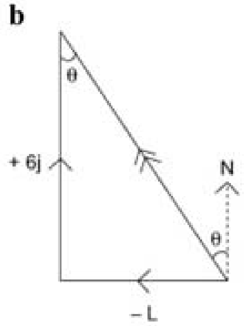
a

Velocity of A is $-\mathbf{i}+6\mathbf{j}$

$\therefore$ Speed of A $= \sqrt{(-1)^2+6^2} = \sqrt{37}\text{ m s}^{-1}$
$= 6.08\text{ m s}^{-1}$ (3 s.f.)

Use the definition that speed is the magnitude of velocity and use the formula for magnitude of a vector.

b



Draw a diagram showing the velocity vector.

From the triangle:

$$\tan\theta = \frac{1}{6}$$
$$\therefore \theta = 9.46^\circ$$

Use trigonometry to find θ, then calculate the bearing (angle measured clockwise from North).

$\therefore$ Bearing $= 360^\circ - 9.46^\circ$
$= 351^\circ$ (3 s.f.)

c At time t:
position vector of A is $\mathbf{r} = 2\mathbf{i} - 10\mathbf{j} + t(-\mathbf{i}+6\mathbf{j})$
position vector of B is $\mathbf{r} = -26\mathbf{i} + 4\mathbf{j} + t(3\mathbf{i}+4\mathbf{j})$

Use displacement = velocity × time.

If A and B collide for some value of t,

$$(2-t)\mathbf{i} + (-10+6t)\mathbf{j} = (-26+3t)\mathbf{i} + (4+4t)\mathbf{j}$$
Equate **i** and **j** components.

Equate components of position vectors and solve equations to find t.

$2 - t = -26 + 3t$ and $-10 + 6t = 4 + 4t$

In both cases $t = 7$

$\therefore$ They do collide at

$2\mathbf{i} - 10\mathbf{j} + 7(-\mathbf{i} + 6\mathbf{j}) = -5\mathbf{i} + 32\mathbf{j}$

Check that position vectors of A and of B are the same when $t = 7$.

The position vector of P is $-5\mathbf{i} + 32\mathbf{j}$.

d The new velocity of B is $\frac{8}{5}(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}

The new velocity is $\frac{8}{5}$ the previous velocity as the previous speed was 5 m s^{-1}(3, 4, 5Δ).

Position vector of B at $t = 7$ is

$$-26\mathbf{i} \quad + \quad 4\mathbf{j} + \frac{8}{5}(3\mathbf{i} + 4\mathbf{j}) \times 7 = 7.6\mathbf{i} + 48.8\mathbf{j}$$

From the vector triangle law $\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$.

$$\therefore \overrightarrow{PB} \quad = \quad \mathbf{b} - \mathbf{p} = 12.6\mathbf{i} + 16.8\mathbf{j}$$

$$|\overrightarrow{PB}| \quad = \quad \text{distance} = \sqrt{12.6^2 + 16.8^2} = 21 \text{ m.}$$

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2 Review Exercise
Exercise A, Question 45

Question:

*The horizontal unit vectors **i** and **j** are due east and due north respectively.*

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})\ \mathrm{m\,s^{-1}}$. Find

a the speed of the ball,

b the position vector of the ball after t seconds.

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m .

c Find the time when the ball is due north of B.

At time $t = 0$, another player starts running due north from B and moves with constant speed $v\ \mathrm{m\,s^{-1}}$. Given that he intercepts the ball,

d find the value of v.

e State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

Solution:

a

$$\text{velocity} = 5\mathbf{i}+8\mathbf{j}\text{ m s}^{-1}$$
$$\therefore \text{speed} = \sqrt{5^2+8^2}\text{ m s}^{-1}$$
$$= \sqrt{89}\text{ m s}^{-1} = 9.43\text{ m s}^{-1}\text{ (3 s.f.)}$$

Use speed = magnitude of velocity. Use formula for magnitude of vector.

b After t seconds, position vector is

$$\mathbf{r} = 2\mathbf{i}+\mathbf{j}+t(5\mathbf{i}+8\mathbf{j})$$

Use displacement = velocity × time.

c When the ball is due north of $10\mathbf{i}+7\mathbf{j}$

$$2+5t = 10$$
$$\therefore 5t = 8 \Rightarrow t = 1.6\text{ s}$$

Equate the **i** component of the ball to 10.

d At $t = 1.6$ ball is at $10\mathbf{i}+13.8\mathbf{j}$
The second player moves from
$10\mathbf{i}+7\mathbf{j}$ to $10\mathbf{i}+13.8\mathbf{j}$ in 1.6 s.
His velocity is $6.8\mathbf{j} \div 1.6$
His speed is $6.8 \div 1.6 = 4.25\text{ m s}^{-1}$.

Find the displacement of the second player. Use velocity = displacement ÷ time.

e Friction on the field – so velocity of ball not constant
or
vertical component of ball's motion
or
time for player to accelerate.

Any of these answers would be valid.

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2 Review Exercise
Exercise A, Question 46

Question:

A destroyer is moving due west at a constant speed of 10 km h^{-1}. It has radar on board which, at time $t = 0$, identifies a cruiser, 50 km due west and moving due north with a constant speed of 20 km h^{-1}. The unit vectors **i** and **j** are directed due east and north respectively, and the origin O is taken to be the initial position of the destroyer. Each vessel maintains its constant velocity.

a Write down the velocity of each vessel in vector form.

b Find the position vector of each vessel at time t hours.

c Show that the distance d km between the vessels at time t hours is given by

$d^2 = 500t^2 - 1000t + 2500$

The radar on the cruiser detects vessels only up to a distance of 40 km. By finding the minimum value of d^2, or otherwise,

d determine whether the destroyer will be detected by the cruiser's radar.

Solution:

a

$$\mathbf{v}_{\text{Destroyer}} = -10\mathbf{i}\ \text{kmh}^{-1}$$
$$\mathbf{v}_{\text{cruiser}} = 20\mathbf{j}\ \text{kmh}^{-1}$$

As it moves *west* its direction is parallel to $-\mathbf{i}$.

The cruiser is moving due North so is a positive multiple of $\mathbf{j}$.

b

$$\mathbf{r}_{\text{destroyer}} = -10\,t\,\mathbf{i} = \mathbf{d}$$
$$\mathbf{r}_{\text{cruiser}} = -50\mathbf{i} + 20\,t\,\mathbf{j} = \mathbf{c}$$

Using $\mathbf{r}_{\text{new}} = \mathbf{r}_{\text{old}} + \mathbf{v}t$

c The vector $\overrightarrow{CD} = \mathbf{d} - \mathbf{c}$

$$= -10\,t\mathbf{i} - (-50\mathbf{i} + 20\,t\mathbf{j})$$
$$= (50 - 10\,t)\mathbf{i} - 20\,t\mathbf{j}$$
$$\therefore |\overrightarrow{CD}| = \sqrt{(50-10\,t)^2 + (-20\,t)^2}$$
$$= \sqrt{2500 - 1000\,t + 100\,t^2 + 400\,t^2}$$
$$\therefore d^2 = 500\,t^2 - 1000\,t + 2500$$

Using the triangle law, or vector subtraction.

Use Pythagoras' Theorem to find the magnitude of $\overrightarrow{CD}$.

d

$$d^2 = 500(t^2 - 2\,t + 5)$$
$$= 500((t-1)^2 + 4)$$

The minimum value of d^2 is when $t = 1$ and

$$d^2 = 500 \times 4$$
$$= 2000$$
$$\therefore d = \sqrt{2000}$$
$$= 44.72$$

The minimum value of d^2 can be found by completion of the square.

$\therefore$ As $44.72 > 40$ cruiser will not be able to detect the destroyer.

An alternative method would be to attempt to solve $d^2 = 40^2$.
This gives a quadratic with no real solutions.

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2 Review Exercise
Exercise A, Question 47

Question:

In this question the vectors **i** and **j** are horizontal unit vectors in the directions due east and due north respectively. Two boats A and B are moving with constant velocities. Boat A moves with velocity $9j \text{ km h}^{-1}$. Boat B moves with velocity $(3i + 5j) \text{ km h}^{-1}$.

a Find the bearings on which B is moving.

At noon A is at the point O and B is 10 km due west of O. At time t hours after noon, the position vectors of A and B relative to O are **a** km and **b** km respectively.

b Find expressions for **a** and **b** in terms of t, giving your answer in the form

$pi + qj$

c Find the time when B is due south of A.

At time time t hours after noon, the distance between A and B is d km. By finding an expression for AB,

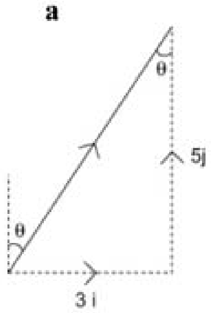
d show that $d^2 = 25t^2 - 60t + 100$.

At noon the boats are 10 km apart.

e Find the time after noon at which the boats are again 10 km apart.

Solution:

a



Let the bearing on which B is moving be α.

Then $\tan\alpha = \frac{3}{5}$

$$\therefore \alpha = \arctan(0.6) = 30.96^\circ$$

Use trigonometry to find α. The bearing is the angle measured clockwise from the North direction.

$\therefore$ bearing is 031° (nearest degree)

b

$$\mathbf{a} = 0 + 9t\,\mathbf{j}$$
$$\mathbf{b} = -10\mathbf{i} + (3\mathbf{i} + 5\mathbf{j})t = (3t - 10)\mathbf{i} + 5t\mathbf{j}$$

using $\mathbf{r}_{\text{new}} = \mathbf{r}_{\text{old}} + \mathbf{v}t$.

c When B is due south of A

$$3t - 10 = 0$$
$$\therefore t = \frac{10}{3} = 3\text{ h }20\text{ min}$$

As A has **i** component zero.

i.e. the time is 15.20

d

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = (3t - 10)\mathbf{i} - 4t\mathbf{j}$$
$$\therefore |\overrightarrow{AB}| = \sqrt{(3t-10)^2 + (-4t)^2} = \sqrt{(9t^2 - 60t + 100 + 16t^2)}$$
$$\therefore d^2 = 25t^2 - 60t + 100$$

Using the 'triangle law' or vector subtraction.

Use Pythagoras to find the magnitude of $\overrightarrow{AB}$.

e When $d = 10$

$100 = 25t^2 - 60t + 100$

Form and solve a quadratic equation in t.

$$\therefore 25t^2 - 60t = 0$$
$$\therefore 5t(5t - 12) = 0$$
$$\therefore t = 0 \text{ or } t = \frac{12}{5} = 2.4\text{ h}$$

Express your answer as a time in hours and minutes.

So the boats are 10 km apart at 14.24.